



Module-boundary element

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Abstract

An unusual type of boundary elements is examined. The suggested boundary element is a three - dimensional one. Such boundary element is called a module-boundary element (MBE). The description of a module-boundary element is given. Theoretical aspects of such approach to solution of hydroelasticity problems are substantiated. Some details of practical application of MBE are presented.

1 Introduction

The solution of hydroelastic oscillation problems of complex three-dimensional structures requires taking into account adjacent fluid influence. An example of such problem is ship hull vibration. The problem is complicated by the fact that the fluid involved into vibration is infinite or half-infinite. The correct solution of the problem is possible if the boundary element method is used. The feature of the technique proposed is an unusual type of boundary elements. The boundary elements are constructed in accordance with the conception of the module element method [1,2]. They have complex three-dimensional configuration. They are called module-boundary elements. Stating and solving a hydroelasticity problem by means of MBE is demonstrated in paper [3].

Investigation connected with the development of the module element method was carried out by Leningrad Shipbuilding Institute and Komsomolsk-na-Amure Polytechnical Institute under supervision of professor V.A.Postnov.

2 Problem statement

Let us examine an elastic shell structure interacting with fluid (Fig.1).

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Here Ω is the half-space occupied by the fluid; B is the area bounding the elastic shell (boundary); B_0 is the free fluid surface; B_f is the surface of contact of the elastic shell and the fluid; B_a is the surface of contact of the shell and air; ν^+ is the outer normal to the boundary contour of the elastic shell; s is the contour coordinate.

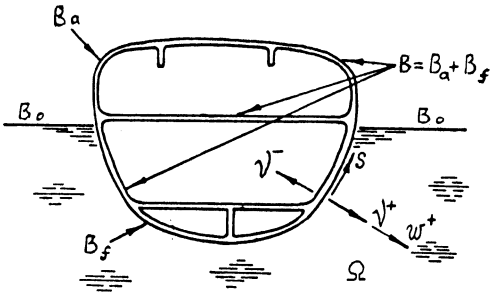


Figure 1: Shell and fluid interaction

A combination of the module element method and the boundary element method is used for solution of the stated hydroelasticity problem. In this case the shell (ship hull) is regarded as an assemblage of module elements (Fig.2,a). The fluid, contacting with the hull along boundary B_f is approximated by surface boundary elements (Fig.2,b). These boundary elements as well as module elements (ME) of the shell are three-dimensional and have complex configuration (Fig.2,d). Term "module-boundary element" is introduced to denote them.

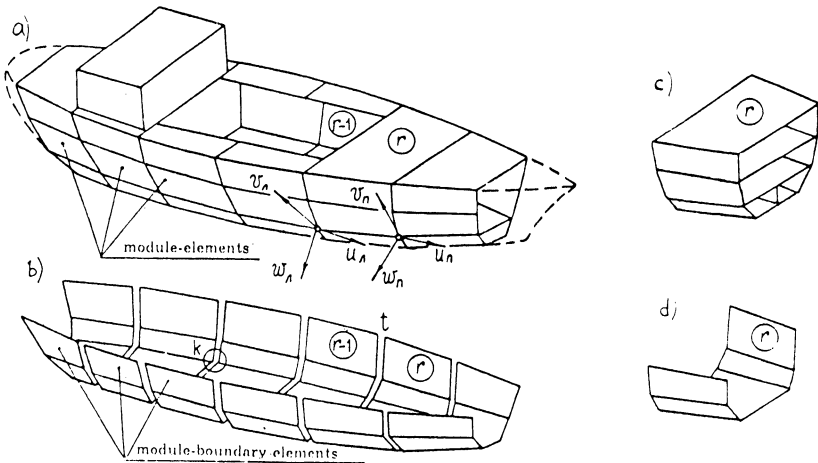


Figure 2: Structural model: a - shell disintegration into ME; b - approximation of fluid surface contacting with MBE; c - r-th ME of shell; d - r-th MBE of fluid.

Let us assume that:

a) the whole fluid surface contiguous to r -th module-element wetted surface is a separate r -th MBE, having the curvature similar to that of the ME of the structure. Thus, there will be as many MBE of the fluid as

there will be ME of the structures contacting with the fluid. These MBE will interact with each other and with ME of the structure in the same cross-sections, i.e. will have common boundaries and nodes of interaction;

b) values of potentials of butt edges where unknown ME displacement will be investigated are assumed as unknown quantities of r -th MBE;

c) potential approximation within MBE is performed in the form analogous to approximation of normal displacement on ME wetted surface. Therefore the form of approximation potential functions in general case can differ from that of approximation displacement functions.

Complying with the last three conditions enables to get conjugation of MBE of the fluid and ME of the shell and to apply finite element procedure for solving the problem.

Equations of dynamic equilibrium of the system "shell-fluid" can be obtained from the Lagrange equation of the second degree:

$$\frac{d}{dt} \cdot \frac{\partial T}{\partial \{\dot{Q}(t)\}} + \frac{\partial \Pi}{\partial \{Q(t)\}} = \{P(t)\}, \quad (1)$$

where $T = T(\dot{u}, \dot{v}, \dot{w}, \Phi)$ is the kinetic energy of the oscillating system "shell-fluid"; $\Pi = \Pi(u, v, w)$ is the potential energy of the shell elastic deformation; $u(x, s, t)$, $v(x, s, t)$, $w(x, s, t)$ - are the displacement components of the shell middle surface; $P(t)$ is the generalized outside forces; $Q(x, s, t)$ is generalized shell displacements; $\Phi(x, s, t)$ is the generalized velocity potential of fluid particles.

Relation between $\{Q\}$ and $\{\Phi\}$ according to the boundary element method is written as

$$[H]\{\Phi\} = -i\omega [G]\{Q\}, \quad (2)$$

where ω is the frequency of oscillation of the regarded system; $[G]$ and $[H]$ are the influence matrices representing the distribution of the velocity potential and its normal derivative (see below for details).

Velocity potential Φ must satisfy the Laplas equation as well as the following boundary conditions:

on the surface of the oscillating
body (condition of impenetrability)

$$\frac{\partial \Phi}{\partial \nu^-} = -\frac{\partial w}{\partial t} \in B_f;$$

on the free fluid surface

$$\Phi = 0 \in B_0;$$

on infinity (emission condition)

$$\lim_{R \rightarrow \infty} \frac{\partial \Phi}{\partial R}; \quad \lim_{R \rightarrow \infty} \Phi = 0;$$

(3)

Presence of "minus" sign in the first condition of eqn (1) is connected with the fact that positive normal direction ν is opposite to positive displacement direction w .

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The final resulting equation system of the problem at issue is written as:

$$\left([K] - \omega^2 ([M] + [M_f]) \right) \{Q\} = \{P\} \quad (4)$$

where $[M]$ and $[M_f]$ are respectively the shell mass matrix and the added fluid mass matrix obtained by calculating the problem of the "shell-fluid" system coupled oscillation,

$$[M_f] = [G]^T [H]^{-1} [L] \quad (5)$$

The case of steady state harmonic oscillation, when it is possible to eliminate the time parameter t by introducing multiplier $e^{i\omega t}$ is given below.

3 Shell module element

A module element is a part of the shell cut out from it by two cross-sections (Fig.2,a). ME has a complex unspecified cross-section contour and can be non-prismatic in general case (Fig.2,c).

Displacements of ME middle surface points are presented as:

$$\left. \begin{aligned} u(x,s) &= \sum_{i=1}^m \left[\left(1 + \frac{x}{l} \right) \varphi_i(s) U_i + \frac{x}{l} \varphi_{m+i}(s) U_{m+i} \right]; \\ v(x,s) &= \sum_{k=1}^n \left[\left(1 - \frac{x}{l} \right) \psi_k(s) V_k + \frac{x}{l} \psi_{n+k}(s) V_{n+k} \right]; \\ w(x,s) &= \sum_{k=1}^n \left[\left(1 - \frac{x}{l} \right) \chi_k(s) V_k + \frac{x}{l} \chi_{n+k}(s) V_{n+k} \right], \end{aligned} \right\} \quad (6)$$

Where m and n are the number of longitudinal and transverse degrees of freedom of ME cross-section; U_i and V_k are the longitudinal and transverse generalized displacements on the ME left end face; U_{m+i} and V_{n+k} are the same on ME right end face; $\varphi_i(s)$, $\psi_k(s)$ and $\chi_k(s)$ are the corresponding coordinate functions. In the given problem coordinate functions may be regarded as generalized oscillation forms. The principles of their construction are given in details in book [1].

4 Module-boundary element

A module-boundary element is a surface boundary element of complex three-dimensional configuration (Fig.2,d). Let us examine some aspects which make MBE features clear.

4.1 Velocity potential

Taking into account that the distance between MBE end cross-sections is small in comparison with the total length of the oscillating

shell area it is possible to assume linear approximation along axis x .

Like in the case with generalized displacements generalized potentials can be written as:

$$\Phi(x, s) = \sum_{k=1}^n \left[\left(1 - \frac{x}{l} \right) f_k(s) \Phi_k + \frac{x}{l} f_{n+k}(s) \Phi_{n+k} \right], \quad (7)$$

where Φ_k and Φ_{n+k} are the unknown amplitudes of k -th generalized potentials on the MBE left and right end faces respectively; $f_k(s)$ and $f_{n+k}(s)$ are the coordinate functions corresponding to them. These coordinate functions show the assumed distribution of velocity potential of the cross-section wetted contour part at isolated amplitude the of k -th and $(n+k)$ -th generalized potentials.

Attention should be paid to the fact that according to eqn (7) the potential is generalized i.e. is formed of a sum of all k -th generalized oscillations, including those, generated by sources, which are away from the regarded oscillating area. It provides fairly simple way of forming coordinate functions of every k -th potential. When choosing the kind of coordinate function $f_k(s)$ the following may be taken into consideration: the distribution of k -th potential around k -th source can be linearized because of close location of the sources to each other. Then it is advantageous to choose functions $f_k(s)$ in the form of triangular pattern (Fig.3, b).

Now let us consider what the source of excitation is. Fluid velocity potential is caused by the oscillation displacement of the shell. Strictly speaking, infinite set of points, forming the oscillating contour part in k -th displacement should be considered the source of k -th potential (Fig.3, a)

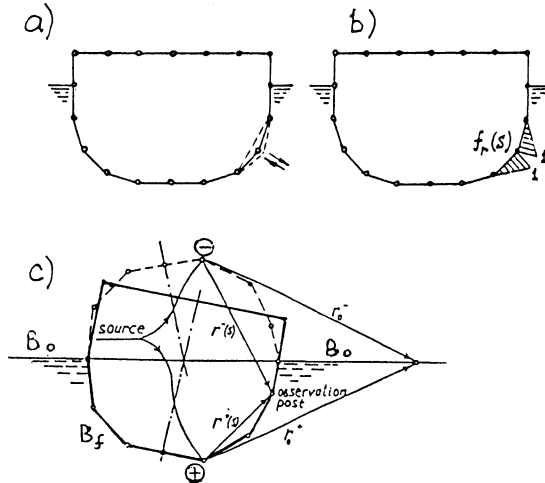


Figure 3: Submerged elastic contour:

a - k -th oscillation; b - the approximation coordinate potential function; c - about construction of a fundamental solution.

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However, the kind of the approximation function having been chosen, the exact nature of the oscillation source is of no principle importance. Therefore, for clarity it may be conventionally assumed that the source of excitation is of point character and is concentrated in the node, having the biggest normal displacement in k -th oscillation.

In the module element method k -th displacement is generalized. It implies simultaneous complex displacement of several contour part (much more complex then that shown in Fig.3,a). In this case several nodal points are the simultaneous sources of excitation. Their set should be regarded as a generalized k -th potential source.

4.2 Connection between generalized displacements and generalized potentials

Let us examine the case when the "shell-fluid" system consists of one ME and one MBE. Then function (2), setting the connection between displacements and potentials is written as:

$$[H_{kh}]\{\Phi_h\} = -\omega[G_{kh}]\{Q_h\}, \quad (8)$$

where

$$\left. \begin{aligned} H_{kh} &= \iint_{B_f} \frac{\partial \Phi_k^*}{\partial \nu_k^-} f_h dB & at \quad k \neq h \\ H_{kh} &= \iint_{B_f} \frac{\partial \Phi_k^*}{\partial \nu_k^-} f_h dB + C_k & at \quad k = h \\ G_{kh} &= \iint_{B_f} \Phi_k^* \chi_h dB & at \quad any \quad k \quad and \quad h \end{aligned} \right\} \quad (9)$$

where $\Phi_k^* = \Phi_k^*(x_k, s_k, x, s)$ is the fundamental solution of the Laplace equation derived for a k -th point with x_k, s_k coordinates. It is an influence function, showing the distribution of the potential from the source originating in k -th generalized displacement; $C_k = C_k(x_k, s_k)$ is the factor of proportion of the potential Φ_k , depending on MBE surface geometry at k -th point which we regard as a source. For constructing fundamental solution Φ_k^* , satisfying boundary conditions on a free fluid surface the technique of mirror reflection of a wetted shell area relative to a free surface is used (Fig.3,c).

In this case

$$\Phi_k^* = \frac{1}{4\pi} \left(\frac{1}{r^+} - \frac{1}{r^-} \right) \quad (10)$$

The meaning of r^+ and r^- radius is clear from Fig.3,c if

three - dimensional character of potential distribution is taken into consideration.

As for definition of C_k factor, it is discussed in detail in book [4]. In this work where we use MBE, as well as in work [4], it is possible to deal without the precise definition of factor C_k . However, in addition to this we shall show here the technique, which will enable, in the case of necessity, to obtain the precise meaning of factor C_k . If we consider physical nature of the factor, it will appear to be the relationship $C_k = F_k / F$, where F is the full sphere surface area around the node "k"; F_k is the sphere surface area which is inside the fluid. In this case it is convenient to assume the sphere radius equal to 1, then $F = 4\pi$ and F_k is the measure of the inner three-dimensional angle γ_k (directed inside the fluid) at point k.

This angle for two-dimensional problems is shown in Fig 4.

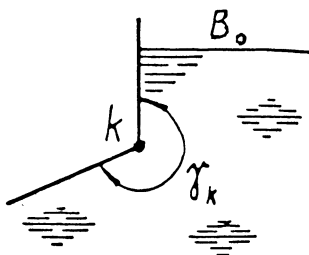


Figure 4: About calculation of factor C_k .

For three - dimensional problems C_k factor can be calculated using the following formula; $C_k = \gamma_k / 4\pi$. It is this relationship by means of which boundary values well known in mathematics and physics, were obtained:

$$C_k = \left\{ \begin{array}{ll} 0 & \text{at } \gamma_k = 0 \text{ (outside of the area } \Omega \text{);} \\ 1/2 & \text{at } \gamma_k = 2\pi \text{ (on the smooth boundary } B_f \text{);} \\ 1 & \text{at } \gamma_k = 4\pi \text{ (inside the area } \Omega \text{).} \end{array} \right\} \quad (11)$$

4.3 Calculation of matrix factors [H] and [G]

Calculation of factors H_{kh} and G_{kh} by means of eqns (9) is the most important and difficult part of the problem. Practical technique of calculating these factors including the cases when the domain, containing a special point is integrated (singularity problem) is given in details in book [1].



5 Conclusion

The above given approach, based on applying module - boundary elements is the most effective in solving hydroelasticity problems of three-dimensional objects having complex configuration.

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