Influence of the constitutive relation in numerical simulations of the perforation of steel plates

S. Dey, T. Børvik, O. S. Hopperstad & M. Langseth
Structural Impact Laboratory (SIMLab), Department of Structural Engineering, Norwegian University of Science and Technology

Abstract

In the present study, the influence of the constitutive relation in numerical simulations of the perforation of steel plates has been studied using the non-linear finite element code LS-DYNA. Two well-known constitutive relations found in the literature are combined with a fracture criterion, and these models have been used in the simulations. The different models have been calibrated and validated for the target material Weldox 460 E using experimental data obtained from tensile tests where the effects of strain rate, temperature and stress triaxiality were taken into account. Numerical simulations of the perforation process indicate that the physical mechanisms can be qualitatively well predicted independently of the chosen constitutive relation, but quantitatively more severe differences appear.

Keywords: high-strength steel, Johnson-Cook, Zerilli-Armstrong, ballistic penetration, LS-DYNA.

1 Introduction

In order to model the mechanical response of structures exposed to ballistic impacts, it is necessary to establish reliable constitutive relations and failure criteria as functions of large strains, high strain-rates, high temperatures, varying stress states and history of loading, in addition to accumulation of damage and mode of failure. A complete description that includes all these phenomena is however difficult to obtain, and certainly the model will become more complex by increasing the number of phenomena involved. To simplify the modelling it is necessary to introduce assumptions according to the problem investigated.
Several constitutive relationships have been proposed for metallic materials under impact loading for use in computational mechanics that vary from being purely empirical to highly theoretical. The empirical models are based on available experimental observations, while the latter are based on the microscopic nature of the material. The multiaxial stress state of the material is usually expressed in terms of the equivalent stress $\sigma_{eq}$, and many constitutive relations define this stress in terms of the accumulated plastic strain $\varepsilon_{eq}$, plastic strain rate $\dot{\varepsilon}_{eq}$ and temperature $T$ as

$$\sigma_{eq} = f(\varepsilon_{eq}, \dot{\varepsilon}_{eq}, T)$$  \hspace{1cm} (1)

The form given in eqn (1) can easily be adapted to most computer codes since it uses variables already available in the codes. Important constitutive relations of this type have been proposed by Johnson and Cook (JC) [1], which was modified by Børvik et al. [2], and Zerilli and Armstrong (ZA) [3, 4]. These relations are selected in the current study since they are intended for impact analysis, have relatively few material constants, and are considered easy to use in computational procedures. A modified version of the Johnson-Cook (JC) fracture criterion [5] proposed by Børvik et al. [2] is used in all simulations, independent of constitutive relation.

Several numerical studies on the perforation of steel plates show that the modified JC models give reliable results [6-9]. Even so, the JC models have received some criticism in the literature over the years [10]. This is partly due to its empirical origin and partly due to the non-coupling between influence of strain rate and temperature during plastic deformation.

In the present study, the effect of constitutive relation for a typical penetration problem has been studied using the non-linear finite element code LS-DYNA [11]. The constitutive models presented above have been calibrated and validated for the target material Weldox 460 E using experimental data obtained from tensile tests where the effects of strain rate, temperature and stress triaxiality were taken into account [2, 9]. Finally, these constitutive models are used in numerical simulations of the perforation process.

## 2 Material models

### 2.1 Johnson-Cook (JC) constitutive relation

Johnson and Cook [1] proposed a phenomenological constitutive relation, which has been frequently used in impact analysis due to its simplicity. Here, a slightly modified version of this constitutive relation proposed by Børvik et al. [2] is used, and the equivalent stress is expressed as

$$\sigma_{eq} = (A + B\varepsilon_{eq}^n)(1 + \dot{\varepsilon}_{eq}^*)^c(1 - T^m)$$  \hspace{1cm} (2)

The model operates with five material constants; $A$, $B$, $C$, $n$ and $m$. The dimensionless strain rate is given by $\dot{\varepsilon}_{eq}^* = \dot{\varepsilon}_{eq} / \dot{\varepsilon}_0$, where $\dot{\varepsilon}_0$ is a user-defined reference strain rate. The homologous temperature $T^*$ is defined by
\[ T^* = \frac{(T-T_r)}{(T_m-T_r)} \]
where the suffixes \( r \) and \( m \) indicate room and melting temperatures, respectively. The various phenomena such as strain hardening, strain rate hardening and temperature softening are uncoupled from each other.

### 2.2 Zerilli-Armstrong (ZA) constitutive relation

While the JC constitutive relation is purely empirical, the ZA constitutive relation is based on the dislocation theory [3, 4]. Each material structure type has a different constitutive behaviour based on the dislocation characteristics for that particular structure. Depending on the structure, the models have six to eight material constants \( \sigma_a, A, B, n, \alpha_0, \alpha_1, \beta_0, \) and \( \beta_1 \). Only the strain rate hardening and temperature softening are coupled in the bcc model, while the strain hardening is uncoupled from the other two phenomena [3]

\[ \sigma_{eq} = \sigma_a + B \exp(-\beta T) + A \varepsilon_{eq}^n \] (3)

where \( \beta = \beta_0 - \beta_1 \ln \dot{\varepsilon}_{eq} \) (4)

The strain hardening becomes strain rate and temperature dependent for hcp metals and steel alloys [4]

\[ \sigma_{eq} = \sigma_a + B \exp(-\beta T) + A \varepsilon_{eq}^n \exp(-\alpha T) \] (5)

where \( \beta \) is defined as eqn (4), and \( \alpha \) equals

\[ \alpha = \alpha_0 - \alpha_1 \ln \dot{\varepsilon}_{eq} \] (6)

Both these versions of the ZA relation will be used in the following simulations.

### 2.3 Johnson-Cook fracture criterion

Johnson and Cook [5] developed a fracture criterion that accounts for strain path, strain rate and temperature in addition to stress triaxiality. The fracture criterion is based on damage evolution, where the damage \( D \) of a material element is expressed as

\[ D = \sum \left( \frac{\Delta \varepsilon_{eq}}{\varepsilon_f} \right) \] (7)

Here, \( \Delta \varepsilon_{eq} \) is the increment of accumulated plastic strain that occurs during an integration cycle and \( \varepsilon_f \) is the fracture strain. Failure occurs when \( D \) equals unity, since no coupling between damage and constitutive relation is considered in this study. The fracture strain is constructed in a similar way as the JC constitutive relation. A slightly modified version of the original model [2] reads

\[ \varepsilon_f = (D_1 + D_2 \exp(D_3 \sigma^*)) (1 + \dot{\varepsilon}_{eq}^*)^{D_4} (1 + D_5 T^*) \] (8)

where \( D_1, ..., D_5 \) are material constants. \( \sigma^* \) is the stress triaxiality ratio defined as \( \sigma_H/\sigma_{eq} \), where \( \sigma_H \) is the mean stress. Again, the various phenomena accounted for in the fracture criterion are uncoupled from each other.
2.4 Identification of material constants

The advantage of the JC constitutive relation is that the material constants can be easily obtained due to the uncoupled formulation. This, however, is at the expense of the potential coupling of effects. Further, the model is purely empirical. Hence, since the ZA constitutive relation is theoretically based, some consider the ZA relation to be a better choice for this type of problem. However, the coupling between the different effects in the ZA constitutive relation will make it more difficult to determine the appropriate material constants.

Table 1: Material constants for the modified JC constitutive relation and the modified JC fracture criterion [9].

<table>
<thead>
<tr>
<th>A (MPa)</th>
<th>B (MPa)</th>
<th>n</th>
<th>C</th>
<th>m</th>
<th>D₁</th>
<th>D₂</th>
<th>D₃</th>
<th>D₄</th>
<th>D₅</th>
</tr>
</thead>
<tbody>
<tr>
<td>499</td>
<td>382</td>
<td>0.458</td>
<td>0.0079</td>
<td>0.893</td>
<td>0.636</td>
<td>1.936</td>
<td>-2.969</td>
<td>-0.014</td>
<td>1.014</td>
</tr>
</tbody>
</table>

Table 2: Material parameters for the ZA constitutive relation [12, 13].

<table>
<thead>
<tr>
<th>σ₀ (MPa)</th>
<th>B (MPa)</th>
<th>β₀</th>
<th>β₁</th>
<th>A (MPa)</th>
<th>n</th>
<th>α₀</th>
<th>α₁</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZAbcc</td>
<td>11</td>
<td>388</td>
<td>1.80·10⁻²</td>
<td>6.91·10⁻²</td>
<td>553</td>
<td>0.2704</td>
<td>-</td>
</tr>
<tr>
<td>ZAhcp*</td>
<td>20</td>
<td>378</td>
<td>2.37·10⁻⁴</td>
<td>6.35·10⁻⁴</td>
<td>563</td>
<td>0.2704</td>
<td>-6.52·10⁻⁴</td>
</tr>
<tr>
<td>ZAbcc (more data)</td>
<td>246</td>
<td>594</td>
<td>5.19·10⁻⁵</td>
<td>2.13·10⁻⁴</td>
<td>553</td>
<td>0.2704</td>
<td>-</td>
</tr>
<tr>
<td>ZAhcp* (more data)</td>
<td>243</td>
<td>496</td>
<td>4.65·10⁻⁵</td>
<td>1.88·10⁻³</td>
<td>579</td>
<td>0.2704</td>
<td>1.17·10⁻⁴</td>
</tr>
</tbody>
</table>

The calibrations for the constitutive models are based on an extensive series of tensile tests that were conducted on Weldox 460 E [2, 9]. The stress-strain relations and fracture strains were obtained for a wide range of strain rates, temperatures, and stress triaxiality levels using both smooth and pre-notched axisymmetric specimens. Each phenomenon such as strain hardening, strain rate hardening, temperature softening, and stress triaxiality, was uncoupled from each other in these tests. Hence, it was relatively easy to calibrate the modified JC constitutive relation and the modified JC fracture criterion [9]. Following values were used: \( \dot{\varepsilon}_0 = 5·10^{-4} \ s^{-1} \), \( T_0 = 293 \ K \) and \( T_m = 1800 \ K \). The material parameters obtained for these two models are given in Table 1. The calibration procedure of the two ZA models was performed differently from the modified JC constitutive relation, since the parameters cannot be decoupled. The calibration is described in detail in [12, 13]. The parameters obtained for the two ZA models are given in Table 2. These are noted as ZAbcc for bcc metals, and ZAhcp* for hcp metals and steel alloys. Further, it is seen that two sets of material parameters are calibrated for each ZA model. As mentioned, there is a coupling between influence of temperature and strain rate in the ZA constitutive relations. Hence, in addition to the experimental data where the various phenomena are uncoupled from each other [2, 9], some more test data from Børvik et al. [14] involving coupling between strain rate and temperature were used in the calibration of the latter material set (noted as ‘more data’ in Table 2.). The validation study
presented in the next section shows that it is necessary to add more material data in order to get reliable results when using the ZA constitutive relation.

3 Validation of material tests

3.1 Material tests

In order to validate the different material models in Section 2, numerical simulations of the tensile tests conducted by Børvik et al. [14] are performed. They conducted tensile tests on Weldox 460 E under the combination of dynamic loading, elevated temperature and stress triaxiality. Further, the tests on smooth specimens from this programme were also used in the calibration of the ZA constitutive relations named ‘more data’. The geometry and dimensions of the specimens that were used in the tests are shown in Figure 1. Three temperatures $T_0$ were considered; 100, 300 and 500 °C. The average engineering strain rates $\dot{\varepsilon}_{avg}$ were between 450 and 550 s$^{-1}$ in the smooth specimens. Three notch radii ($R_0$) were chosen; 0.4, 0.8 and 2.0 mm in addition to smooth specimens. It is referred to the original paper [14] for more details about the experimental programme carried out.

3.2 Numerical model and set-up

The different constitutive models presented in the previous section were implemented as user-defined material models in the explicit non-linear FEM code LS-DYNA. Beside of using the modified JC fracture criterion, element erosion was initiated when the homologous temperature reached 0.9, indicating a maximum temperature of 1649 K in the simulations. This extra criterion was implemented in order to avoid numerical problems in severely distorted elements, which may occur in perforation problems.

![Figure 1: Geometry and dimensions of the smooth (left) and pre-notched (right) specimens used in the material tests given in mm [14].](image)

Both smooth and pre-notched axisymmetric specimens were meshed with the same configuration as the experimental specimens shown in Figure 1. The notched specimens have initial notch root radius $R_0$ equal to the experimental values; 0.4, 0.8 and 2.0 mm, while the initial minimum radius $a_0$ was 1.5 mm in all models. The specimens were modelled using four-node axisymmetric elements with one integration point and stiffness-based hourglass control. 20 elements across the specimen radius were chosen based on previous studies [15].
This gave an element size in the critical gauge region of approximately 0.075 x 0.075 mm², and a total of 3400 elements (3591 nodes) for smooth specimens and 4560 elements (4809 nodes) for pre-notched specimens.

The loading was defined by describing the elongation at one end of the specimen against time, using the measured elongation-time curve from typical experiments, while the other end was fixed. Only the specimen is modelled in the numerical simulations, and it follows that the calculated force is disturbed by an elastic wave that propagates along the specimen length and reflects from the specimen ends. Hence, severe oscillations occurred in some of the numerical force-time curves, while this was not the case for experimental curves. The force oscillations were reduced by filtering the force signals. More details about the filtering and numerical procedures can be found in [15].

3.3 Validation study

The force-elongation curves from the simulations are compared with the experimental data of Børvik et al. [14]. The results are shown in Figure 2 for only one temperature (T₀ = 300 °C), and is representative for comparison of the different material models. All results are however shown in [13].

Figure 2: Experimental versus numerical force-elongation curves for smooth and pre-notched specimens at 300 °C.
Firstly, it is seen that the modified JC constitutive model (both constitutive relation and fracture criterion) is in reasonably good agreement with the experimental results for both the maximum force as well as the elongation to fracture. Secondly, it is easily observed that both the ZA models deviate strongly from the experimental results when the calibration is based on the data where the effects of temperature and strain rate are uncoupled. On the other hand, it is seen that both these models predict the behaviour very well when the coupled data on strain rate and temperature are included. The maximum force is especially well predicted. Hence, the ZA models require more experimental data than the JC constitutive criterion in order to get reliable results. Further, hardly any differences are seen in the behaviour between the bcc model and the hcp* model. Another observation is that the ZA models show less temperature softening than the modified JC constitutive relation. Overall, all three constitutive relations are found to give reasonable and similar results when properly calibrated.

4 Validation of component tests

4.1 Component tests

Perforation tests have been performed on Weldox 460 E steel plates with blunt projectiles [8]. The targets, having a nominal thickness of 12 mm and a free span diameter of 500 mm, were clamped in a circular frame. The projectiles have a nominal mass, diameter and length of 197 g, 20 mm and 80 mm, respectively. A compressed gas gun was used to launch projectiles within the velocity range from 150 m/s to 350 m/s [8]. The initial and residual velocities of the projectile were measured, while the perforation process was captured using a digital high-speed camera system. The ballistic limit velocity, which is defined as the average between the highest impact velocity \( v_i \) not giving perforation and the lowest \( v_i \) giving complete perforation, was obtained based on the test data. Results from these tests will be compared with numerical simulations in the next section.

4.2 Numerical simulations

The impact tests for blunt projectiles were analysed in LS-DYNA. The target was modelled using the same material models as presented in Section 3. Since it was essential to take the coupled effect of temperature and strain rate into account, only the material parameters for the ZA constitutive relations noted as ‘more data’ in Table 2 will be used in the following. The projectile of hardened tool steel was modelled as a bilinear elastic-plastic von Mises material with isotropic hardening without fracture using Material Type 3 in LS-DYNA. The material constants are given in Table 3 along with other relevant material constants that are needed for the target.

The geometry of the target and projectile is identical to that used in the experimental tests. The same type of elements was used as in the validation study in Section 3. Perforation problems with blunt projectiles involving shear localisation and plugging, are highly sensitive to the mesh density. This
sensitivity is however assumed not to be pathological [6], since the numerical solution seems to converge monotonically towards a limit solution when the number of elements over the target thickness becomes sufficiently large. Due to today’s computer capacity, finer mesh densities than 120 elements over the target thickness could however not be considered in order to obtain the ballistic limit velocity [9].

Table 3: Relevant material constants needed for the target and the projectile.

<table>
<thead>
<tr>
<th></th>
<th>E (GPa)</th>
<th>υ</th>
<th>ρ (kg/m³)</th>
<th>C_p (J/kgK)</th>
<th>χ</th>
<th>α (K⁻¹)</th>
<th>σ₀ (MPa)</th>
<th>E_t (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>target</td>
<td>210</td>
<td>0.33</td>
<td>7850</td>
<td>452</td>
<td>0.9</td>
<td>1.2·10⁻⁵</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>projectile</td>
<td>204</td>
<td>0.33</td>
<td>7850</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1900</td>
<td>15000</td>
</tr>
</tbody>
</table>

In the simulations, the projectile was given an initial velocity identical to the one used in a corresponding experiment and the residual velocity of the projectile was registered. From this, the ballistic limit velocity \( v_{bl} \) was estimated based on 6-8 runs using the analytical model by Recht-Ipson [16]

\[
v_r = a(v_i^n - v_{bl}^n)^{1/p}
\]

The method of least squares was then used to fit the model constants \( a, p \) and \( v_{bl} \) to the simulated residual projectile velocities.

Figure 3: Comparison of the different material models and experiments: Residual velocity versus initial velocity (left) and ballistic limit velocity versus mesh density (right).

Figure 3 (left) shows the residual velocity versus impact velocity curves from simulations using 120 elements over the target thickness. The numerical results are compared with the experimental data. It is seen that the modified JC constitutive relation predicts the ballistic limit velocity considerably better than the two ZA models. Further, Figure 3 (right) indicates that the ballistic limit velocity is highly dependent on the mesh density when the modified JC
It is also seen that the mesh size dependency is less distinct for the two versions of the ZA model. In addition, the two ZA models predict almost the same ballistic limit velocity except for 60 elements over the target thickness.

Figure 4 shows high-speed camera images of a perforation test compared with fringe plots from two numerical analyses, where the modified JC constitutive relation and the ZAhcp* model were used, respectively. The mesh density was 120 elements over the target thickness. All series are taken at impact velocities 1-3 % above the respective ballistic limits. The experimental test shows that perforation occurs by plugging. For this type of failure the deformation localises in narrow shear bands where the shear strain, shear strain rate and temperature may locally be very high [9]. The localised plastic zone is clearly seen in the numerical analyses and the deformation mainly develops in a narrow, almost horizontal zone in the target. This physical mechanism is well captured for both constitutive relations. However, further investigation shows that the JC constitutive relation has a stronger localisation and less global bending. Moreover, around 33 % more elements fail in the analysis when using the ZAhcp* model.

Figure 4: High-speed camera images of an experimental test compared with numerical analyses. The target plotted as fringes of effective plastic strain in the user-defined range $\varepsilon_{eq} = 0$ (light grey) and $\varepsilon_{eq} = 2$ (black).
5 Concluding remarks

The overall conclusion of this study is that although both constitutive relations predict the physical mechanism of the perforation process well, the modified JC constitutive relation seems to predict the ballistic limit velocity considerably better than the two constitutive relations of Zerilli and Armstrong. This is interesting, since it was seen that numerical simulations of tensile tests predict the experimental results with reasonably good agreement for all three constitutive relations. However, at very high strain rates, there is a significant difference in the predicted stress-strain rate values between the JC constitutive relation and the ZA constitutive relation. For calibration purposes, the JC model requires less tensile tests in order to predict the correct behaviour. In other words, from an engineering point of view, it may seem reasonable to use the modified JC constitutive relation for the type of problem studied herein.

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References


