A generalized boundary element model for fluid-structure interaction modelling

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Abstract

A generalized extended boundary element model is proposed for handling complex acoustic problems in the frequency domain. The model relies on a mixed direct/indirect boundary integral representation allowing to handle closed structures with thin appendages. The formulation uses pressure, jump-of-pressure, mean pressure and/or velocity variables along the boundary surface. A special attention is devoted to the treatment of both continuous and discontinuous boundary conditions. In contrast with earlier implementations of this mixed integral representation, the present solution procedure relies on variational solution scheme. By this way, the symmetry of the discrete problem is preserved. This property makes easier the required coupling with a finite element structural model in order to solve complex fluid-structure interaction problems.

1 Introduction

Boundary element formulations have become more and more popular for solving acoustic radiation problems. This success is mainly due to their flexibility (no need for generating a volumic mesh) and to the availability of reliable software products. Although initial integral formulations [1] were restricted to homogeneous acoustic fluids, closed boundary surfaces and nodal collocation solution schemes, the most elaborated methods refer now to multi-fluids, open or closed ribbed surfaces [2] using indirect representations (single and double layer potentials) together with (Galerkin) variational solution methods.
The present paper is related to the development of an extended acoustic boundary element model enabling the treatment of more complex problems. The complexity is related to the geometry (structures consisting of main bodies and thin components) together with the consideration of generalized boundary conditions. An attractive feature of this enhanced formulation is the possibility for mixing pressure and jump-of-pressure variables in order to handle more efficiently coated submerged structures with thin fins [3].

2 Boundary integral representation

The acoustic pressure \( p \) at a point \( X \) inside the acoustic domain bounded by the surface \( S \) of a structure can be represented by the Kirchoff-Helmholtz integral stated as:

\[
p(X) = p_i(X) + \int_S \left\{ p(Y) \frac{\partial G(X, Y)}{\partial n_Y} - \frac{\partial p(Y)}{\partial n_Y} G(X, Y) \right\} dS(Y)
\]

where \( p_i(X) \) is the incident pressure field, \( G(X, Y) \) is the Green’s function and \( n_Y \) is the (outward) normal at point \( Y \) along the boundary surface \( S \).

Irrespective of the actual boundary conditions, this integral representation can be reformulated when the boundary surface (Figure 1) is partitioned into two sub-surfaces: \( S_1 \) (in contact with the acoustic fluid on one side) and \( S_2 \) (in contact with the acoustic fluid on both sides). The related expression is obtained through a limit process [3] which involves the mean surface \( S_2 \) (provided the thickness of the related component is small versus the acoustic wavelength).

The generalized result is:

\[
p(X) = p_i(X) + \int_{S_1} \left\{ p(Y) \frac{\partial G(X, Y)}{\partial n_Y} - \frac{\partial p(Y)}{\partial n_Y} G(X, Y) \right\} dS_1(Y)
+ \int_{S_2} \left\{ \mu(Y) \frac{\partial G(X, Y)}{\partial n_Y} - \sigma(Y) G(X, Y) \right\} dS_2(Y)
\]

where \( \mu(Y) = p^+(Y) - p^-(Y) \) is the jump of pressure variable while \( \sigma(Y) = \frac{\partial p^+(Y)}{\partial n_Y} - \frac{\partial p^-(Y)}{\partial n_Y} \) is the jump of the normal gradient of the pressure. These variables are the so-called single and double layer potentials traditionally used with the indirect boundary integral representation.

An earlier application of this process (with a nodal collocation solution scheme) for the case of a thin plate can be found in [6,7]. More recently Wu [9] proposed a similar scheme for handling mixed regular and thin bodies within the general frame of a direct boundary element method. The formulation relies on the coupling between the conventional Helmholtz integral equation and the
hyper-singular thin-body integral equation while the solution procedure refers to a traditional nodal collocation scheme.

One drawback of the mixed formulation is the handling of non-physical variables on one part of the boundary surface. The evaluation of the related acoustic variables (pressure and velocity) should imply a specific post-processing. This feature is a 'key' characteristic of the so-called 'indirect' boundary integral formulations. This particular difficulty can be solved by performing some changes of variables based on the use of actual boundary conditions.

![Generalized boundary surface](image)

**Figure 1: Generalized boundary surface**

### 3 Boundary conditions

The acoustic (exterior and/or interior) problem can be formulated with reference to the boundary surface \( S = \Omega \cup \Omega \) represented at Figure 1. For the simplicity, we assume that the only boundary conditions encountered along the surface \( S \) are generalized admittance boundary conditions stated as:

\[
\frac{\partial p}{\partial n} = -Ap - ip\omega \nu_0 \text{ on } S_1 \\
\frac{\partial p^*}{\partial n} = -A^* p^* - ip\omega \nu_0 \text{ on } S_2^* \text{ and } \frac{\partial p^-}{\partial n} = +A^- p^- - ip\omega \nu_0 \text{ on } S_2^- \quad (3) \quad (4)
\]

The above relations allow for discontinuous admittance boundary conditions along a moving support (the admittance values on both sides of the thin fin are different).

### 4 Selection of boundary variables

The boundary variables have to be selected according to the boundary conditions and keeping in mind the requirement of an easy access to acoustic surface quantities. Restricting our attention to the generalized admittance boundary conditions, it appears that the normal gradient of the pressure along
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$S_1$ can be expressed as a function of $p$ using (3) while the single layer potential along $S_2$ can be expressed from (4) as a combination of two other variables:

$$\sigma(Y) = -\bar{A}(Y)p(Y) - \tilde{A}(Y)\Delta p(Y) \text{ on } S_2$$

(5)

where $\bar{A}$ and $\tilde{A}$ are given by $\bar{A} = (A^+ + A^-)/2$ and $\tilde{A} = (A^+ - A^-)/2$ while $\bar{p} = (p^+ + p^-)$ and $\Delta p = (p^+ - p^-)$ is the jump of pressure. Substitution of (5) into the integral representation (2) leads to an updated representation which has a specific feature: the new boundary variables along thin components enable the direct evaluation of boundary pressures on both sides using

$$p^+ = (\bar{p} + \Delta p)/2 \text{ and } p^- = (\bar{p} - \Delta p)/2$$

(6)

Following this way, one drawback of the indirect boundary integral formulation (need for a specific postprocessing task for getting acoustic boundary pressures) is eliminated.

The use of the integral representation (2) and its derivative form allow to derive a set of three auto-adjoint boundary integral equations [4] which can be written in a compact form as:

$$A(X,Y)\{X(Y)\} = b(X)$$

(7)

where the boundary operator $A$ is a $3\times 3$ operator while $X(Y) = (p(Y), \Delta p(Y), \bar{p}(Y))^T$.

5 Discrete model

The discrete model relies on a Garlerkin boundary element model which is extended in order to handle coupled elasto-acoustic problems. In a matrix form, the coupled systems appears as [4]:

$$\begin{bmatrix}
Z_S - \rho \omega^2 A \\
C/2 + B_1 \\
C + B_2 \\
B_3
\end{bmatrix}
\begin{bmatrix}
C^T/2 + B_1^T \\
C^T + B_2^T \\
B_3^T
\end{bmatrix}
\begin{bmatrix}
U \\
P \\
\Delta P \\
\bar{P}
\end{bmatrix} =
\begin{bmatrix}
F_S - C^T P_1 \\
G_1 \\
G_2 \\
G_3
\end{bmatrix}$$

(8)

where $Z_S$ is the structural impedance matrix (usually derived form an appropriate FE model), $A$, $B_i$ and $D_{ij}$ are frequency dependent acoustic matrices while $C$ is a geometrical coupling matrix.

Various limit cases can be considered:
Case 1: \( S = S_1 \) and admittance condition on one side (structure ‘wetted’ on one side)

\[
\begin{bmatrix}
Z_S - \rho \omega^2 A & \frac{C^T}{2} + B_1^T \\
C/2 + B_1 & -D_{11}/\rho \omega^2
\end{bmatrix}
\begin{bmatrix}
U \\
P
\end{bmatrix}
= \begin{bmatrix}
F_S - C^T P_1 \\
G_1
\end{bmatrix}
\]  

(9)

This formulation (direct/variational) requires the evaluation of three acoustic matrices \((A, B_1 \text{ and } D_{11})\). The \( A \) matrix has to be superimposed to the structural impedance matrix so that the upper left sub-matrix is no longer sparse and real-valued (as it is if no damping is considered for the structural model).

Case 2: \( S = S_2 \) and zero admittance values on both sides (structure ‘wetted’ on both sides)

\[
\begin{bmatrix}
Z_S & \frac{C^T}{2} \\
C & -D_{22}/\rho \omega^2
\end{bmatrix}
\begin{bmatrix}
U \\
\Delta P
\end{bmatrix}
= \begin{bmatrix}
F_S - C^T P_1 \\
G_2
\end{bmatrix}
\]  

(10)

This attractive formulation (indirect method) preserves the sparseness of the structural operator and requires only one acoustic matrix [2].

Case 3: \( S = S_2 \) and different admittance conditions on both sides (structure ‘wetted’ on both sides)

\[
\begin{bmatrix}
Z_S & C^T & 0 \\
C & -D_{22}/\rho \omega^2 & -D_{23}/\rho \omega^2 \\
0 & -D_{23}/\rho \omega^2 & -D_{33}/\rho \omega^2
\end{bmatrix}
\begin{bmatrix}
U \\
\Delta P \\
P
\end{bmatrix}
= \begin{bmatrix}
F_S - C^T P_1 \\
G_2 \\
G_3
\end{bmatrix}
\]  

(11)

The handling of different admittance values on both sides implies the use of two boundary variables. The size of the coupled system is increased versus an indirect formulation with zero admittance values along the boundary surface but it has still the same interesting features.

Case 4: \( S = S_1 \cup S_2 \) and zero admittance values on both sides of thin components

\[
\begin{bmatrix}
Z_S - \rho \omega^2 A & \frac{C^T}{2} + B_1^T & C^T + B_2^T \\
C/2 + B_1 & -D_{11}/\rho \omega^2 & -D_{12}/\rho \omega^2 \\
C + B_2 & -D_{12}/\rho \omega^2 & -D_{22}/\rho \omega^2
\end{bmatrix}
\begin{bmatrix}
U \\
P \\
\Delta P
\end{bmatrix}
= \begin{bmatrix}
F_S - C^T P_1 \\
G_1 \\
G_2
\end{bmatrix}
\]  

(12)
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This formulation reduces to a previously developed mixed method [4] which has been applied to submerged elastic structures.

6 Solution procedure

The solution methodology to be used for each case has to be selected carefully according to the specific features of the related discrete model. As it can be seen from inspection of (8), (9) and (12), the discrete system is penalized by the superimposition of a full complex (and frequency dependent) A matrix with the sparse structural impedance matrix if the structure is wetted on one side. It should be stressed that this penalty is only induced by the symmetry requirement for the related coupled problem. This disadvantage is only effective when the structural model is solved in terms of physical coordinates. Usually a modal reduction scheme is selected for the structural model so that the related matrices are of a reduced order. An effective solution could rely in this case on the so-called ‘structural methodology’ where the fluid degrees of freedom (acoustic boundary variables) are condensed on the structural ones. This process (based on the selective elimination of these variables) leads to set up modal acoustic impedance matrices. These frequency dependent matrices characterize completely the fluid domain(s).

7 Conclusions

An extended boundary integral representation has been proposed for handling complex acoustic problems characterized by a mixture of regular and thin bodies. A special attention has been devoted to the treatment of generalized discontinuous admittance boundary conditions. The procedure is characterized by the choice of appropriate boundary variables. The solution procedure relies on a variational scheme applied to the global elasto-acoustic problem. Various limit cases have been considered and the solution strategy has been discussed.

References


