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Radiation and scattering of acoustic waves from axisymmetric bodies in a half space using boundary element method

Benjamin Soenarko
Department of Engineering Physics
Bandung Institute of Technology
Jl.Ganesha 10, Bandung 40132
Indonesia
Email: tfitb@melsa.net.id

Abstract

This paper presents a BEM (Boundary Element Method) formulation suitable for solving acoustic radiation and scattering problems involving axisymmetric bodies in a half space. The method is based on the Helmholtz Integral Equation wherein some modifications are introduced by taking advantage of the axisymmetric properties of the body in question. This work proposes some further development to the previous published papers involving axisymmetric bodies in that the present formulation includes the half space phenomenon. By using appropriate Green's function, the integral over the infinite plane is avoided. Then the integral consisting of the acoustic velocity potential and the half space Green's function is reduced to line integral over the generator of the body and an integral over the angle of revolution by virtue of the axisymmetric properties. The integration over the angle is performed partly analytically in terms of elliptic integrals and partly numerically using simple Gaussian quadrature formula. The present formulation is also extended to include axisymmetric bodies mounted in an infinite baffle. A modified geometrical coefficient in the Helmholtz Integral Equation is presented in which the integration is simplified in the same way. Simplification proposed in this paper offers conveniency in the discretization scheme and reduces computing time compared to the three dimensional formulation. Test cases are shown involving spherical and cylindrical shape bodies in which comparison with analytical solution where available or three dimensional calculation is also demonstrated.

1. Introduction

The Boundary Element Method (BEM), as it is known, has been widely developed by many researchers and used extensively for the solution of problems involving radiation and scattering of acoustic waves from arbitrary shape bodies [1-9]. The BEM has a major advantage in that it reduces the dimension of the problem being solved by one. Exploration of the feature of the BEM applicable to various kind of problems has been growing, enhancing its utilization. The problems involving axisymmetric bodies have remarkable simplification on the BEM formulation, in which further reduction of the dimension of the problem is obtained. By taking advantage of the axisymmetric properties of the body, the surface integrals are reduced to line integrals along the generator and integrals over the angle of revolution. Thus the numerical evaluation of the integrals are confined in one dimensional (line) elements on the generator.

In a previous paper we presented a special formulation of the boundary integral equation concerning axisymmetric bodies with axisymmetric boundary conditions [7]. Further development includes the case involving axisymmetric bodies with non-axisymmetric boundary conditions [10]. For this problem, the author presented Fourier series expansion technique to describe the boundary condition and solved the boundary value problem using superposition principle with regard to each term of the series.

The present paper proposes a simplified formulation for problems involving axisymmetric bodies and boundary conditions in a half space. Starting with three dimensional boundary integral equation with appropriate Green's function, the integration over the infinite plane is avoided. Then by using axisymmetric properties of the body the surface integrals are reduced to line integrals along the generator. Following the same mechanism given by Ref.[7], the evaluation of the integrals involves numerical part and analytical part in terms of elliptic integrals. When the body is in contact with the infinite plane a modification of the formulation is needed concerning the geometrical coefficient for the contact points. Using a proper limiting process, a modified expression for the geometrical coefficient of the contact points is obtained. It turns out that this coefficient contains an integral over the contact plane and the surface of the body. On the other hand, the integral involving the acoustic variables in the Helmholtz integral equation does not include the contact plane, but takes only the surface of the body above the infinite plane into account. The proposed formulation can be used for solving problems involving axisymmetric body in a half space where the body is located above the infinite plane or mounted on the infinite plane.

2. The boundary element formulation

Consider an axisymmetric body of surface S_0 with incoming wave ϕ_i in a half space as shown in Fig.1.

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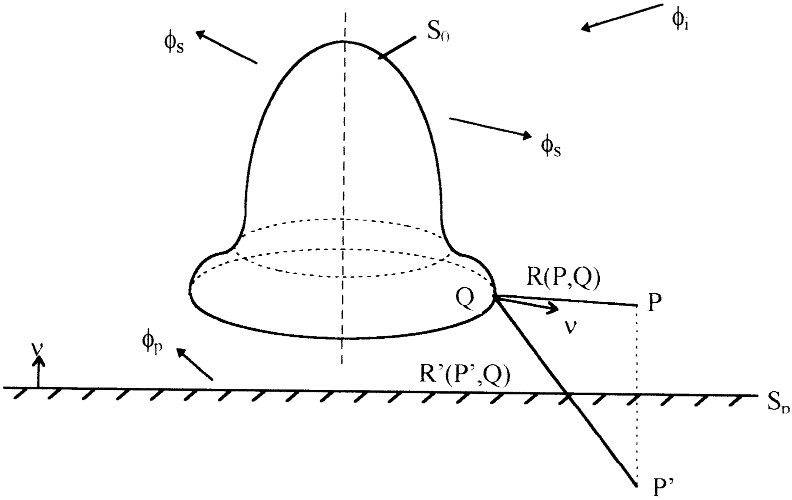

Fig.1: An axisymmetric body in a half space

The scattered wave from the surface S_0 is denoted by ϕ_S, while ϕ_P denotes the reflected wave from the infinite plane S_p, ν is the unit normal of the surface S_0, P' is the image of point P, R(P,Q) is the distance between field point P and point Q on the surface S_0, and R'(P',Q) is the distance between P' and Q. The total velocity potential ϕ_T at any point P is given by [11]:

$$C(P)\phi_T(P) = \int_{S_0}[\phi_T(Q)\frac{\partial\psi_H(P,Q)}{\partial\nu} - \psi_H(P,Q)\frac{\partial\phi_T}{\partial\nu}(Q)]dS(Q) + 4\pi\phi_I(P) \quad (1)$$

where

$$\psi_H(P,Q) = \frac{e^{-ikR(P,Q)}}{R(P,Q)} + R_p\frac{e^{-i'kR(P',Q)}}{R'(P',Q')},$$

$$\phi_I = \phi_i + \phi_p, \quad \phi_T = \phi_s + \phi_I$$

and R_p is the reflection coefficient of S_p, $k=\omega/c$ is the wave number. Omitting subscript T for simplicity, Eq.(1) can be rewritten:

$$\begin{aligned} C(P)\phi(P) = &\int_{S_0}[\phi(Q)\frac{\partial}{\partial\nu}(\frac{e^{-ikR(P,Q)}}{R(P,Q)}) - \frac{e^{-ikR(P,Q)}}{R(P,Q)}\frac{\partial\phi}{\partial\nu}(Q)]dS(Q) \\ &+ \int_{S_0}[\phi(Q)\frac{\partial}{\partial\nu}(R_p\frac{e^{-i'kR(P',Q)}}{R'(P',Q)}) - R_p\frac{e^{-i'kR(P',Q)}}{R'(P',Q)}\frac{\partial\phi}{\partial\nu}]dS(Q) \\ &+ 4\pi\phi_I(P) \end{aligned} \quad (2)$$

The coefficient C(P) has the value 4π for P in the acoustic medium outside of S_0, $C(P) = 0$ for P inside of S_0, $C(P) = 2\pi$ for P on smooth surface S_0, and for P on a general non-smooth surface S_0 where there is no unique tangent at such location of P, the value of C(P) is determined by [11]:

$$C(P) = 4\pi + \int_{S_0} \frac{\partial}{\partial v}\left[\frac{1}{R(P,Q)}\right] dS(Q) \tag{3}$$

When the body is in contact with the infinite plane, Eq.(2) still applies but there are three cases that should be considered concerning the location of the field point P which affects the value of C(P) in Eq.(2). The first case is the situation where the field point P is located outside of S_0. For this case, it can be shown that C(P) has the value 4π. The second case includes the situation where P is located on the surface S_0, wherein the portion S_B of the body is in contact with the infinite plane. Using a proper limiting process [12] the following result for C(P) is obtained:

$$C(P) = 4\pi + \int_{S_0+S_B} \frac{\partial}{\partial v}\left(\frac{1}{R(P,Q)}\right) dS(Q) \tag{4}$$

The most crucial situation is the case wherein the field point P is at the point of contact between S_0 and S_B. By taking a limiting process involving S_0, the infinite plane S_p, the semi infinite boundary of the acoustic medium, a small spherical surface surrounding P, and using the Green's identity with Helmholtz differential equation, the following result is obtained [12]:

$$C(P) = (1 + R_p)\left[2\pi + \int_{S_0+S_B} \frac{\partial}{\partial v}\left(\frac{1}{R(P,Q)}\right) dS(Q)\right] \tag{5}$$

It must be noted that the integral calculation for the determination of C(P) includes the whole surface of the body (portion of the body above the infinite plane and the contact plane of the body and the infinite plane), whereas the integral involving the acoustic variables (the right hand side of Eq.(2)) includes only the portion of the body above the infinite plane.

2.1. Axisymmetric formulation

Using cylindrical coordinate system (ρ,θ,ϕ), Eq.(2) becomes:

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$$
\begin{aligned}
C(P)\phi(P) = & \int_L \phi(Q)\Big[\int_0^{2\pi} \frac{\partial}{\partial \nu}\Big(\frac{e^{-ikR(P,Q)}}{R(P,Q)}\Big)d\theta(Q)\Big]\rho(Q)dL(Q) \\
& - \int_L \frac{\partial \phi(Q)}{\partial \nu}(Q)\Big[\int_0^{2\pi}\Big(\frac{e^{-ikR(P,Q)}}{R(P,Q)}\Big)d\theta(Q)\Big]\rho(Q)dL(Q) \\
& + \int_L \phi(Q)\Big[\int_0^{2\pi} \frac{\partial}{\partial \nu}\Big(R_p \frac{e^{-ikR'(P',Q)}}{R'(P',Q)}\Big)d\theta(Q)\Big]\rho(Q)dL(Q) \\
& - \int_L \frac{\partial \phi}{\partial \nu}(Q)\Big[\int_0^{2\pi} R_p \frac{e^{-ikR'(P',Q)}}{R'(P',Q)} d\theta(Q)\Big]\rho(Q)dL(Q) + 4\pi\phi_I(P)
\end{aligned} \tag{6}
$$

For the purpose of evaluating Eq.(6) define

$$
\begin{aligned}
& K^A(P,Q) = \int_0^{2\pi} \frac{e^{-ikR(P,Q)}}{R(P,Q)} d\theta(Q), K^B(P,Q) = \int_0^{2\pi} \frac{\partial}{\partial \nu}\Big(\frac{e^{-ikR(P,Q)}}{R(P,Q)}\Big)d\theta(Q), \\
& \overline{K}^A(P',Q) = \int_0^{2\pi} R_p \frac{e^{-ikR'(P',Q)}}{R'(P',Q)} d\theta(Q), \overline{K}^B(P',Q) = \int_0^{2\pi} \frac{\partial}{\partial \nu}\Big(R_p \frac{e^{-ikR(P',Q)}}{R'(P',Q)}\Big)d\theta(Q)
\end{aligned} \tag{7}
$$

so that Eq.(6) can be expressed as:

$$
\begin{aligned}
C(P)\phi(P) = & \int_L [\phi(Q) K^B(P,Q) - \frac{\partial \phi(Q)}{\partial \nu} K^A(P,Q)]\rho(Q)dL(Q) \\
& + \int_L [\phi(Q) \overline{K}^B(P',Q) - \frac{\partial \phi(Q)}{\partial \nu} \overline{K}^A(P',Q)]\rho(Q)dL(Q) \\
& + 4\pi\phi_I(P)
\end{aligned} \tag{8}
$$

Following the same procedure described in Ref.[7] to remove singularities, it can be shown that evaluations of $K^A(P,Q)$, $K^B(P,Q)$, $\overline{K}^A(P',Q)$, $\overline{K}^B(P',Q)$ and $C(P)$ can be performed partly numerically and partly analytically in terms of elliptic integrals. Equation (8) is the boundary integral equation or the BEM formulation applicable to any axisymmetric body in a half space.

3. Test cases

To verify the proposed formulation a number of test cases were run. The first test case is the problem of acoustic radiation from a hemispherical body of radius A mounted on an infinite baffle. The body is vibrating uniformly over its surface. The baffle is assumed to be rigid. The result of the proposed axisymmetric formulation is shown in Fig.2, in which comparison with analytical solution proposed by Ref.[13] and the three dimensional conventional BEM result is presented. It can be seen that good agreement was obtained between those results.

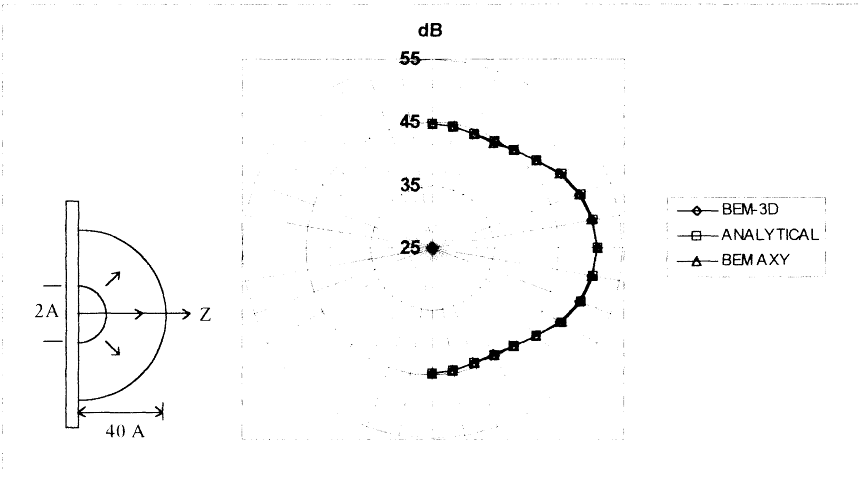

Fig.2: Radiation pattern of a hemispherical body on an infinite baffle at kA=3 plotted at radius 40A (BEM AXY: present formulation).

The second test case considered is radiation from a finite cylinder mounted on a rigid infinite baffle. The radius of the cylinder is A=5cm and its height is L=5cm. A uniform radial velocity is prescribed on the (curved) lateral surface of the cylinder. The ends of the cylinder are motionless. This problem does not have analytical solution available. The result of the present formulation is depicted in Fig.3 where three dimensional BEM result is also shown for comparison. It can be observed that good agreement between the two results was obtained.

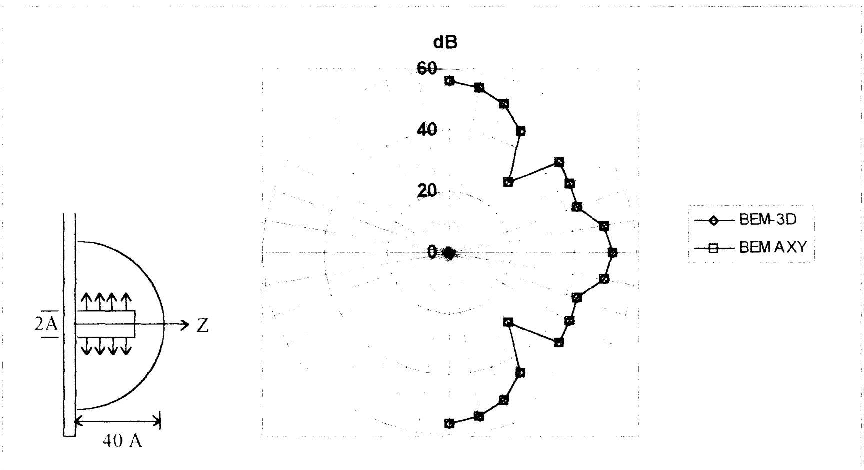

Fig.3: Radiation from a cylinder in an infinite baffle at kA=1 plotted at radius 40A (BEM AXY: present formulation)

The next test case considered is the scaterring from a hemispherical dome in an infinite baffle. The source of the incoming wave is a ring source on the baffle surrounding the dome. The dome and the baffle are assumed to be rigid. The result of the present formulation is shown in Fig.4 along with the analytical solution proposed by Ref.[14]. It can be observed that the axisymmetric BEM result agrees very well with the analytical solution.

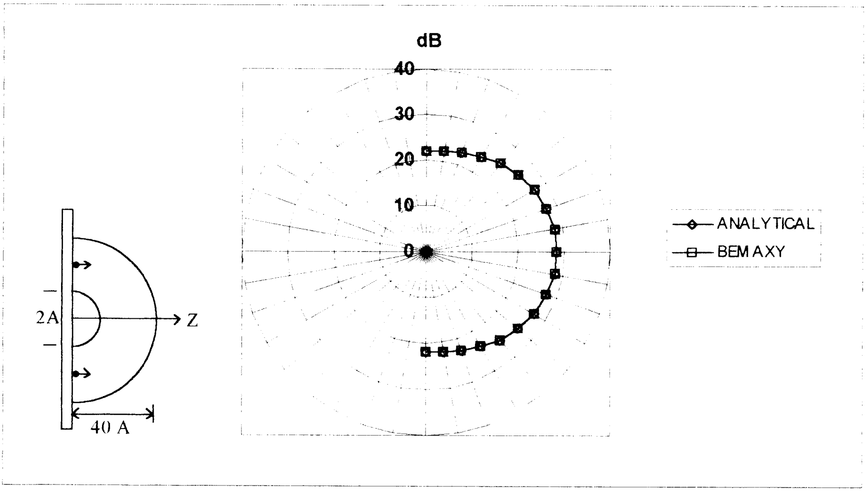

Fig.4: Scattering from a hemispherical dome in an infinite baffle with a ring source at kA=1, plotted at radius 40A (BEM AXY: present formulation)

The scattering problem is also run using a plane wave whose direction is parallel with the axis of symmetry as the incident wave. This problem is solved using the present axisymmetric formulation and the three dimensional BEM formulation. Figure 5 depicts the polar plot at a distance of 40 radii from the center of a hemispherical dome mounted in an infinite baffle, in which good agreement between the two solutions were obtained.

The last test case is the problem of scattering of a plane wave from a circular cylinder in an infinite baffle. The direction of the plane wave is parallel with the axis of symmetry. Figure 6 shows the result of the proposed formulation along with the result of the three dimensional BEM calculation. It can be seen that both solutions yield good agreement. Comparing Fig.6 and Fig.5 one may observe that in the area beyond 20 degrees from the axis of symmetry the scattered wave is dominated by the effect of the reflection from the infinite plane.

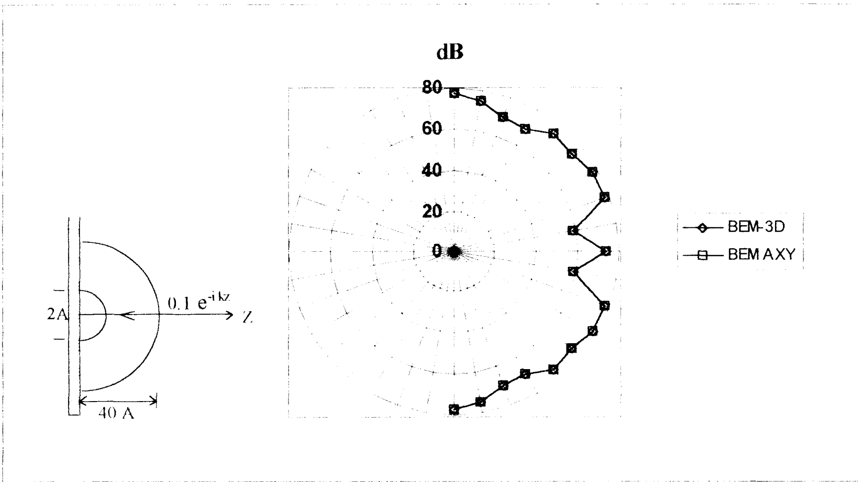

Fig.5: Scattering of a plane from a hemispherical dome in an infinite baffle at kA=1, plotted at radius 40A (BEM AXY: present formulation)

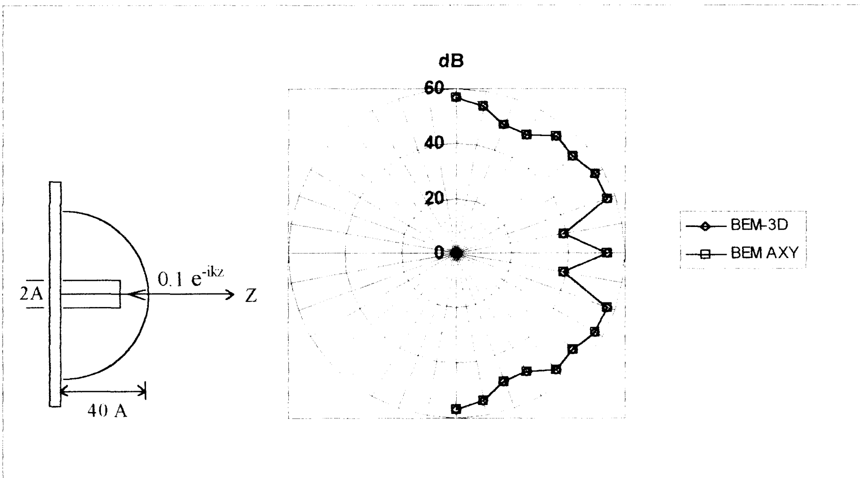

Fig.6: Scattering of a plane wave from a cylinder in an infinite baffle at kA=1 plotted at radius 40A (BEM AXY: present formulation)

4. Summary

A boundary element formulation involving axisymmetric body in a half space has been presented. By using axisymmetric property of the body the surface integral is reduced to line integral along the generator which simplifies the evaluation. The half space formulation is obtained using appropriate Green's function such that the integral over the infinite plane is avoided, in which the image of the field point is taken into account. It turns out the evaluation of the line integral can be performed partly analytically in terms of elliptic integral and partly numerically using Gaussian quadrature. Example problems were shown involving spherical and cylindrical bodies in an infinite baffle. Good agreements were obtained between the results of the proposed formulation and the available analytical solution as well as the three dimensional BEM formulation. The proposed formulation can be used for problems involving axisymmetric bodies in a half space such as acoustic barrier or sound radiator in a wall.

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Key words: boundary element, axisymmetric, half plane

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