GRALE2D – an explicit finite element code for two-dimensional plane and axi-symmetric multi-material ALE simulations

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Abstract

This paper describes the Euler-Lagrange coupling in GRALE2D, an explicit finite element code for two-dimensional plane and axi-symmetrical multi-material ALE simulations, which is being developed at the Swedish Defence Research Agency (FOI). A brief overview of the functionalities of the code is followed by a description of the implemented method for Euler-Lagrange coupling together with a numerical example demonstrating the performance of the code in a typical ballistic application. Since GRALE2D works with selectively reduced integrated elements, hourglass stabilization is not necessary for the treatment of materials with shear strength.

1 Introduction

GRALE2D is an explicit multi-material ALE finite element software for two-dimensional plane and axi-symmetric problems developed at the Swedish Defense Research Agency. The main objective with the software is to simulate the mechanics of warheads, with special emphasis on liner collapse and jet formation of shaped charges and projectile formation of EFP warheads. However, the software is general and can be used for other purposes as well. Most commercially available software are focused on three-dimensional problems while there is a lack of well-supported two-dimensional hydro codes. This was the motivation for our group to develop a hydro code supporting Lagrangian and Eulerian elements as well as Euler-Lagrange coupling.

The only implemented element type is a four node, selectively reduced integrated, iso-parametric, bi-linear quadrilateral. That is, there are no zero energy
modes. An operator split technique is applied to solve the governing equations posed in the ALE system of reference and the advection method is first order accurate.

GRALE2D is mainly based on classical hydrocode algorithms. However, the penalty approach to Euler-Lagrange coupling is relatively uncommon. Hence, this article is devoted to describing the implemented coupling method (Section 2) and to display its performance in an application example (Section 3).

2 Euler-Lagrange coupling algorithm

The Euler-Lagrange coupling algorithm in GRALE2D communicates forces between two overlapping element grids, one Eulerian (or ALE) and one Lagrangian. The materials represented by the Eulerian and Lagrangian element grids are normally referred to as fluid and structure, respectively.

2.1 Penalty formulation

Working with a penalty formulation, the magnitude of the contact pressure $p$ at a point is proportional to the distance $d$ the fluid and the structure penetrate each other, see fig. 1.

$$p = kd$$

$k$ is a user defined penalty factor [pressure/unit distance of penetration].

2.2 Penetrations

In GRALE2D normal penetrations $d_i$ are calculated at a finite number of contact points located on the boundary of the structure, where each contact point represents a specific contact area $A_i$. The resulting contact force $F_i$ at each coupling point is defined as

$$F_i = kA_i d_i$$

A situation with a discretized contact surface is schematically depicted in fig. 2. The contact is here assumed frictionless, such that the spring forces act in the direction of the contact surface normal $\hat{n}$.

$$\hat{n} = [n_x \ n_y \ n_z]^t$$

2.3 Force distribution

Working with finite elements, both the fluid and the structure are spatially discretized. The contact surface of the structure is simply composed by element faces located on the surface of the discretized body. The surface of the fluid is somewhat
more complicated. In a multi-material analysis the fluid boundary cuts through the elements of the Eulerian or ALE mesh and in GRALE2D it is defined by a linear interface reconstruction algorithm.

The discrete contact forces $F_i$ are distributed to the nodes of the fluid and structure elements, in which the contact points are located, using the element interpolation functions. The Eulerian interpolation functions are denoted $N_{fi, i=1,4}$ and the Lagrangian ones $N_{si, i=1,2}$. The distribution of forces is schematically visualized in fig. 3.

2.4 Time step size

Using the explicit central difference time integration scheme the critical time step size $\Delta t_c$ is limited by the highest eigenfrequency $\omega_{max}$ of the system, c.f. Hughes [1].

$$\Delta t_c = \frac{2}{\omega_{max}} \quad \text{(4)}$$

One can show that

$$\omega_{max} \leq \omega_{max}^c \quad \text{(5)}$$
where \( \omega_{max} \) is the highest eigenfrequency of a single element in the system. Actually, \( \omega_{max} \) is generally a good estimation of \( \omega_{max} \). In GRALE2D \( \omega \) is estimated for each individual element based on the element size and on the local speed of sound. The element tangent stiffness matrices \( k^e \) are never explicitly formed.

The presence of a penalty based Euler-Lagrange coupling interface complicates
the situation. The coupling interface adds stiffness to the system and, consequently, influences the eigenfrequency spectra of the model. To estimate the influence on the critical time step size GRALE2D makes a conservative assumption and lets the contact springs become part of the local Eulerian and Lagrangian elements to which they are attached. Half the spring is assumed part of the Lagrangian element and half part of the Eulerian element. The system is further stiffened by constraining the end points of these springs. The situation is visualized in fig. 4.

Figure 4: Contact spring assumed part of the Eulerian and Lagrangian elements.

Looking at the Eulerian element, the updated tangent stiffness matrix $\bar{k}^e$ becomes

$$\bar{k}^e = k^e + \begin{bmatrix} n_x^2 k^s & n_x n_y k^s & n_x n_z k^s \\ n_y n_x k^s & n_y^2 k^s & n_y n_z k^s \\ n_z n_x k^s & n_z n_y k^s & n_z^2 k^s \end{bmatrix} \tag{6}$$

where

$$k^s = 2kA_i \begin{bmatrix} N_1^f N_1^f & N_1^f N_2^f & N_1^f N_3^f & N_1^f N_4^f \\ N_2^f N_1^f & N_2^f N_2^f & N_2^f N_3^f & N_2^f N_4^f \\ N_3^f N_1^f & N_3^f N_2^f & N_3^f N_3^f & N_3^f N_4^f \\ N_4^f N_1^f & N_4^f N_2^f & N_4^f N_3^f & N_4^f N_4^f \end{bmatrix} \tag{7}$$

From eqn 6 one can define an upper bound for the maximum row sum $S$ of $\bar{k}^e$.

$$\tilde{S} \leq S + 2kA_i \max(N_1, N_2, N_3, N_4) \tag{8}$$

where $S$ is the maximum row sum of $k^e$. Neither $k^e$ nor $S$ is explicitly calculated. However, knowing $\omega^e$ and working with a lumped mass matrix, where one quarter
of the element mass \( m^e \) is located at each node, Gerschgorin’s eigenvalue theorem [2] can be used to obtain an upper bound for the highest eigenfrequency \( \bar{\omega}^e \) of the modified element.

\[
(\bar{\omega}^e)^2 \leq (\omega^e)^2 + \frac{kA_i}{2m^e} \max(N_1^f, N_2^f, N_3^f, N_4^f) \tag{9}
\]

The same procedure to estimate an upper bound for the modified eigenfrequency can be carried out for the Lagrangian element and, finally, the critical time step size is estimated as

\[
\Delta t_c \approx \frac{2}{\bar{\omega}^e_{\text{max}}} \tag{10}
\]

where \( \bar{\omega}^e_{\text{max}} \) is the highest estimated element eigenfrequency in the modified system.

![Diagram](image)

**Figure 5**: Artificial contact mass added to the coupling springs.

### 2.5 Contact density

From eqn 9 one can see that contact springs added to elements with small masses might cause high eigenfrequencies and, consequently, dropping time steps. Actually, large spatial fluctuations in the density can be devastating for the critical time step size. The problem can be decreased by introducing a contact density \( \rho_c \) [mass/unit contact area] as an artificially added mass that is distributed to the contact springs. Hence, the springs can be treated as individual elements in the eigenfrequency calculation according to fig. 5. The contact spring eigenfrequency \( \omega_c \) becomes

\[
\omega_c = 2\sqrt{\frac{k}{\rho_c}} \tag{11}
\]

and \( \Delta t_c \) can be estimated as
\[ \Delta t_c \approx \frac{2}{\max[\omega_{c,\text{max}}, \omega_c]} \] (12)

The contact interface density is a kind of mass scaling and one has to be careful not to significantly violate the physics of the problem.

Figure 6: CAD drawing of EFP-charge and starting geometry for simulation.

3 Application example

To illustrate the Euler-Lagrange coupling, the formation of an EFP is considered. The warhead is constructed of a high explosive (Octol 70/30), a cylindrical cover (Al 2024) and a copper liner with constant curvatures. A plane wave lens is used to initiate the HE. To simplify the calculations, the plane wave generator is not included in the simulation. Instead, a line initiation is prescribed and the nodes
adjacent to the plane wave generator are constrained in the axial direction. A multi-material Eulerian grid is used to describe the high explosive and the cover and a Lagrangian grid describes the liner. A CAD-drawing of the charge and the starting geometry for the simulation is shown in fig. (6). When the projectile is formed, the elements on the circumference become greatly distorted. To avoid small time-steps, the liner was divided into two parts, a central part and a small ring-shaped part around the circumference. After the projectile has left the Eulerian grid, the outer part of the liner is deleted.

A characteristic length of a computational element is 0.5 mm and the longitudinal sound speed of copper is 4.76 km/s. The critical time step based on the CFL-condition is thus of the order of 0.1 \( \mu s \). The maximum interface pressure between the HE and the liner is approximately 50 GPa. If the maximum allowed penetration is one half element size, the penalty stiffness \( k \) must be at least 200 GPa/mm. The contact density in the simulation was set to \( \rho_c = 10^{-2} \text{ g/mm}^2 \), which implied a critical time step of 3.16 ns. This is much smaller than what is required by the CFL-criterion and has a great negative impact on the performance.

The simulation was run to 140 \( \mu s \), after which the penetrator moved as a rigid body. The computed shape agreed rather well with the experiments. The computed velocity (2720 m/s) agreed with the experimentally deduced within experimentally uncertainty. Flash X-ray radiographs from the experiments and the results from the simulations are shown in fig. (7).

### 4 Discussion

Earlier experience and the application example above indicates two important issues for the future development efforts. Firstly, a higher order advection scheme is important for the reduction of energy losses and other dissipation related numerical errors. Secondly, an alternative constraint based Euler-Lagrange coupling
algorithm should be implemented. The current penalty based method adds stiffness to the system and reduces the critical time step in applications where high contact pressures require large penalty stiffnesses.

References