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On coastal waves and related upwellings in the Narva bay

J. Laanearu
Department of Mechanics, Tallinn Technical University, Estonia

Abstract

Large vertical displacements of the coastal waters in the Narva Bay (the Baltic Sea), observed during 2001-2002, were found dependent on the winds and topography. The Ekman drift of surface water associated with the eastern wind led to the longshore mixing in the southwestern side of the bay, but vertical motion in the vicinity of the Narva River mouth was suppressed. The quasi-geostrophic shallow water equations for the reduced gravity case are used to investigate the long-wave characteristics for internal motion. In the half-triangular cross section, wide in terms of the baroclinic Rossby deformation radius which is based on the potential depth, the internal wave trapped to the vertical sidewall moves with the corrected Kelvin-wave speed, and the wave speed at the other side of the section is based on the coastal slope. Simple mechanisms of the thermocline rise (and fall) near the Narva Bay coasts are analyzed for the observed stratification, wind and topography.

1 Introduction

Different initial conditions can yield the coastal wave solutions from the shallow-water equations of the rotating system, which set up the flows along the boundaries (cf. Gill [4]). Recently the coastal motions having the geostrophic cross-flow characteristics have been observed in the Baltic Sea. Stipa [11] investigated the stability of the coastal current along the northern shore of the Gulf of Finland. The deep-water motion through the Irbe Strait, associated with the thermohaline front between the Baltic proper and the Gulf of Riga, in the spring of 1995 had the form of coastal jet (Lilover *et al.* [9]; Laanearu *et al.* [7]). Lass and Talpsepp [8] found that the offshore scale of the boundary current in the Arkona Basin, a small sub-basin in the southern Baltic, exceeds the baroclinic Rossby radius, and having the approximate width of the coastal slope region. Large variations in the thermocline depth near the shore can lead to formation of the internal Kelvin waves (cf. Bennett [2]). In the shallow sea, such as the Baltic, nearshore topography is important and the coastal waves generated from the upwelling situations are expected to deviate in many aspects from the Kelvin wave in a basin of constant depth.

The Narva Bay is a large sub-basin of the Gulf of Finland (Fig 1), the depth of which increases more-or-less smoothly northwestward (the maximum is around 50 metres). The bay is topographically well opened to the rest of the gulf and because the wind induced upwelling and downwelling processes are frequent. The coast of the bay is a part of the sedimentary rock, the cliff of which evidently has an influence on the local winds. The bay receives the freshwater flux from the Narva River (entering the sea near Narva Jõesuu), the water of which originates mainly from the lake of Peipsi. The annual mean of the river volume flux is around 400 $m^3 s^{-1}$, but this magnitude can be significantly larger during the floods.

The present work concentrates on simple modelling of the vertical and longshore motions in a two-layer system, such as the Narva Bay. Section 2 deals with analytical framework of the "generalized Kelvin wave" problem. Section 3 reviews the stratification situations as well as the low-frequency motions observed in the bay. In the final part, the results are briefly discussed and applied to a typical upwelling.

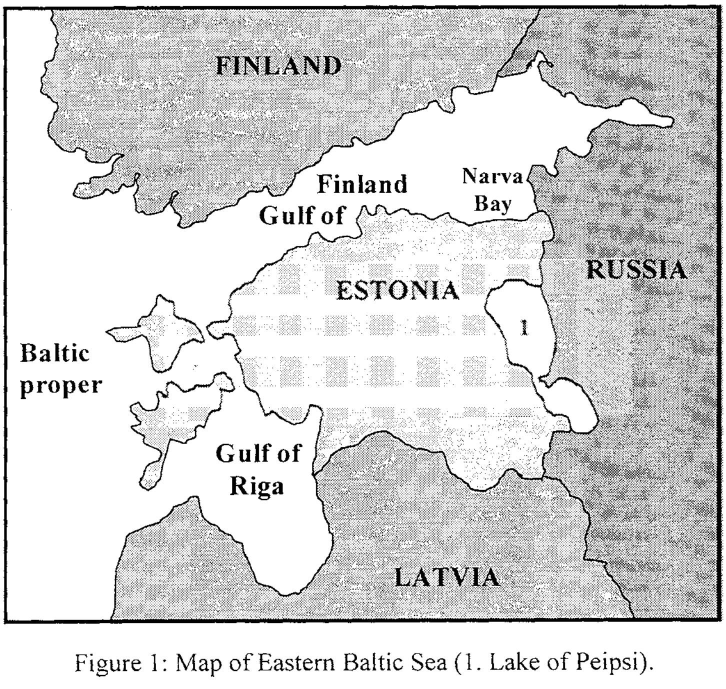

Figure 1: Map of Eastern Baltic Sea (1. Lake of Peipsi).

2 Long-wave characteristics

The shallow-water approximation is used to examine motion, the length scale of which along the coast is significantly larger than the Rossby deformation radius. For the two-layer system the gravity is replaced by the reduced gravity. The quasi-geostrophic shallow water equations are

$$fv = g'\frac{\partial\eta}{\partial x}, \tag{1}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + fu = -g'\frac{\partial\eta}{\partial y} + F^{(y)}, \tag{2}$$

$$\frac{\partial d}{\partial t} + \frac{\partial}{\partial x}(ud) + \frac{\partial}{\partial x}(vd) = 0. \tag{3}$$

Interface elevation is $\eta = d + h$, where d is deep-fluid depth and h bottom elevation (Fig 2). The along- and cross-coast velocity components are v and u, respectively. The Coriolis parameter is f and the reduced gravity is $g' = g(\delta\rho/\rho_0)$, where $\delta\rho$ is the density jump between the layers and ρ_0 is the reference density. Unspecified forcing is represented by $F^{(y)}$. The potential vorticity of the motion is given by $(f + \partial v/\partial x)/d = f/D_\infty$, where D_∞ is the potential depth (i.e. the fluid depth, for which the relative vorticity of flow can be neglected, Borenäs and Pratt [3]). This potential vorticity relationship and eqn (1) yield the following expression for the cross-current structure

$$\frac{g'}{f^2}\left(\frac{\partial^2 d}{\partial x^2} + \frac{\partial^2 h}{\partial x^2}\right) - \frac{d}{D_\infty} = -1. \tag{4}$$

In the case of the half-triangular section (Fig 2), the solution to eqn (4) is

$$d(x) = D_\infty + \frac{(D_\infty - d_0)\cosh(w/R_\infty) - D_\infty}{\sinh(w/R_\infty)} \cdot \\ \cdot \sinh(x/R_\infty) + (d_0 - D_\infty)\cosh(x/R_\infty), \tag{5}$$

where w is the flow width in "channel" and $R_\infty = \sqrt{g' D_\infty}/f$. The along-channel fluid velocity according to eqn (1) is

$$v(x) = \sqrt{\frac{g'}{D_\infty}}\frac{(D_\infty - d_0)\cosh(w/R_\infty) - D_\infty}{\sinh(w/R_\infty)} \cdot \\ \cdot \cosh\left(\frac{x}{R_\infty}\right) + \sqrt{\frac{g'}{D_\infty}}(d_0 - D_\infty)\sinh\left(\frac{x}{R_\infty}\right) + \frac{g'}{f}K, \tag{6}$$

where K is the bottom slope. The half of the sum and difference of the results evaluated by eqn (2) at the channel edges, respectively, gives the characteristic form of the problem:

$$\frac{\partial R_\pm}{\partial t} + c_\pm \frac{\partial R_\pm}{\partial y} = F^{(y)} \tanh^2\left(\frac{w}{2R_\infty}\right), \tag{7}$$

where $R_\pm$ and $c_\pm$ represent the Riemann invariants and the characteristic speeds, respectively. According to the boundary conditions, which state no cross-flow at the vertical wall and allow motion of the free-edge position on the sloping bottom, the characteristic speeds are

$$c_\pm = (\alpha_\pm - 1)\sqrt{\frac{g'}{D_\infty}\frac{d_0}{2}\coth\left(\frac{w}{2R_\infty}\right)} + \frac{g'}{f}K, \tag{8}$$

where

$$\alpha_\pm = \pm\sqrt{\frac{\frac{1}{4d_0D_\infty}(d_0 - 2D_\infty)^2 \tanh^2\left(\frac{w}{2R_\infty}\right)\left(1-\tanh^2\left(\frac{w}{2R_\infty}\right)\right)+\frac{1}{2}}{\frac{d_0}{4D_\infty}\left(\coth^2\left(\frac{w}{2R_\infty}\right)-1\right)+\frac{1}{2}}}$$

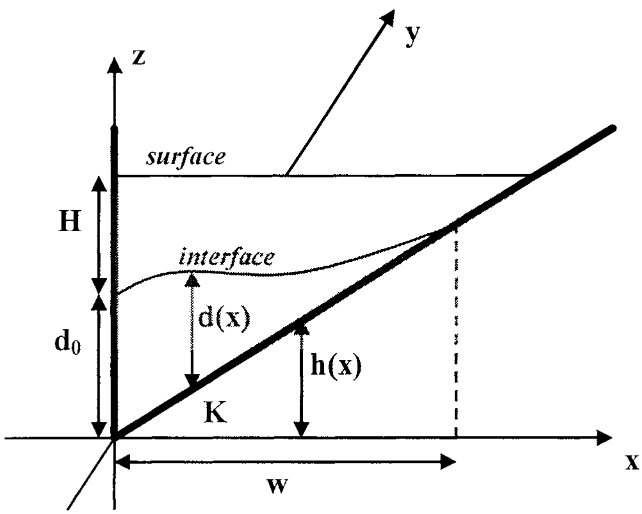

Figure 2: Sketch of cross-section and notations.

Unfortunately no explicit integral for the Riemann invariants was found. However, these characteristics can be determined implicitly from

$$R_{\pm} = f \int_{w} (1+\alpha_{\pm}) \tanh^2\left(\frac{w}{2R_{\infty}}\right) \cdot \left(\frac{d_0}{4D_{\infty}}\left(\coth^2\left(\frac{w}{2R_{\infty}}\right)-1\right)+\frac{1}{2}\right) \mathrm{d}w \tag{9a}$$

or

$$R_{\pm} = \frac{1}{2}\sqrt{\frac{g}{D_{\infty}}} \int_{d_0} (\alpha_{\pm}-1) \tanh\left(\frac{w}{2R_{\infty}}\right) \mathrm{d}d_0 . \tag{9b}$$

Equation (7) gives two non-linear waves characterized by the Riemann invariants, which are conserved following the characteristic speeds for a non-forced situation. For a cross-section much wider than the baroclinic Rossby deformation radius, the long-wave characteristics can be simplified, i.e. using proxies $\tanh(w/(2R_{\infty})) \rightarrow 1$ and $\coth(w/(2R_{\infty})) \rightarrow 1$ in eqns (8) and (9a,b), which also yields $\alpha_{\pm} = \pm 1$. The "channel wave" is thus split into the independent coastal wave, either trapped along the vertical sidewall or the sloping bottom, which propagate in the opposite direction, with the coast on the right in northern hemisphere. However, in the Narva Bay, no topographic underwater ridges exist, i.e. the bay depth increases more-or-less smoothly northwestward, therefore the characteristics of coastal waves only will be examined.

In the wide channel: $w/R_{\infty} \gg 1$, the characteristic speed of the wave at the vertical sidewall (left side in Fig 2) is

$$c_{-} = -\sqrt{\frac{g'}{D_{\infty}}d_0} + \frac{g'}{f}K . \tag{10}$$

The corresponding Riemann invariant here is derived approximately, using $\alpha_{-} = -1$ inside the integral of eqn (9b). Thus this expression considerably simplified and the integration yields

$$R_{-} \approx -\sqrt{\frac{g'}{D_{\infty}}}d_0 \tanh\left(\frac{w}{2R_{\infty}}\right) . \tag{11}$$

The left-wall wave can propagate in the negative y- direction if the first term in eqn (10) is larger from the other term including the bottom slope. For the specific case, with $K = d_0 f / \sqrt{g' D_{\infty}}$, any disturbance remains "frozen" in the flow, implying that c_{-} is zero. Note that the characteristic speed of the wave, wall on the left-side, reduces to the non-linear Kelvin-wave speed in the case of flat bottom.

The wave at the other side of the cross section describes the free edge motion on the sloping bottom and has properties quite different from those of the left-side wave. For a wide cross section the characteristic form (8) of this wave (at the right-side slope in Fig 2) yields the speed

$$c_+ = \frac{g'}{f} K . \tag{12}$$

Using $\alpha_+ = 1$ inside the integral of eqn (9a), an approximate expression for the Riemann invariant is given by

$$R_+ \approx \sqrt{\frac{g'}{D_\infty}} d_0 \tanh\left(\frac{w}{2R_\infty}\right) + wf - \\ - 2\sqrt{g'D_\infty} \tanh\left(\frac{w}{2R_\infty}\right) . \tag{13}$$

It is interesting to emphasize that the right-side wave can propagate only in the positive y- direction. However, this interpretation is not entirely correct, since eqn (12) contains a hidden advection property. In the case $K = 0$, the characteristic speed of the wave is zero, implying a possibility of the standing wave. Note that in the rectangular channel "frontal wave" occurs when the flow becomes separated from the vertical wall (cf. Pratt *et al.* [10]).

The above-presented theoretical results of the coastal waves are comparable with the Kelvin-wave characteristics in a basin of constant depth. The reason is that the potential vorticity of the motion in the half-triangular channel was initially constant, and it will stay uniform until forcing has no curl. However, the formalism introduced here is applicable for the constant potential vorticity motion through the cross section of vertical sidewalls with sloping bottom, fully triangular, convex slope, etc.

3 Observational evidence

During hydrographic observations in the western part of the Narva Bay in 2001-2002, the thermohaline and dynamical fields in the vicinity of the Narva River mouth as well as in the coastal and open-sea areas were measured. Meteorological parameters were recorded at the two coastal stations surrounding the bay.

The thermohaline characteristics of the surface water in the Narva Bay had a wide range of variability. In the open-sea area, the surface water was well mixed, the thickness of which varied between 10 and 25 metres. But in the vicinity of the Narva River mouth the surface water conformed poorly to the unstratified situation. On the basis of CTD measurements, the upper mixed layer was separated from the bay deep-water in early spring by the halocline and during summer by the thermocline, Laanearu and Lips [6]. The surface water was comparatively warm from July, because the temperature

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was commonly higher than 20 °C. The surface-water salinity in the bay was less than 5 psu, having permanent minimum close to the river mouth area. However, the observed thermohaline fields showed that the water buoyancy was related rather independently to the salinity and temperature.

An upwelling situation along the southwestern coast of the bay was observed during a cruise on 11 July 2001. The interpolated structure of the temperature at 5-metre depth in Fig 3 shows the existence of the comparatively cold water with an average around 10 °C, westward from Sillamäe. On basis of the CTD measurements the comparatively high-saline surface water along the coast of the bay also existed, with an average around 5 psu. Concerning the wind conditions two days before the observations, the moderate easterly winds (at the speed around 7 m s^{-1}) changed to a stronger southern wind on July 11.

The thermohaline conditions together with the observed currents at the buoy stations showed the upwelling situations also in the vicinity of the Narva River mouth, e.g. during 24-29 July 2001, 24-29 August 2002. Rapid temperature changes were apparent together with the winds blowing several days from the eastern sectors. The upwelled thermocline not only produces the vertical motion near the shore, but can also be a source of the internal waves (cf. Csanady [1]). However, the Ekman drift of the surface water and the large-amplitude effects of coastal waves are important since it is not uncommon that the water originating from the deep layers intersects the surface in the Gulf of Finland.

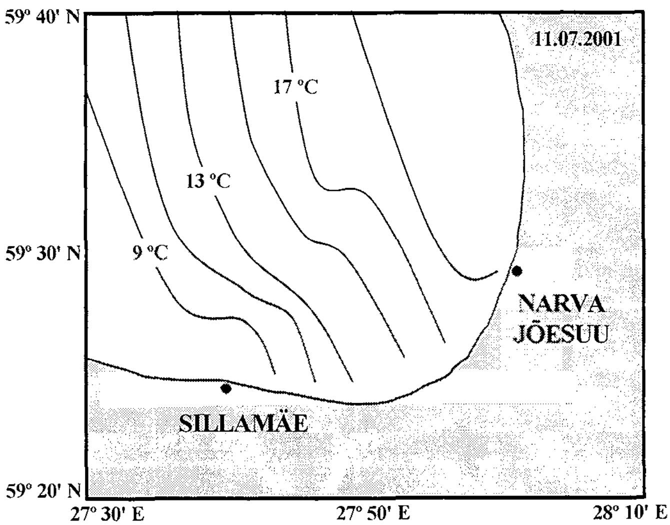

Figure 3: Temperature at 5-metre depth in the Narva Bay.

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4 Application and concluding remarks

The temperature mainly determined the density stratification in the Narva Bay during summer, *viz.* the upper mixed layer above the deep water. Such a two-layer system obeys eqns (1)-(3). In eqn (2) the forcing function $F^{(y)}$ is set equal to $\tau^{(y)}/(\rho_0 H)$, where $\tau^{(y)}$ is longshore wind stress and H mixed-layer depth. If the interface variations at the deeper side of the half-triangular section (Fig 2) are neglected, i.e. the Ekman transport occurs mainly on the sloping bottom, the problem by eqn (7) can be evaluated only for the right-side motions:

$$\frac{\partial w}{\partial t} = \frac{\tau^{(y)}}{fH\rho_0} - \frac{g'}{f} K \frac{\partial w}{\partial y}. \tag{14}$$

This differential equation describes the topographically relevant case in which the rise and fall of the thermocline is due to a longshore wind, which produces an Ekman drift of the surface water toward or away from the shore, and the coastal wave. It is interesting to note that the potential depth does not appear in the final characteristic form of the problem. Eqn (14) is very useful when investigating the vertical motion as well as longshore adjustment of the interface. For instance assuming the balance between the time-dependent term and the wind stress term, a time for the thermocline intersecting the surface due to the Ekman transport can be estimated by

$$T = \frac{f\rho_0 H^2}{K\tau^{(y)}}. \tag{15}$$

The following parameters characterized the summer upwelling in the western side of the Narva Bay: $\tau^{(y)} = 5.2\times10^{-2}$ N m^{-2}, $H = 10$ m, $\delta\rho = 2$ kg m^{-3} ($f = 1.25\times10^{-4}$ s^{-1}, $K = 3.2\times10^{-3}$). Thus, the interface would intersect the surface without adjustment along the coast within 21 hours after the wind at the speed of 7 m s^{-1} is switched on. The upwelling conditions in the vicinity of the Narva River mouth were different. The reason is that the stronger stratification would give the faster interface adjustment along the coast. However, the moderate easterly winds observed, which produced the Ekman drift of the surface water toward the coast, had duration of around two days. This period was too short to make the deep water intersecting the surface in the vicinity of the river mouth.

In eqns (1)-(3) solution for the coastal motions in the rotating basin depends on the forcing function in many aspects. In nature the wind has a wide range of space and time scales. On the basis of the observed surface currents it became evident that the strong winds of short duration grated the near-inertial oscillations in the sea. (The wind induced surface-water motion is not simple, because the momentum of flow depends on the wind strength as well as direction and on the gust duration, cf. Käse [5]). The Ekman drift observed in the Narva Bay was associated with the wind blowing several days in one direction (Laanearu and Lips [6]). The dynamics associated with the upwelling is not only related to the cross-shore bottom, but also the longshore topography is important, allowing incline the interface locally. In the case of the sloping bottom the motion has stronger cross-shore flow component underlying the difference from the Kelvin wave propagation. However, the Kelvin-type wave would propagate eastward along the coast of the Narva Bay, and non-linear effects are associated with steepening of the motion depth as well as width. In conclusion, the upwellings in the Narva Bay were not only response to the wind along the whole coast, but the longshore topography also favored the adjustment by the coastal waves.

Acknowledgements

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References

[1] Csanady, G.T., Baroclinic boundary currents and long edge-waves in basins with sloping shores. *J. Phys. Oceanogr.*, 1, pp. 92–104, 1971.

[2] Bennett, J.R., A theory of large-amplitude Kelvin waves. *J. Phys. Oceanogr.*, 3, pp. 57–60, 1973.

[3] Borenäs, K.M. & Pratt, L.J., On the use of rotating hydraulic models. *J. Phys. Oceanogr.*, 24, pp. 108–123, 1994.

[4] Gill, A.E., Adjustment under gravity in a rotating channel. *J. Fluid Mech.*, 77, pp. 603–621, 1976.

[5] Käse, R.H., Calculations of the energy transfer by the wind to near inertial waves. *Deep-Sea Res.*, 26, pp. 227-232, 1979.

[6] Laanearu, J. & Lips, U., Observed thermohaline fields and low-frequency currents in the Narva Bay. *Accepted to Proc. Estonian Acad. Sci. Eng.*, 2003.

[7] Laanearu, J., Lips, U. & Lundberg, P., On the application of the hydraulic theory to the deep-water flow through the Irbe Strait. *J. Mar. Syst.*, 25(3–4), pp. 323–332, 2000.

[8] Lass, H.U. & Talpsepp, L., Observations of coastal jets in the Southern Baltic. *Cont. Shelf. Res.*, 13, pp. 189–203, 1993.

[9] Lilover, M.J., Lips, U. & Laanearu, J. & Liljebladh, B., Flow regime in the Irbe Strait. *Aquatic Sciences*, 60 (3), pp. 253–265, 1998.

[10] Pratt, L.J., Helfrich, K.R. & Chassignet, E.P., Hydraulic adjustment to an obstacle in a rotating channel. *J. Fluid Mech.*, 404, pp. 117–149, 2000.

[11] Stipa, T., Instabilities and along-shore variability in the Finnish coastal current. *Estonian Marine Institute Report Series*, 10, pp. 62–66, 1999.