Influence of localised damage around crack tip on fracture propagation

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Abstract

A model for evaluation of Mixed Mode crack propagation, considering the mechanical damage effects, has been previously defined by the Author. According to this model, a computer program, using a composite finite element named octagonal core with singular radial nodes, has been developed and tested comparing numerical simulations with experimental results. This paper describes an application of the numerical code in order to analyse the localisation of damage around the crack tip for Mode I and Mixed Mode problems.

The results of numerical simulations show that the strain-rate has a greater influence on damage distribution than on crack extension.

1 Introduction

The damage crack model can describe the fracture behaviour under loading and slow crack growth in Mixed Mode problems. The crack growth depends on the strain energy density and on mechanical damage of the material around the crack tip. The material damage is assumed to be related to the strain and stress fields.

The finite element computer code, named SFEA, and developed by Author, has been designed for applying the damage crack model with an evolutive analysis technique that allows a complete simulation until final collapse. The code has been tested comparing numerical simulations with experimental results, as reported in [1,2].

The numerical simulation results are strain-rate dependants; for
Mode I problems, the behaviour has been discussed by Carpinteri [3] and Papakaliatakis [4]. In order to investigate the relation between numerical results and experimental behaviour, may be meaningful to analyse the interaction between damage and crack growth at the beginning of propagation from a notch.

2 Theoretical Foundations of Damage Crack Model

2.1 Stress Field around Crack Tip

The stress field in the vicinity of the crack tip can be expressed, according to the general solution of Westergaard, assuming a system of polar co-ordinates \( r \) and \( \theta \), oriented in the crack direction and with the origin in the crack tip, as:

\[
\sigma_r = \frac{1}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left( K_I \left(1 + \sin^2 \frac{\theta}{2}\right) + K_{II} \left(\frac{3}{2} \sin \theta - 2 \tan \frac{\theta}{2}\right)\right)
\]

(1.a)

\[
\sigma_\theta = \frac{1}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left( K_I \left(\cos^2 \frac{\theta}{2}\right) + K_{II} \left(\frac{3}{2} \sin \theta\right)\right)
\]

(1.b)

\[
\tau_{r\theta} = \frac{1}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left( K_I \sin \theta + K_{II} (3 \cos \theta - 1)\right)
\]

(1.c)

The stress intensity factors \( K_I \) and \( K_{II} \) are the only parameters related to the geometry and load conditions.

The criterion based on the maximum circumferential stress proposed by Erdogan and Sih [5] is used in order to determine the direction of propagation at the extremity of the crack: it is based on the hypothesis that the crack extends starting from its tip, orthogonally to the direction of maximum circumferential stress \( \sigma_\theta \).

Numerical simulations evaluating the crack propagation through this criterion are reported by Ballatore et al. [6] and by Carpinteri and Valente [7].

2.2 Damage Constitutive Law

Damage of the material is computed on the basis of a uniaxial bilinear elastic-softening stress-strain relation (Fig. 1). Stress may increase up to the ultimate strength corresponding to point \( U \), while strain increases proportionally, the slope of line \( OU \) is the elastic modulus \( E \). Then, only
strain may increase, while stress decreases linearly down to zero corresponding to point F. If the loading is relaxed when the representative point is in A, the unloading is assumed to occur along the line AO, so that the new bilinear constitutive relation is the line OAF. No permanent deformation is allowed by such a model, but only the degradation of the elastic modulus.

Figure 1: Bilinear elastic-softening stress and strain relationship.

The above described model can be extended to the three-dimensional stress conditions, using, as a measure of damage, the value of the absorbed strain energy density \( (dW/dV) \), corresponding in the uniaxial condition to the OUAB area. The effective elastic modulus \( E^* \), is related to the decreased critical value of strain energy density, \( (dW/dV)_{c*} \), corresponding in the uniaxial condition to the OAF area.

A comparative study of Polanco-Loria and Sørensen [8] reports the comparisons between the diagrams \( \sigma - \varepsilon \) corresponding to different constitutive laws: the bilinear elastic-softening law give intermediate results in good agreement with other different and more complex models.

2.3. Strain Energy Density Criterion

The eqn (1) can be expressed in vectorial form with the following notation:

\[
\{\sigma\} = \frac{1}{\sqrt{2\pi r}} \left( K_1 \{\Theta_I(\theta)\} + K_\Pi \{\Theta_{\Pi}(\theta)\} \right)
\]  

(2)

The vectors \( \{\Theta_I\} \) e \( \{\Theta_{\Pi}\} \) contain the trigonometric terms of Westergaard solution for Mode I and Mode II respectively.

Consequently, representing the material elastic properties by the Hessian matrix \([H]\), the strain energy density can be expressed as a
function of the polar co-ordinates $r$ and $\theta$, with the origin in the crack tip:

$$
\frac{dW(r, \theta)}{dV} = \frac{1}{4\pi r} \left( K_I \{\Theta_I\}^T + K_{II} \{\Theta_{II}\}^T \right) [H]^{-1} \left( K_I \{\Theta_I\} + K_{II} \{\Theta_{II}\} \right)
$$

(3)

In order to evaluate the crack growth increment at each loading step, the strain energy density criterion is applied, as proposed by Sih [9,10]. It is based on the fundamental assumptions, according to Beltrami's criterion, that the material failure extends from the crack tip until the strain energy density is higher than the critical value $(dW/dV)_c^*$.

3 Finite Element Model

A detailed description of the finite element code SFEA is reported in [1]. During the evolutive analysis, for each simulation step, the following phases are executed: (1) stress and strain fields calculation by means of a linear elastic finite element code, using for each element an elastic modulus corresponding to the local damage; (2) determination of the new damage level in each element; (3) recalculation of stress and strain fields using for each element the new elastic modulus; (4) evaluation of the crack growth; (5) automatic remeshing corresponding to the new geometry, consequent to the crack propagation.

3.1 Singularity of the Stress and Strain Fields

The Westergaard solution shows that, around the crack tip, the strain and stress fields are characterised by a $1/r$ singularity in any direction.

Using quadrilateral isoparametric finite elements, with two intermediate nodes on each side, a singularity of the same order as that predicted by Westergaard, can be obtained (as demonstrated by Sih [11] and Harrop [12]) by arranging the nodes, on two sides converging to the crack tip, at 1/9 and 4/9 of the length of the side, as represented in Fig. 2b. The element characterised by this disposition of nodes is denominated singular element and gives exact results in any point.

A composite finite element, denominated octagonal core with singular radial nodes, represented in Fig. 2a, is placed in correspondence of each crack tip. The octagonal core is constituted by 12 quadrilateral elements, 4 central and 8 peripheral: the elements may have different elastic moduli, due to the variability of material damage around the crack tip, as consequence of eqn. (3).

Each of the four central elements has a vertex coincident with the crack tip of the fracture while the other three vertexes are disposed on an
ideal circumference of radius $R_i$ so that the external sides constitute the perimeter of an octagon. The external vertexes of the eight peripheral elements are arranged on an ideal circumference of radius $R_e$ so that constitute an octagon. For numerical results accuracy the best ration between $R_e/R_i$ is 3. The value of the external radius $R_e$ must be selected in a such way to include the zone around the crack tip, in which is relevant the entity of the stress concentration. Besides, for geometrical congruency, it must be shorter than the grid pattern interval.

Figure 2: Structure of the octagonal core and magnification of a central element with unequally spaced nodes.

3.3 Material Damage

The damage is represented assigning to each element the effective elastic modules $E^*$, corresponding to the absorbed strain energy density integrated to the whole element. The values $E^*$ assigned to elements are selected with an interval of $E/100$, so that a number $n$ represent the damage level as

$$E^* = E \left( \frac{100-n}{100} \right)$$

4 Numerical Results Discussion

All the numerical simulations have been performed considering a specimen with a single notch having the following dimensions: length
L = 70 cm; height h = 15 cm; thickness t = 15 cm; depth of the notch a = 5 cm.

The concrete mix presents the following relevant mechanical properties: $E = 379000 \text{ kg cm}^{-2}$; $\sigma_u = 24.7 \text{ kg cm}^{-2}$; $\varepsilon_f = 2.58 \times 10^{-3}$; from the previous values, results $\varepsilon_u / E = 0.60 \times 10^{-4}$.

4.1 Mode I

In Fig. 3 is shown the deformed mesh of the beam subject to flexural load condition. Two roller support are symmetrically located at a distance $l = 60 \text{ cm}$ distance and a deflection $\delta$ is imposed on central axis.

The calculations have been performed in a single step, assuming different values of deflection $\delta$, varying from 0.001 cm to 0.012 cm. The ratio $P_d / P_e$, representing the loss of structural strength for damage effect, is plotted in Fig. 4a. The load $P_d$ has been evaluated assuming for each element the effective elastic modulus $E^*$, corresponding to its damage level. The load $P_e$ has been evaluated considering the initial elastic modulus $E$. The values of the crack increment $\Delta a$, produced by the imposed deflection, are plotted in Fig. 4b.

Figure 3: Deformed mesh for three points bending test.

Figure 4: Variation of damage effects and crack growth with applied deflection for Mode I problem.
The maximum circumferential stress given by eqns (1) corresponds to the direction of the notch axis, and also numerical results of computer code are in agreement (as shown by Ballatore [1]). In Fig. 5, for two different values of deflection, the magnifications of elements around the initial notch are displayed with the indication of damage level of each. The damage extension is symmetric, but the most damaged elements are not corresponding to the propagation direction.

\[ \delta = 0.006 \text{ cm} \quad \delta = 0.012 \text{ cm} \]

Figure 5: Damage level in the elements around the crack tip for Mode I problem for two different deflections.

4.2 Mixed Mode

In Fig. 6 is shown the deformed mesh of the beam with a single notch subjected to a four point shear test. The distance between the two shearing forces, skew-symmetric respect the notch axis, is \( c = 10 \text{ cm} \).

The calculations have been performed: (1) in a single step, assuming different values of displacement \( \delta \), varying from 0.001 cm to 0.006 cm; (2) in incremental mode by step of 0.001 cm until a displacement \( \delta = 0.006 \text{ cm} \) has been reached.

The ratio \( P_d/P_e \), representing the loss of structural strength for damage effect, is plotted in Fig. 7a: single step results are represented by continuous line and incremental mode results by dashed one. The values of the crack increment \( \Delta a \), produced by the imposed deflection, are plotted in Fig. 7b, with the same convention of Fig. 7a. The maximum circumferential stress, evaluated by computer code, is always oriented at about 47° to left of the notch axis.

The incremental mode calculation results indicate a relevant
reduction of the structural stiffness and a loss of structural strength compared with the single step results. The crack extension results obtained by the two different calculation techniques are quite similar.

Figure 6: Deformed mesh of beam subjected to four point shear test.

Figure 7: Variation of damage effects and crack growth with applied deflection for Mixed Mode problem.

Figure 8: Damage level in the elements around the crack tip for Mixed Mode problem.
In Fig. 8, for two different values of displacement, the magnifications of elements around the initial notch are displayed, with the indication of damage level of each. The damage extension is not symmetric and is localised only on the side on which the fracture will propagate: the most damaged elements are located on one edge of the notch.

The crack growth, corresponding to the displacement $\delta = 0.006$ cm and evaluated both in a single step and in incremental mode, is displayed in Fig. 9 by the magnification of surrounding elements, with the indication of recalculated damage level.

In single step calculation, the damage is prevailingly localised on a side of crack and the crack length is lightly shorter.

In the incremental simulation, the damage distribution becomes quite similar to the Mode I condition: in effect, after the deviation of about $47^\circ$ from the notch axis at the first step, in the next steps the trajectory is nearly oriented along the axis of existing fracture.

\[
\begin{align*}
\delta = 0.006 \text{ cm in one step} \\
\delta = 0.006 \text{ cm in six steps}
\end{align*}
\]

Figure 9: Damage level in the elements around the crack tip after crack growth for Mixed Mode problem.

References


