Stress and displacement discontinuity boundary elements in an anisotropic medium

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Abstract

Functions, which permit us to determine stresses and displacements at an arbitrary point in infinite anisotropic elastic medium due to different three-dimensional (3D) stress or displacement discontinuities that are distributed on flat infinite band-type boundary elements, are presented. In case of transversely isotropic medium the formulae represent the closed form of the solutions for arbitrary oriented boundary elements. The analytical functions can be used in the Indirect Boundary Element Method (IBEM) and the Displacement Discontinuity Method (DDM) computer codes to solve the generalized plane strain problems with 3D boundary conditions, and elastic homogeneous medium with arbitrary anisotropy may be considered.

1 Introduction

One of the main steps for creating the IBEM or the DDM computer codes is the integration of concentrated forces or dipoles on boundary elements. Because the solution for a concentrated force or a dipole has a singularity at the point of application, usually the numerical integration of concentrated forces or dipoles on boundary elements leads to non satisfactory results near boundaries. It is possible to improve results by using the analytical integration that was carried out in the papers [2-7] and in the books [1, 8, 14] for different types of stress and displacement discontinuity elements. These analytical influence functions produce distinct advantages over most other approaches employing numerical integration (the analytical influence functions are exact while unknown errors occur during numerical integration; storage requirement is lowed and computation speed is raised; analytical influence functions can be examined exactly). The result of the study herein is a generalization of the solutions mentioned above. The new analytical functions can be used to solve the generalized plane strain problems with 3D boundary conditions, and elastic homogeneous medium with arbitrary anisotropy may be considered. The functions are based on S.G.Lekhnitskii's [12] analytical solution, and they can be used in the IBEM and the DDM computer codes for geomechanics, for mechanics of composite materials, for optimal structural design, etc.

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2 Single concentrated force

Let us consider homogeneous, infinite, linear elastic, anisotropic medium without body forces and infinite straight line L in this medium. The line L is parallel to the axis Oz of the Cartesian system of coordinates $Oxyz$. (Figure 1 (a)). The line L is loaded by 3-dimensional force $\mathbf{f}\ \{ f_x, f_y, f_z \}$ that, being evenly distributed along the line L, does not depend on the coordinate z. Here $\mathbf{f}$ is the force acting at every part of such a line L with the unit length; f_x, f_y, f_z are projections of the vector $\mathbf{f}$ on axes Ox, Oy and Oz, respectively. Since for prearranged conditions any plane xy, perpendicular to axis Oz, is at the same state, it is possible to consider the plane Oxy only. In this case we can call the force $\mathbf{f}$ as a concentrated force applied at an arbitrary point $C(x_C, y_C)$ of the continuous infinite plane Oxy (Figure 1 (b)). The infinite line L mentioned above contains the point C.

The known formulae [9, 12, 13] can be used for determining stresses and displacements at an arbitrary point $W(x_W, y_W)$ of the anisotropic plane Oxy (Figure 1 (b)) due to the concentrated force $\mathbf{f}\ \{ f_x, f_y, f_z \}$ applied at an arbitrary point $C(x_C, y_C)$:

$$\sigma_x = 2\mathrm{Re}[\mu_1^2\Sigma_1 + \mu_2^2\Sigma_2 + \mu_3^2\lambda_3\Sigma_3], \quad \sigma_y = 2\mathrm{Re}[\Sigma_1 + \Sigma_2 + \lambda_3\Sigma_3],$$

$$\sigma_z = -(a_{13}\sigma_x + a_{23}\sigma_y + a_{34}\tau_{yz} + a_{35}\tau_{xz} + a_{36}\tau_{xy}) / a_{33}, \qquad (1)$$

$$\tau_{yz} = -2\mathrm{Re}[\lambda_1\Sigma_1 + \lambda_2\Sigma_2 + \Sigma_3], \quad \tau_{xz} = 2\mathrm{Re}[\mu_1\lambda_1\Sigma_1 + \mu_2\lambda_2\Sigma_2 + \mu_3\Sigma_3],$$

$$\tau_{xy} = -2\mathrm{Re}[\mu_1\Sigma_1 + \mu_2\Sigma_2 + \mu_3\lambda_3\Sigma_3],$$

$$u_x = 2\mathrm{Re}\sum_{k=1}^{3} p_k\Pi_k, \quad u_y = 2\mathrm{Re}\sum_{k=1}^{3} q_k\Pi_k, \quad u_z = 2\mathrm{Re}\sum_{k=1}^{3} r_k\Pi_k.$$

Here Re[ ] means real part of the complex number between brackets; Π_j and Σ_j are complex functions

$$\Pi_j = A_j \ln(\omega_{Wj} - \omega_{Cj}), \qquad \Sigma_j = \frac{A_j}{\omega_{Wj} - \omega_{Cj}}, \qquad (2)$$

$$\omega_{Wj} = x_W + \mu_j y_W, \quad \omega_{Cj} = x_C + \mu_j y_C \quad (j = 1, 2, 3).$$

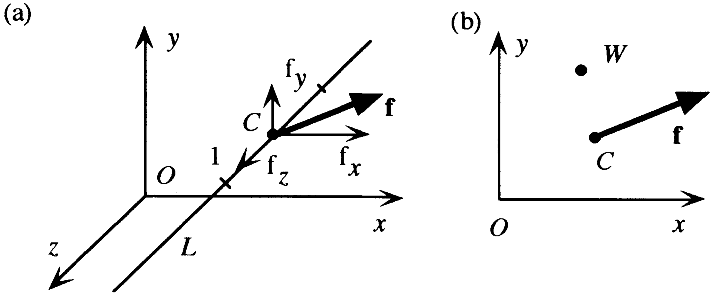

Figure 1: Calculation scheme: (a) isometric view and (b) points C and W.

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Coefficients A_j are the solution of the following system of linear equations

$$\begin{bmatrix} 1 & 1 & \lambda_3 & -1 & -1 & -\bar{\lambda}_3 \\ \mu_1 & \mu_2 & \mu_3\lambda_3 & -\bar{\mu}_1 & -\bar{\mu}_2 & -\bar{\mu}_3\bar{\lambda}_3 \\ \lambda_1 & \lambda_2 & 1 & -\bar{\lambda}_1 & -\bar{\lambda}_2 & -1 \\ p_1 & p_2 & p_3 & -\bar{p}_1 & -\bar{p}_2 & -\bar{p}_3 \\ q_1 & q_2 & q_3 & -\bar{q}_1 & -\bar{q}_2 & -\bar{q}_3 \\ r_1 & r_2 & r_3 & -\bar{r}_1 & -\bar{r}_2 & -\bar{r}_3 \end{bmatrix} \cdot \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \\ \bar{A}_1 \\ \bar{A}_2 \\ \bar{A}_3 \end{Bmatrix} = \frac{i}{2\pi} \begin{Bmatrix} -f_y \\ f_x \\ f_z \\ 0 \\ 0 \\ 0 \end{Bmatrix}, \quad (3)$$

where i is imaginary unit; π=3.14...; the bar denotes the complex conjugate.

In the formulae (1), all parameters with the exception roots μ_j of the sixth order equation can be defined analytically [9, 12, 13]. Parameters μ_j can be calculated by using known numerical methods when a medium with an arbitrary anisotropy is considered. In case of a transversely isotropic medium, the formulae represent the closed form of the solutions for an arbitrary oriented line L.

3 Displacement discontinuity point cells

In the paper, any element looks like an infinite flat band that is parallel to the axis Oz (Figure 2 (a)). The loads **g** or the displacement discontinuities $\boldsymbol{D}$ being distributed on the band do not depend on the coordinate z. The straight segment AB belongs to both the band and the plane Oxy. Further in the paper we will call the segment AB as an element, however, keeping in mind that the loads **g** or displacement discontinuities $\boldsymbol{D}$ are distributed on the infinite band.

To construct point displacement discontinuity cells we can use the same method as in the paper [5]. Let us distinguish 3 such cells: the first one provides the displacement discontinuity D_ξ ($D_\eta = 0$, $D_z = 0$) at the point C of the cell action on the element AB (Figure 2 (b)); the second one provides the displacement discontinuity D_η ($D_\xi = 0$, $D_z = 0$) and the last provides the displacement discontinuity D_z ($D_\xi = 0$, $D_\eta = 0$).

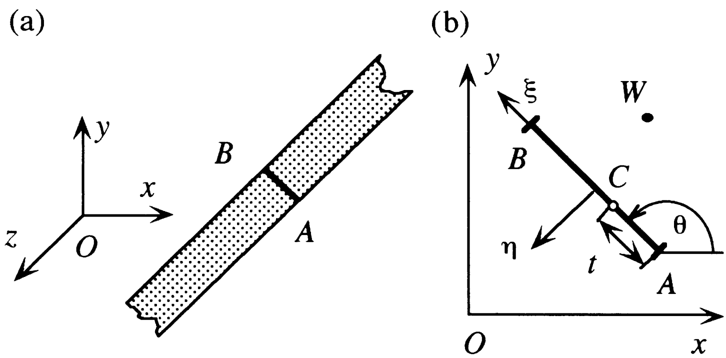

Figure 2: An isometric view on the infinite band (a) and the element AB on the plane Oxy (b).

Omitting some intermediate steps we can write down the final expressions for the complex functions Π_{Dj} and Σ_{Dj}. These functions after substituting them into eqns (1) instead of Π_j and Σ_j allow the definition of stresses and displacements at an arbitrary point W due to the point displacement discontinuity (DD) cells actions at the point C on the element AB

$$\Pi_{Dj} = \sum_{\alpha=\xi,\,\eta,\,z} \frac{B_{\alpha j}\mu_j + C_{\alpha j}}{\omega_{Wj} - \omega_{Cj}} \cdot D_\alpha, \quad \Sigma_{Dj} = -\sum_{\alpha=\xi,\,\eta,\,z} \frac{B_{\alpha j}\mu_j + C_{\alpha j}}{(\omega_{Wj} - \omega_{Cj})^2} \cdot D_\alpha, \tag{4}$$

$$\omega_{Wj} = x_W + \mu_j y_W, \quad \omega_{Cj} = x_C + \mu_j y_C \quad (\alpha = \xi,\ \eta,\ z;\ j = 1,\ 2,\ 3).$$

Coefficients $B_{\alpha j}$ and $C_{\alpha j}$ are defined from the solutions of the system (3), where values f_x, f_y, f_z are substituted by $Q_{y\,y\alpha}$, $Q_{x\,y\alpha}$, $Q_{z\,y;\alpha}$ and $Q_{x\,y\alpha}$, $Q_{x\,x;\alpha}$, $Q_{z\,x;\alpha}$, respectively ($\alpha = \xi,\ \eta,\ z$). The last values are defined as the solutions of the following systems of linear equations.

Point ξ-DD cell. $D_\alpha = D_\xi, D_\eta = D_z = 0$ in the expression (4).

$$\begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{14} & \beta_{15} & \beta_{16} \\ \beta_{12} & \beta_{22} & \beta_{24} & \beta_{25} & \beta_{26} \\ \beta_{14} & \beta_{24} & \beta_{44} & \beta_{45} & \beta_{46} \\ \beta_{15} & \beta_{25} & \beta_{45} & \beta_{55} & \beta_{56} \\ \beta_{16} & \beta_{26} & \beta_{46} & \beta_{56} & \beta_{66} \end{bmatrix} \cdot \begin{Bmatrix} Q_{x\,x\xi} \\ Q_{y\,y\xi} \\ Q_{z\,y;\xi} \\ Q_{z\,x;\xi} \\ Q_{x\,y\xi} \end{Bmatrix} = \begin{Bmatrix} -\sin\theta\cos\theta \\ \sin\theta\cos\theta \\ 0 \\ 0 \\ \cos^2\theta - \sin^2\theta \end{Bmatrix}. \tag{5}$$

Point η-DD cell. $D_\alpha = D_\eta$, $D_\xi = D_z = 0$ in the expression (4).

$$\begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{14} & \beta_{15} & \beta_{16} \\ \beta_{12} & \beta_{22} & \beta_{24} & \beta_{25} & \beta_{26} \\ \beta_{14} & \beta_{24} & \beta_{44} & \beta_{45} & \beta_{46} \\ \beta_{15} & \beta_{25} & \beta_{45} & \beta_{55} & \beta_{56} \\ \beta_{16} & \beta_{26} & \beta_{46} & \beta_{56} & \beta_{66} \end{bmatrix} \cdot \begin{Bmatrix} Q_{x\,x\eta} \\ Q_{y\,y\eta} \\ Q_{z\,y;\eta} \\ Q_{z\,x;\eta} \\ Q_{x\,y\eta} \end{Bmatrix} = \begin{Bmatrix} \sin^2\theta \\ \cos^2\theta \\ 0 \\ 0 \\ -2\sin\theta\cos\theta \end{Bmatrix}. \tag{6}$$

Point z-DD cell. $D_\alpha = D_z$, $D_\xi = D_\eta = 0$ in the expression (4).

$$\begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{14} & \beta_{15} & \beta_{16} \\ \beta_{12} & \beta_{22} & \beta_{24} & \beta_{25} & \beta_{26} \\ \beta_{14} & \beta_{24} & \beta_{44} & \beta_{45} & \beta_{46} \\ \beta_{15} & \beta_{25} & \beta_{45} & \beta_{55} & \beta_{56} \\ \beta_{16} & \beta_{26} & \beta_{46} & \beta_{56} & \beta_{66} \end{bmatrix} \cdot \begin{Bmatrix} Q_{x\,x\,z} \\ Q_{y\,y\,z} \\ Q_{z\,y;z} \\ Q_{z\,x;z} \\ Q_{x\,y\,z} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \cos\theta \\ -\sin\theta \\ 0 \end{Bmatrix}. \tag{7}$$

Here θ is the angle between the axes Ox and η (Figure 2 (b)); $\beta_{ij} = a_{ij} - a_{i3}a_{j3}/a_{33}$ ($i, j = 1, 2, 4, 5, 6;\ a_{ij} = a_{ji}$); values a_{ij} are rigidity coefficients from the generalized Hook's law.

4 Stress discontinuity elements

The results presented in this Section were produced by integrating single concentrated forces actions (ref. Section 2) on the elements. Some details of the integration method and numerical examples can be found in the book [1] and in the papers [10, 11]. In the paper, t is the distance between point A and point C on the element (Figure 2 (b)), L_{AB} is the length of the segment AB and parameters ω_{Wj}, ω_{Aj}, ω_{Bj}, Λ_j, Ω_j are defined as it follows

$$\omega_{Wj} = x_W + \mu_j y_W \ , \quad \omega_{Aj} = x_A + \mu_j y_A \ , \quad \omega_{Bj} = x_B + \mu_j y_B \ , \tag{8}$$

$$\Lambda_j = \frac{\omega_{Wj} - \omega_{Bj}}{\omega_{Wj} - \omega_{Aj}} \ , \qquad \Omega_j = \frac{\omega_{Wj} - \omega_{Aj}}{\omega_{Bj} - \omega_{Aj}} \ , \qquad (j = 1,\ 2,\ 3) \ ,$$

where x_W, y_W, x_A, y_A and x_B, y_B are coordinates of the points W, A and B, respectively.

4.1 Polynomially Distributed Stress Discontinuities

The load $\mathbf{g}$ changes as polynomial function along the segment AB and at point C on the element,

$$\mathbf{g}(t) = \sum_{k=0}^{m} \mathbf{g}_k \cdot (t / L_{AB})^k \ , \qquad \mathbf{g}_k \equiv \mathbf{g}_k \ \{ g_{xk},\ g_{yk},\ g_{zk} \} = \text{const} \ , \tag{9}$$

where t is the distance between point A and point C; constants g_{xk}, g_{yk} and g_{zk} $(k = 0,\ 1,\ 2,\ \ldots\ ,\ m)$ are projections of the vectors $\mathbf{g}_k$ on axes Ox, Oy and Oz, respectively.

Complex functions Π_j and Σ_j in the eqns (1) are

$$\Pi_j = \sum_{k=0}^{m} \frac{A_{jk}}{k+1} \left\{ \ln\Lambda_j + \ln\Omega_j + \ln(\omega_{Bj} - \omega_{Aj}) - \Omega_j^{k+1} \left[\ln\Lambda_j + \sum_{l=1}^{k+1} \frac{\Omega_j^{-l}}{l} \right] \right\} ,$$

$$\Sigma_j = \frac{1}{\omega_{Bj} - \omega_{Aj}} \sum_{k=0}^{m} A_{jk} \left[\frac{1}{k+1} \cdot \frac{1}{\Omega_j} - \Omega_j^k \left(\ln\Lambda_j + \sum_{l=1}^{k+1} \frac{\Omega_j^{-l}}{l} \right) \right] .$$

Coefficients $A_{jk} = A_j$ $(j = 1,\ 2,\ 3;\ k = 0,\ 1,\ \ldots\ ,\ m)$ are defined from the solutions of the system (3), where values f_x , f_y , f_z are substituted by $f_{xk} = L_{AB} \cdot g_{xk}$, $f_{yk} = L_{AB} \cdot g_{yk}$, $f_{zk} = L_{AB} \cdot g_{zk}$, respectively ($k$=0, ... , m).

4.1.1 Constant Stress Discontinuity Element

In the formula (9) $\mathbf{g}(t) = \mathbf{g}_0 \equiv \mathbf{g}_0 \ \{ g_{x0} \ , g_{y0} \ , g_{z0} \} = \text{const}$.

Complex functions Π_j and Σ_j in the eqns (1) are

$$\Pi_j = A_{j0}[\ln(\omega_{Bj} - \omega_{Aj}) + \ln\Omega_j + (1 - \Omega_j)\ln\Lambda_j - 1] \ , \quad \Sigma_j = -\frac{A_{j0}}{\omega_{Bj} - \omega_{Aj}} \ln\Lambda_j \ .$$

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4.1.2 Linear Stress Discontinuity Element

In the formula (9) $\mathbf{g}(t) = (t / L_{AB}) \cdot \mathbf{g}_1$, $\mathbf{g}_1 \equiv \mathbf{g}_1 \{ g_{x1} , g_{y1} , g_{z1} \} = \text{const}$.

Complex functions Π_j and Σ_j in the eqns (1) are

$$\Pi_j = \frac{A_{j1}}{2}\left[\ln(\omega_{Bj} - \omega_{Aj}) + \ln\Omega_j + (1-\Omega_j^2)\ln\Lambda_j - \Omega_j - \frac{1}{2}\right],$$

$$\Sigma_j = -\frac{A_{j1}}{\omega_{Bj} - \omega_{Aj}}\,(\Omega_j \ln\Lambda_j + 1)\ .$$

4.1.3 Parabolic Stress Discontinuity Element

In the formula (9) $\mathbf{g}(t) = (t / L_{AB})^2 \cdot \mathbf{g}_2$, $\mathbf{g}_2 \equiv \mathbf{g}_2 \{ g_{x2}, g_{y2}, g_{z2} \} = \text{const}$.

Complex functions Π_j and Σ_j in the eqns (1) are

$$\Pi_j = \frac{A_{j2}}{3}\left[\ln(\omega_{Bj} - \omega_{Aj}) + \ln\Omega_j + \ln\Lambda_j - \Omega_j^3\ln\Lambda_j - \Omega_j^2 - \frac{\Omega_j}{2} - \frac{1}{3}\right],$$

$$\Sigma_j = -\frac{A_{j2}}{\omega_{Bj} - \omega_{Aj}}\left(\Omega_j^2\ln\Lambda_j + \Omega_j + \frac{1}{2}\right).$$

4.1.4 *k*-order (*k*>2) stress discontinuity element

In the formula (9) $\mathbf{g}(t) = (t / L_{AB})^k \cdot \mathbf{g}_k$, $\mathbf{g}_k \equiv \mathbf{g}_k \{ g_{xk}\ g_{yk}\ g_{zk} \} = \text{const}$.

Complex functions Π_j and Σ_j in the eqns (1) are

$$\Pi_j = \frac{A_{jk}}{k+1}\left[\ln\Lambda_j + \ln\Omega_j + \ln(\omega_{Bj} - \omega_{Aj}) - \Omega_j^{k+1}\ln\Lambda_j - \sum_{l=1}^{k}\frac{\Omega_j^{k+1-l}}{l} - \frac{1}{k+1}\right],$$

$$\Sigma_j = -\frac{A_{jk}}{\omega_{Bj} - \omega_{Aj}}\left(\Omega_j^k \ln\Lambda_j + \sum_{l=1}^{k-1}\frac{\Omega_j^{k-l}}{l} + \frac{1}{k}\right).$$

4.2 Reverse root stress discontinuity element

For the element, $\mathbf{g}(t) = \mathbf{g}_{rr}\sqrt{\dfrac{L_{AB}}{L_{AB} - t}}$, $\mathbf{g}_{rr} \equiv \mathbf{g}_{rr} \{ g_{xrr} , g_{yrr} , g_{zrr} \} = \text{const}$.

Complex functions Π_j and Σ_j in the eqns (1) are

$$\Pi_j = 2A_{jrr}\left[\ln(\omega_{Bj} - \omega_{Aj}) + \ln\Omega_j + \sqrt{1-\Omega_j}\ln\frac{\sqrt{1-\Omega_j}+1}{\sqrt{1-\Omega_j}-1} - 2\right],$$

$$\Sigma_j = \frac{A_{jrr}}{\omega_{Bj} - \omega_{Aj}} \cdot \frac{1}{\sqrt{1-\Omega_j}} \cdot \ln\frac{\sqrt{1-\Omega_j}-1}{\sqrt{1-\Omega_j}+1}\ .$$

Coefficients $A_{jrr} = A_j$ ($j = 1, 2, 3$) are defined from the solutions of the system (3), where values f_x, f_y, f_z are substituted by $f_{xrr} = L_{AB} \cdot g_{xrr}$, $f_{yrr} = L_{AB} \cdot g_{yrr}$, $f_{zrr} = L_{AB} \cdot g_{zrr}$, respectively.

5 Displacement discontinuity elements

The results presented in this Section were produced by integrating displacement discontinuity point cells actions (ref. Section 3) on the elements. In the Section coefficients $B_{\alpha j}$ and $C_{\alpha j}$ are defined by using the solutions of the systems of equations (5) - (7), $j = 1, 2, 3$ and $\alpha = \xi, \eta, z$.

5.1 Polynomially Distributed Displacement Discontinuities

The displacement discontinuity $\boldsymbol{D}$ $\{D_\xi, D_\eta, D_z\}$ changes as polynomial function along the segment AB and at point $C(x_C, y_C)$ on the element,

$$\boldsymbol{D}(t) = \sum_{k=0}^{m} \boldsymbol{D}_k (t / L_{AB})^k, \ \boldsymbol{D}_k \equiv \boldsymbol{D}_k \{ D_{\xi k}, D_{\eta k}, D_{zk} \} = \text{const} , \tag{10}$$

where t is the distance between point A and point C; constants $D_{\xi k}$, $D_{\eta k}$ and D_{zk} $(k = 0, 1, 2, \dots, m)$ are displacement discontinuities in directions ξ, η and z, respectively.

Complex functions Π_j and Σ_j in the eqns (1) are

$$\Pi_j = \frac{L_{AB}}{\omega_{Bj} - \omega_{Aj}} \sum_{\alpha=\xi,\eta,z} (B_{\alpha j}\mu_j + C_{\alpha j}) \times$$
$$\times \sum_{k=0}^{m} \Omega_j^k \left(\frac{\Omega_j^{-k-1}}{k+1} - \ln\Lambda_j - \sum_{l=1}^{k+1} \frac{\Omega_j^{-l}}{l} \right) \cdot D_{\alpha k} \quad ,$$

$$\Sigma_j = -\frac{L_{AB}}{(\omega_{Bj} - \omega_{Aj})^2} \sum_{\alpha=\xi,\eta,z} (B_{\alpha j}\mu_j + C_{\alpha j}) \times$$
$$\times \sum_{k=0}^{m} \Omega_j^{k-2} \left[\begin{array}{l} k\Omega_j \ln\Lambda_j - \sum_{l=1}^{k+1} \left(1 - \frac{k}{l+1}\right) \Omega_j^{-l} + \\ + \frac{1}{\Lambda_j} + k - 1 + \Omega_j^{-k} \sum_{l=1}^{2} \frac{l}{k+l} \cdot \Omega_j^{1-l} \end{array} \right] \cdot D_{\alpha k} \ .$$

5.1.1 Constant displacement discontinuity element

In the formula (10) $\boldsymbol{D}(t) = \boldsymbol{D}_0 \equiv \boldsymbol{D}_0 \{ D_{\xi 0}, D_{\eta 0}, D_{z 0} \} = \text{const}$.

Complex functions Π_j and Σ_j in the eqns (1) are

$$\Pi_j = -\frac{L_{AB}}{\omega_{Bj} - \omega_{Aj}} \ln\Lambda_j \sum_{\alpha=\xi,\eta,z} (B_{\alpha j}\mu_j + C_{\alpha j}) D_{\alpha 0} \quad ,$$

$$\Sigma_j = -\frac{L_{AB}}{(\omega_{Bj} - \omega_{Aj})^2} \cdot \frac{1}{\Lambda_j \Omega_j^2} \sum_{\alpha=\xi,\eta,z} (B_{\alpha j}\mu_j + C_{\alpha j}) D_{\alpha 0} \quad .$$

5.1.2 Linear displacement discontinuity element

In the formula (10) $\boldsymbol{D}(t) = \boldsymbol{D}_1(t / L_{AB})$, $\boldsymbol{D}_1 \equiv \boldsymbol{D}_1 \{D_{\xi 1}, D_{\eta 1}, D_{z1}\} = \text{const}$.

Complex functions Π_j and Σ_j in the eqns (1) are

$$\Pi_j = -\frac{L_{AB}}{\omega_{Bj} - \omega_{Aj}}(\Omega_j \ln\Lambda_j + 1) \sum_{\alpha=\xi,\eta,z}(B_{\alpha j}\mu_j + C_{\alpha j})D_{\alpha 1},$$

$$\Sigma_j = -\frac{L_{AB}}{(\omega_{Bj} - \omega_{Aj})^2}\left(\ln\Lambda_j + \frac{1}{\Lambda_j \Omega_j}\right) \sum_{\alpha=\xi,\eta,z}(B_{\alpha j}\mu_j + C_{\alpha j})D_{\alpha 1}.$$

5.1.3 Parabolic displacement discontinuity element

In the formula (10) $\boldsymbol{D}(t) = \boldsymbol{D}_2(t / L_{AB})^2$, $\boldsymbol{D}_2 \equiv \boldsymbol{D}_2 \{D_{\xi 2}, D_{\eta 2}, D_{z2}\} = \text{const}$.

Complex functions Π_j and Σ_j in the eqns (1) are

$$\Pi_j = -\frac{L_{AB}}{\omega_{Bj} - \omega_{Aj}}\left(\Omega_j^2 \ln\Lambda_j + \Omega_j + \frac{1}{2}\right) \sum_{\alpha=\xi,\eta,z}(B_{\alpha j}\mu_j + C_{\alpha j})D_{\alpha 2},$$

$$\Sigma_j = -\frac{L_{AB}}{(\omega_{Bj} - \omega_{Aj})^2}\left(2\Omega_j \ln\Lambda_j + \frac{1}{\Lambda_j} + 1\right) \sum_{\alpha=\xi,\eta,z}(B_{\alpha j}\mu_j + C_{\alpha j})D_{\alpha 2}.$$

5.1.4 *k*-order (*k*>2) displacement discontinuity element

In the formula (10) $\boldsymbol{D}(t) = \boldsymbol{D}_k(t / L_{AB})^k$, $\boldsymbol{D}_k \equiv \boldsymbol{D}_k \{D_{\xi k}, D_{\eta k}, D_{zk}\} = \text{const}$.

Complex functions Π_j and Σ_j in the eqns (1) are

$$\Pi_j = -\frac{L_{AB}}{\omega_{Bj} - \omega_{Aj}} \cdot \left(\Omega_j^k \ln\Lambda_j + \sum_{l=1}^{k-1}\frac{\Omega_j^{k-l}}{l} + \frac{1}{k}\right) \sum_{\alpha=\xi,\eta,z}(B_{\alpha j}\mu_j + C_{\alpha j})D_{\alpha k},$$

$$\Sigma_j = -\frac{L_{AB}}{(\omega_{Bj} - \omega_{Aj})^2} \cdot \left(\begin{array}{l} k\,\Omega_j^{k-1}\ln\Lambda_j + \dfrac{\Omega_j^{k-2}}{\Lambda_j} + \\ + \displaystyle\sum_{l=1}^{k-2}\frac{k-l}{l}\cdot\Omega_j^{k-1-l} + \frac{1}{k-1} \end{array}\right) \sum_{\alpha=\xi,\eta,z}(B_{\alpha j}\mu_j + C_{\alpha j})D_{\alpha k}.$$

5.2 Root displacement discontinuity element

For the element, $\boldsymbol{D}(t) = \boldsymbol{D}_{rr}\sqrt{\dfrac{L_{AB} - t}{L_{AB}}}$, $\boldsymbol{D}_{rr} \equiv \boldsymbol{D}_{rr} \{D_{\xi rr}, D_{\eta rr}, D_{zrr}\} = \text{const}$.

Complex functions Π_j and Σ_j in the eqns (1) are

$$\Pi_j = \frac{L_{AB}}{\omega_{Bj} - \omega_{Aj}}\left(2 + \sqrt{1 - \Omega_j}\ln\frac{\sqrt{1 - \Omega_j} - 1}{\sqrt{1 - \Omega_j} + 1}\right) \sum_{\alpha=\xi,\eta,z}\left(B_{\alpha j}\mu_j + C_{\alpha j}\right)D_{\alpha rr},$$

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$$\Sigma_j = -\frac{L_{AB}}{\left(\omega_{Bj} - \omega_{Aj}\right)^2} \left[\frac{1}{2} \frac{1}{\sqrt{1-\Omega_j}} \ln \frac{\sqrt{1-\Omega_j} - 1}{\sqrt{1-\Omega_j} + 1} + \frac{1}{\left(\sqrt{1-\Omega_j} + 1\right)\left(\sqrt{1-\Omega_j} - 1\right)} \right] \sum_{\alpha=\xi,\eta,z} \left(B_{\alpha j} \mu_j + C_{\alpha j}\right) D_{\alpha r} \; .$$

6 Conclusions

Functions, which permit us to determine stresses and displacements at an arbitrary point in anisotropic elastic medium due to different three-dimensional stress or displacement discontinuities that were distributed on flat infinite band-type boundary elements, have been derived. In case of transversely isotropic medium the formulae represent the closed form of the solutions for arbitrary oriented boundary elements.

The influence functions are exact and in closed form and, thus, have advantages over their numerical counterparts. The new analytical functions can be used in the IBEM and the DDM computer codes to solve the generalized plane strain problems with 3D boundary conditions, and elastic homogeneous medium with arbitrary anisotropy may be considered.

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