An integrated system for damage tolerance design of aircraft panels
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Abstract

This paper describes an object oriented interactive system for damage tolerance design of Aircraft panels. The use of the Dual Boundary Element Method allows for the solution of single or multi-site damage problems under mixed mode conditions. Examples of application featuring multi-site damage problems in stiffened panels and fuselage lap joints are presented.

1 Introduction

Aircraft structural components such as stiffened panels must satisfy damage tolerance requirements. A structural component is damage tolerant if it can withstand reasonable loads without catastrophic failure or excessive deformation after the occurrence of serious fatigue damage. Fatigue crack growth analysis is central to damage tolerance assessment. It involves determining how cracks will propagate during the Aircraft operational life. The Boundary Element Method (BEM), has emerged as a powerful numerical technique for fracture mechanics. Its most attractive feature is the reduction of the

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dimensionality of the numerical model. The high stress gradients near the crack tip can be modelled more efficiently, in comparison with the FEM, as the necessary concentration of grid points is confined to one less dimension.

The recently developed Dual Boundary Element Method (DBEM), as presented by Portela, Aliabadi and Rooke [1], is capable of analysing configurations involving any number of edge and embedded cracks in any given geometry under mixed mode load conditions. The use of only one region, an intrinsic feature of the DBEM, eliminates re-meshing difficulties typical with FEM and other boundary element methods. Salgado and Aliabadi (reference [2]) later extended the DBEM to deal with multiple crack growth analysis and stiffened panels. Both continuously and discretely attached stiffeners are considered and the stress intensity factors calculated are shown to be very accurate.

The DBEM is the core of an object-oriented interactive environment for Damage Tolerance Design that was recently developed by Salgado and Aliabadi (see [3]). The environment provides an enhanced level of abstraction, which combined with the integration of all damage Tolerance design activities in a single computer program, allows the engineer to concentrate on the real design problem, speeding up the production of solutions for design alternatives and therefore, creating conditions for optimum solutions to be found.

In this paper, an overview of the I-DTD system is presented, with emphasis to the role played by the DBEM. Two examples of application are presented. The first features a multi-site damage problem in a stiffened panel. The other presents a multi-site damage problem in a fuselage lap joint.

## 2 System Description

The system was designed mainly in an object-oriented fashion. Object-orientation allows for the structural configuration to be defined in terms of it's real features instead of the DBEM model features that represent them. The user can manipulate the structural configuration components in a simple and intuitive manner using well known mouse activated operations. The knowledge that allows the numerical model to be constructed from the structural configuration objects is encapsulated in the objects themselves. The system provides an enhanced level of abstraction that makes it unique when compared with BEM or FEM systems with integrated pre and post processing facilities.

### 2.1 The Crack growth simulation task

Although there are, undoubtedly, benefits in designing scientific code in an object-oriented manner (see references [4],[5]), they do not seem yet attractive enough to compensate for the extra work and loss of run-time efficiency
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(see reference [6]). The approach adopted in the I-DTD system was to use an existing DBEM program which is called by the system when the crack growth simulation is required.

2.1.1 DBEM formulation for problems involving stiffened panels

The boundary integral displacement equation, for a source point \( x' \) at the boundary \( \Gamma \) of a finite sheet, subjected to a set of boundary tractions and displacement constraints and under the action of body forces \( b_j(X) \) applied at field points \( X \) continuously distributed over \( n \) straight lines \( L_n \) inside the domain it is given by:

\[
\sum_{ij} c_{ij}(x') u_j(x') + \int_{\Gamma} T_{ij}(x', x) u_j(x) d\Gamma(x) = \int_{\Gamma} U_{ij}(x', x) t_j(x) d\Gamma(x) + \sum_{n} \int_{L_n} U_{ij}(x', X) b_j(X) dL_n(X)
\]

where \( T_{ij}(x', x) \) and \( U_{ij}(x', x) \) are the Kelvin traction and displacement fundamental solutions respectively, \( u_j(x) \) and \( t_j(x) \) are displacements and tractions at boundary field points \( x \), \( \int \) stands for principal value integral and \( c_{ij} \) is a coefficient that can be determined by rigid body movement considerations.

The corresponding traction boundary integral equation, presented below, can be obtained by differentiation of equation (1), application of Hooke’s law and multiplication by the outward normal,

\[
\frac{1}{2} t_j(x') + n_i(x') \int_{\Gamma} S_{ijk}(x', x) u_k(x) d\Gamma(x) = n_i(x') \int_{\Gamma} D_{ijk}(x', x) t_k(x) d\Gamma(x) + \sum_{n} \int_{L_n} D_{ijk}(x', X) b_k(X) dL_n(X)
\]

where \( S_{ijk}(x', x) \) and \( D_{ijk}(x', x) \) contain derivatives of \( T_{ij}(x', x) \) and \( U_{ij}(x', x) \) respectively and \( n_i(x') \) denotes the \( i \)-th component of the unit outward normal to the boundary at the source point \( x' \).

The boundary \( \Gamma \) and the body forces loci \( L_n \) are discretized into a set of elements. By taking the collocation point to lie at each of the boundary element nodes, and using equation (1), a system of simultaneous linear equations is constructed. The coefficients of the unknown nodal boundary values are in the form of integrals over the boundary elements and are evaluated numerically. The line integrals over the body forces loci are evaluated analytically. If a crack is present, equation (2) is used instead of (1), when collocating at one of the crack surfaces.

If one or more stiffeners are continuously bonded to a cracked sheet, and this configuration is subjected to a set of boundary loads and displacement
constraints, the sheet and the stiffener will share interaction forces along the connection lines. These interaction forces can be treated in the boundary integral equations (1) and (2) as the body forces distributed along lines. Instead of known forces applied at the lines, the interaction forces are now considered as new unknowns of the problem. The condition that the displacements $u_i^s$ of the sheet and $u_i^n$ at position $y$ of the $n$-th stiffener are compatible with the shear deformation of the adhesive layer connecting them, is expressed by the following relation:

$$u_i^s - u_i^n = \phi^nb_i^n$$  \hspace{1cm} (3)

where $\phi^n$ is the coefficient of shear deformation of the adhesive layer.

Displacement compatibility equations can be generated by taking points $X^{n(y)}$ corresponding successively to each of the nodes on each stiffener. Other necessary equations can be constructed from the condition that the stiffeners are in equilibrium.

If the stiffeners are discretely attached to the sheet, then instead of continuously distributed along the attachment lines, the interaction forces are considered as concentrated at points, corresponding to the fasteners positions and the integrals over the stiffener elements in equations (1) and (2) reduce to summations. That is:

$$\sum_n \int_{L_n} (\text{kernel})b(X)dL_n(X) \text{ reduces to } \sum_n \sum_m (\text{kernel})b_{mn}$$

where $m_n$ stands for the nodes at the $n$-th stiffener locus.

2.1.2 Evaluation of the stress intensity factors

Once the system of equations is assembled and solved for the unknown boundary displacements $u_i$, boundary tractions $t_i$, interaction forces $b_i$, the stress intensity factors are calculated. The path independent J-integral is used for that purpose. It can be defined for mixed-mode problems as:

$$J^m = \int_S (W^{m}n_1 - t^m_ju^m_j)dS$$  \hspace{1cm} (4)

where $W$ is the strain energy per unit volume, $n_1$ is the component in direction 1 of the outward normal to the path $S$, $t_j(=\sigma_{ij}n_j)$ and $u_j$ are the components of the interior tractions and displacements and $m$ stands for mode I and mode II. The J-integral is related to the stress intensity factor; under plane stress conditions, the relationship is:

$$J^I = \frac{K^I_I}{E} \text{ and } J^{II} = \frac{K^{II}_I}{E}$$  \hspace{1cm} (5)
2.1.3 Crack growth analysis

Crack growth directions are calculated using the maximum principal stress criteria (reference [7]), According to that criteria, the local crack extension direction is determined by the condition that the crack-tip shear stress is zero, that is:

\[ K_I \sin \theta_t + K_{II}(3 \cos \theta_t - 1) = 0 \]

where \( \theta_t \) is an angular coordinate centred at the crack tip and measured from the crack axis ahead of the crack tip. However, The maximum principal stress criteria does not take into account the discreteness of the crack-extension modelling. Thus, the direction of the crack-extension increment must be corrected to take the size of the increment into account. A prediction-correction scheme described by Portela [8] is adopted in the work presented here.

In multiple-site damage problems it is necessary, after each iteration, to determine not only the direction of the crack extension but also the relative sizes of the increments (i.e. the relative growth rates between the various crack tips). The fatigue crack growth properties of the structural material \( \left( \frac{da}{dN} \right) \) are usually given as a function of the effective stress intensity factor range \( \Delta K_{eff} \). For the sake of simplicity, Paris law (reference [9]) is used. That is:

\[ \frac{da}{dN} = C(\Delta K_{eff})^m. \]  

(6)

where \( C \) and \( m \) are the Paris material parameters and \( \Delta K_{eff} \) is given by Tanaka [10] as:

\[ \Delta K_{eff}^2 = \Delta K_I^2 + 2\Delta K_{II}^2 \]  

(7)

For constant amplitude fatigue load with stress amplitude ratio \( R \), \( \Delta K_I = K_I(1 - R) \) and \( \Delta K_{II} = K_{II}(1 - R) \).

At the end of each iteration, the effective stress intensity factors are calculated and the crack growth equation (6) is integrated to determine, for each crack tip(i), the number of cycles (\( \Delta N_i \)) necessary to grow an arbitrary reference size (RS).

\[ \Delta N_i = \frac{1}{C} \int_{a_i}^{a_i+RS} \frac{1}{\Delta K_{eff}^m} da. \]  

(8)

The integration in (8) is performed assuming that the stress intensity factors remain constant as the crack grows from the initial size \( a_i \) to \( a_i + RS \).

It is then assumed that the fast growing crack tip - i.e. the one for which the smallest number of cycles \( \Delta NC = Min(\Delta N_i) \) was calculated - will
grow the chosen reference size $RS$. The crack growth increment size of the other cracks is then calculated by integrating the inverse of the crack-growth equation,

$$\Delta a_i = C \int_{NC}^{NC+\Delta NC} \Delta K_{eff}^m dN. \quad (9)$$

now using the minimum number of cycles previously determined ($\Delta NC$) to calculate the growth length. Again, the stress intensity factors are assumed to remain constant. At this point, we have a first estimate of the increments size.

The boundary element mesh is then updated to include the new increments and a structural analysis is performed, at the end of which, new stress intensity factors are calculated. The increments size are then re-calculated, now taking into account the variation of the stress intensity factors when integrating equations (8) and (9). The increments are re-meshed and a new structural analysis is performed. This procedure is repeated until convergence for the increments size is achieved. Actually, the increments direction and size are calculated simultaneously.

Re-meshing after each crack increment is introduced by adding new elements to the crack boundary. No change is required in the existing mesh. Moreover, due to the use of discontinuous elements at the crack boundaries, only matrix coefficients due to the new elements have to be calculated and added as new rows and columns, to the existing (LU-decomposed) matrix. Only the new rows and columns have to be decomposed during the current increment and the computational time is substantially reduced. Figure (1) illustrates the way in which each increment is added to the system matrix.

![Figure 1: System matrix after crack extension analysis](image-url)
3 Examples of application

3.1 Stiffened Panel

This example presents a multi-site damage problem in a stiffened panel under mixed mode loading conditions. The stiffeners are connected to the panel by means of rivets. The panel contains three cracks emanating from rivet holes under stiffeners. The cracks longitudinal axis are not aligned. The panel is subjected to a constant normal traction in the direction parallel to the stiffeners as well as shear forces with magnitude equivalent to 25% of the normal tractions. Both normal and shear forces are applied at the top and bottom edges of the panels. Crack growth is performed using constant amplitude load cycles. The I-DTD system screen with four views containing the results is presented in figure 2. The top left window displays the BEM model geometry. The location of the crack tips is indicated in this view. The top right window shows the crack paths. It can be seen that, initially, the cracks growth direction is determined exclusively by the global stresses applied to the panel. However, as the cracks grow, they start to influence each other as can be seen by the change in the growth direction of tips 1,2,4 and 5. The window on the bottom left corner shows the stress intensity
factors plotted against the analysis increment number. The stress intensity factors are slightly higher for tips 1 and 2 than for tips 4 and 5. The stress intensity factors for the tips 3 and 6 are considerably lower. The Crack Growth Diagram presented in the bottom right window reflects the behaviour of the stress intensity factors, showing that tips 1 and 2 grow at slightly higher rates than tips 4 and 5 while tips 3 and 6 grow at lower rates.

### 3.2 Fuselage Lap Joint

This example presents a multi-site damage problem in a section of a fuselage Lap Joint. There are cracks of different sizes emanating from the rivet holes. The configuration is subjected to a constant traction at the upper edge of the sheet. Displacement constraints are applied to the lower half of the rivet holes to simulate a rigid pin. Fatigue crack growth is performed assuming constant amplitude load cycles. The problem geometry is presented in figure 3 while the calculated crack paths are presented in figure 4.

### 4 Conclusion

An integrated system for Damage Tolerance Design of Aircraft panels was presented. The I-DTD system supports all tasks involved in Damage Tolerance Design, from the definition of the structural configuration to the specialized post-processing (Crack Growth Diagram, Residual Strength Diagram, etc.). The use of the system allows the engineer to concentrate on the real design issues rather than the numerical model complexities or the integration of disparate pieces of software. He/she is free to experiment with the design variables. They can be easily changed using well known mouse operations. Optimum design solution can be obtained from such experiments.

The Dual Boundary Element Method plays a central part in the I-DTD system. Problems involving multiple cracks under mixed mode loading conditions can be solved. Stiffeners can be continuously or discretely attached to the sheet. The flexibility of the attachments can be taken into account. Important structural components frequently present in Aircraft such as: Stiffened panels and Lap-Joints, can be analysed in an inexpensive personal computer.
Figure 3: I-DTD system screen showing Lap joint with cracks emanating from rivet holes.

Figure 4: I-DTD system screen displaying crack paths for Lap joint example.
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References


