Modelling, simulation and experimental investigation of plates subjected to blast loading conditions

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Abstract

This paper deals with experimental investigation, modelling, and simulation of plates subjected to blast loading conditions. Experiments are performed on thin clamped circular aluminium plates in a shock tube. The main advantage of this experimental technique is that the front wave impinging on the structure is plane and yields a uniformly distributed pressure pulse. The non-linear shell theory of Schmidt and Reddy [1] is used as a basis of a finite element algorithm for the simulation of the transient, geometrically non-linear elastic-viscoplastic response. Numerical solutions are obtained by using the isoparametric Lagrangian 9-node shell finite element and the central difference method for the time integration of the non-linear equations of motion. Special emphasis is focused on the evolution of deflections, bending moments, membrane forces, and equivalent plastic strain rates under blast loading conditions. The obtained results allow for an explanation of the evolution of the experimentally observed final conical shape by flexural waves which originate at the boundary and travel towards the plate centre at different speeds.

Keywords: blast loading, shock tube, viscoplasticity, vibrations, plates.

1 Introduction

Experimental investigations of structures subjected to blast loading conditions are frequently performed by shock waves generated by explosives (e.g. gaseous mixtures, charges of plastic explosives) that detonate at some distance of the structure (see Idczak et al. [2, 3], Renard and Penetier [4], Penetier [5], among others). These experimental techniques lead to a complex non-uniform time-space evolution of the pressure distribution on plane structural elements thus
requiring a sophisticated modelling and simulation of the blast loading conditions. The main advantage of the shock tube technique used in this paper is that the shock wave front is plane, thus yielding a uniformly distributed pressure pulse on plates which can be measured easily by only one pressure transducer integrated in the specimen mounting ring. This allows for a precise modelling of the transient loading conditions for simulations of the experimentally observed plate response.

Comparative experimental investigations of blast loaded plates and comparative numerical simulations have been given already in our earlier papers [6, 7]. The focus of this work is on a more detailed analysis of the evolution of bending moments, membrane forces and equivalent plastic strain-rates of blast loaded plates. The results allow for an explanation of the evolution of the experimentally observed final conical shape by flexural waves which originate at the boundary and travel towards the plate centre at different speeds.

2 Experimental set up

For the experiments a shock tube is used in order to subject circular aluminium plates (diameter 553 mm, thickness 2 mm) to impulsive loadings, see Fig. 1. The high pressure chamber (HPC) and the low pressure chamber (LPC) are separated from each other by a diaphragm. The pressure in the HPC is increased until the diaphragm is ruptured. A shock wave travels through the LPC and impinges on the plate specimen clamped at the end of the tube between two ring flanges leading to a plane high-pressure pulse. The time history of the pressure acting on the plate and of the centre deflection is recorded by a piezoelectric pressure sensor and a capacitor, respectively.

![Figure 1: Shock tube.](image)

3 Structural modelling

Consider a plate or shell with volume $V$, total bounding surface $B$, midsurface $M$ and thickness $h$ subjected to blast loading conditions.
The principle of virtual displacements of 3-D elastokinetics reads

$$\int \left\{ s^{ij} \delta E_{ij} - \rho \left( B^i - A^i \right) \delta V^i \right\} d\mathcal{V} - \int \left\{ * s^{ij} + D^i \right\} \delta V^i d\mathcal{B} = 0$$

(1)

where $V_i$ denote the covariant components of the displacement vector in an arbitrary point of the body, $s^{ij}$ and $E_{ij}$ are the components of the second Piola-Kirchhoff stress and Green strain tensor, $\rho$ is the density of the undeformed body, and $B^i$, $A^i$ are components of the body force and acceleration vectors per unit volume of the undeformed body, respectively. Furthermore, $* s^{ij}$ and $D^i$ are components of the prescribed external stress vector and of the damping force vector, respectively, per unit area of the undeformed shell boundary surface $\mathcal{B}$. For viscous damping the damping force vector components may be written as $D^i = -D^{ij} \dot{V}_j$ with $D^{ij}$ denoting the components of the damping tensor. Here and throughout the paper the Einstein summation convention is used with Latin indices ranging from 1 to 3 and Greek indices ranging from 1 to 2.

The displacement vector $\mathbf{V}$ of any point in the shell space can be referred to the contravariant base vector triad of the reference surface as

$$\mathbf{V} = v_\alpha a^\alpha + v_3 n.$$  

(2)

Refined nonlinear first-order shear deformation shell theories (FOSD), which account for small strains (of $O(\eta)$, $\eta \ll 1$), small rotations (of $O(\eta)$) about the normal, and moderate rotations (of $O(\eta^{1/2})$) of the normal, have been derived by Schmidt and Reddy [1]. They are based on the hypothesis of a linearly varying displacement field through the thickness, i.e.

$$v_\alpha^{(0)} = v_\alpha + \theta^1 v_\alpha^{(1)}, \quad v_3^{(0)} = v_3^{(0)},$$  

(3)

where $v_\alpha^{(0)}$ and $v_3^{(0)}$ denote the tangential and normal displacement components at the reference surface, $v_\alpha^{(1)}$ stands for the rotations of the normal and $v_3^{(1)}$ is the distance from the reference surface. Based on the above assumptions the Green strain tensor components can be approximated, to within a negligible small relative error margin (of $O(\eta)$), by the series expansions

$$E_{\alpha\beta}^{(0)} = E_{\alpha\beta}^{(0)} + \theta^1 E_{\alpha\beta}^{(1)} + \left( \theta^1 \right)^2 E_{\alpha\beta}^{(2)}, \quad E_{\alpha 3}^{(0)} = E_{\alpha 3}^{(0)} + \theta^1 E_{\alpha 3}^{(1)} + \theta^2 E_{\alpha 3}^{(2)}, \quad E_{33}^{(0)} = E_{33}^{(0)}.$$  

(4)

The nonlinear moderate rotation FOSD shell strain-displacement relations of Schmidt and Reddy [1] specified for the kinematical hypothesis (3) with all
transverse normal strain effects neglected yield the following five-parameter variant of the 2-D strain measures

\[ E_{\alpha\beta}^{(0)} = \theta_{\alpha\beta}^{(0)} + \frac{1}{2} \theta_{\alpha}^{(0)} \theta_{\beta}^{(0)}, \]
\[ E_{\alpha\beta}^{(1)} = \frac{1}{2} \left( \varphi_{\alpha\beta}^{(1)} + \varphi_{\beta\alpha}^{(1)} \right) - \frac{1}{2} \left( b_{\alpha} b_{\beta} + b_{\beta} b_{\alpha} \right), \]
\[ E_{\alpha\beta}^{(2)} = \frac{1}{2} \left( b_{\alpha} b_{\beta} \varphi_{\alpha\beta}^{(1)} + b_{\beta} b_{\alpha} \varphi_{\beta\alpha}^{(1)} \right) + \frac{1}{2} \left( \varphi_{\alpha} b_{\beta} v_{\alpha} + \varphi_{\beta} b_{\alpha} v_{\beta} \right), \]
\[ E_{\alpha3}^{(0)} = \frac{1}{2} \left( \varphi_{\alpha} + \varphi_{\alpha} \right) + \frac{1}{2} \theta_{\alpha}^{(0)} \theta_{3\alpha}, \]
\[ E_{\alpha3}^{(1)} = \frac{1}{2} \left( \varphi_{\alpha} + \varphi_{\alpha} \right) + \frac{1}{2} \theta_{\alpha}^{(0)} \theta_{3\alpha}, \]
\[ E_{33}^{(0)} = 0. \]

Here the following abbreviations have been used:

\[ \theta_{\alpha\beta}^{(0)} = \frac{1}{2} \left( \varphi_{\alpha\beta}^{(0)} + \varphi_{\beta\alpha}^{(0)} \right) - b_{\alpha\beta}^{(0)}, \]
\[ \varphi_{\alpha\beta}^{(0)} = \varphi_{\alpha\beta}^{(0)} b_{\alpha\beta}^{(0)}, \]
\[ \varphi_{\alpha} = \varphi_{\alpha}, \]
\[ \theta_{\alpha}^{(0)} = \theta_{\alpha}. \]

The notations (.) stand for partial and covariant differentiation on the reference surface and \( b_{\alpha\beta}, b_{\alpha} \) denote the covariant and mixed components of the curvature tensor.

The transformation of the principle of virtual displacements of 3-D elastokinetics eqn (1) into a 2-D formulation using the kinematical hypothesis eqn (3) yields

\[ \sum_{\alpha, \beta} \left[ \sum_{n=0}^{(n)} L^{\alpha} \delta E_{\alpha\beta}^{(n)} + \sum_{n=0}^{(n)} L^{3} \delta E_{\alpha3}^{(n)} - \sum_{n=0}^{(n)} \left( B^{\alpha} - \theta^{\alpha} + D^{\alpha} + p^{\alpha} \right) \delta V_{\alpha}^{(n)} - \left( B^{3} - \theta^{3} + D^{3} + p^{3} \right) \delta V_{3}^{(n)} \right] dM \]
\[ - \sum_{C} \left[ \sum_{n=0}^{(n)} L^{\alpha\beta} \nu_{\alpha} \nu_{\beta} \delta V_{\alpha}^{(n)} + L^{\alpha\beta} t_{\alpha} \nu_{\beta} \delta V_{3}^{(n)} + L^{\alpha\beta} \nu_{\alpha} \delta V_{3}^{(n)} \right] dC = 0. \]

Here, \( L^{ij} \) are the gross stress resultants, \( B^{i}, I^{i}, \) and \( D^{i} \) denote the gross body, inertia and damping couples, while \( p^{i}, \) and \( L^{ij} \) are the gross surface and boundary load couples, respectively, of the n-th order. They are obtained by integration of the respective physical quantities through all layers and can be found in detail in Librescu and Schmidt [8], Schmidt and Reddy [1].

Furthermore, in (7) \( C \) denotes the boundary curve of the reference surface \( M, \) \( \nu_{\alpha} \) and \( t_{\alpha} \) are the components of the unit outward normal and tangent vector of \( C, \) while indices \( v \) and \( t \) denote normal and tangential displacement and rotation components, respectively.

For viscoplastic transient analysis the constitutive equations of Chaboche [9] are employed. Details of the material parameter identification by uniaxial tension
tests are available in Stoffel et al. [7]. A layered shell model is used which permits to trace the evolution of the material properties in each individual layer. Numerical solutions are obtained by using the isoparametric Lagrangian 9-node shell finite element and the central difference method for the time integration of the non-linear equations of motion (see [7], [10-12]).

Figure 2: Time history of pressure and centre displacement.

Figure 3: Impulsively versus statically deformed plate shapes.

4 Results

Fig. 2 shows the evolution of the pressure and centre deflection, respectively, recorded during a typical shock tube experiment using aluminium plate specimens as described in Chapter 2. One can observe a very good agreement between the results predicted by FE simulation of the elastic-viscoplastic transient response and the experimental observation.
In the above experiment the peak pressure during blast loading was 3.5 bar. Fig. 3 shows the resulting final conical shape of the plate in comparison with the shape of a plate loaded quasi-statically with a pressure of 3.5 bar. For a better comparison of the different shapes also a plate is included that was loaded quasi-statically with a higher pressure of 7.2 bar resulting in the same centre deflection as in the case of the above dynamically deformed plate.

The observed conical shape in Fig. 3 is explained in this study by flexural waves travelling from the boundary towards the plate centre at different speeds.

Figs. 4 and 5 show FE simulation results for the evolution of the bending moment and membrane force, respectively, at the plate centre, the plate boundary and an additional point located at the distance of 137 mm from the centre.

Figure 4: Evolution of the bending moment.

It can be observed from Fig. 4 that immediately after the arrival of the shock wave high bending moments occur only close to the boundary. Then the bending action spreads inwards and reaches the plate centre with a delay. Note also that in the plate centre oscillations of the bending moment occur. First small oscillations are visible, than the amplitudes increase, and finally the bending moment tends to zero. This indicates that the movement of the centre during the deformation is much more complicated than it is suggested by Fig. 2. Note also the rapid increase of the membrane force in Fig. 5 which occurs almost simultaneously in all points. This reflects the von Kármán effect and indicates that the blast loading problem turns into a geometrically non-linear one already during the impulse duration.

The above observations can be explained by flexural waves which originate at the boundary and travel towards the plate centre at different speeds. In the
present case the impulsive transversal loading of the boundary, that occurs as reaction to the pressure impulse on the plate, generates such flexural waves with a wide spectrum of frequencies. According to Doyle [13] the propagation velocity of flexural waves increases with increasing frequency. Furthermore, spectral analysis of the investigated plate vibration indicates that wave amplitudes decrease with increasing frequency. Consequently, those waves with high frequency and small amplitude reach the plate centre first. This results in small amplitude bending oscillations in the plate centre as can be observed in Fig. 4. With a considerable delay the flexural waves with the highest amplitude and lowest frequency reach the plate centre at last what can be seen in Fig. 4, too. At the boundary these oscillations do not occur, because the waves originate there all at the same instant of time.

![Graph](image)

Figure 5: Evolution of the membrane force.

The above observations are also reflected in the evolution of the equivalent plastic strain rate shown in Fig. 6. At the boundary the negative moment causes stretching at the bottom of the plate. Together with the action of the tensile membrane forces (see Fig. 5) and shear forces plastic zones develop first at the boundary bottom layer.

Furthermore it can be seen in Fig. 6 that the equivalent plastic strain rate in the plate centre depends on the oscillations of the bending moment. The local minimum between 8.3ms and 8.4ms in Fig. 4 causes stretching at the bottom of the plate centre and together with the action of tensile normal forces and shear forces plastic strains occur, see Fig. 6. At 8.4ms the bending moment changes its sign resulting in an unloading of the bottom zone. Consequently the equivalent plastic strain rate becomes zero, see Fig. 6. Additional plastic strains develop
during the negative bending moment history associated with stretching of the centre bottom layer at about 8.6ms.

Figure 6: Evolution of the equivalent plastic strain rate at the bottom of the plate.

Figure 7: Evolution of the deformed shape.

The influence of the oscillating bending moment is also visible in the deformed shape of the plate at several instants of time. The shock wave impinges
on the plate at the time \( t = 7.9\text{ms} \). In Fig. 4 the bending moment changes its sign at \( t = 8.4\text{ms} \), i.e. 0.5ms after the arrival of the shock wave. The corresponding shape of the plate at this time is shown in Fig. 7 (the legend indicates the time, which has passed after the plate was loaded impulsively). After \( t = 8.4\text{ms} \) the bending moment in Fig. 4 becomes positive causing a stretching at the top of the plates centre, which is visible in the shape 0.6ms after the impulsive loading (see Fig. 7). Likewise, at \( t = 8.6\text{ms} \) the bending moment in Fig. 4 becomes negative causing a stretching at the bottom of the plates centre, which is visible in the shape 0.7ms after the impulsive loading (see Fig. 7).

This time evolution of the bending moments, normal and shear forces leads first to a trapezoidal shape of the deformed plate (see Fig. 7). Later the bending moment becomes zero leading to a membrane state of deformation in the midpoint, but not at the boundary (see Fig. 4). This results in the evolution of a final conical shape (see Fig. 7).

5 Conclusions

The results presented in this paper demonstrate that simulation of blast loaded structures cannot be based on bending or non-linear membrane theories but require models which take into account the complicated membrane-bending interaction already during the impulse duration. Furthermore, the results of the present investigations allow for an explanation of the evolution of the experimentally observed final conical shape by flexural waves travelling towards the plate centre. This observation improves earlier simplified models of plate deformation which are discussed in detail e.g. by Cristescu [14].

References


