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Constriction resistance of conducting spots in an electric contact surface

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ABSTRACT

In electric or electronic circuits, there are always contact surfaces such as relay and switch. When an electric current flows through the contact boundary surface between two conducting bodies, the real contact area where the current can pass is far smaller than the nominal or apparent contact area, and the current constricts to real contact conducting spots. The resistance of conducting spots is called the constriction resistance. In this parer the boundary element method is introduced into the computation of the constriction resistance, and the superiority of the method is particularly demonstrated in the computation. At first the constriction resistance of a circular spot is computed, the computed values are compared with the exact solution, and the high accuracy of the computed values is made sure. Next the constriction resistance of a triangular, square, hexagonal and octagonal spot is calculated. As the result, it is clarified that the difference among the constriction resistances is small when the spots have an eqaul area.

INTRODUCTION

In electric contact surfaces between two conducting bodies, the real contact area where currents actually pass is far smaller than the nominal or apparent contact area. Nevertheless currents can flow through contact surface without any difficulty. This cause is the strong current constriction effect at real contact spots, as illustrated in Fig. 1. The constriction effect is a very important effect to the contact conduction, and the resistance owing to the effect is called the constriction resistance. In the problem for electric contact, the computation of the constriction resistance is a substaintial problem. By the finite element method (FEM) the constriction

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resistance has been calculated [1–6]. In this paper the boundary element method (BEM) is introduced into the computation, and the superiority of the BEM is demonstrated very well.

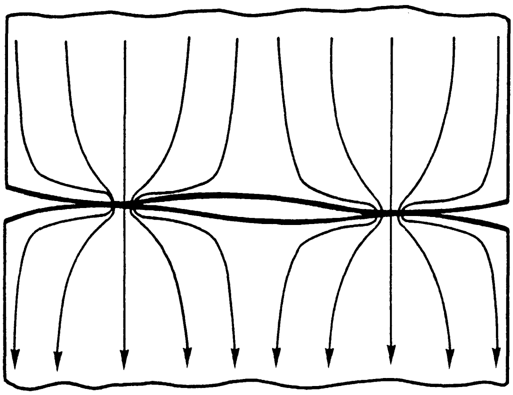

Figure 1: Current constriction to conducting spots in a contact surface.

In our previously published papers [7–11] the BEM was applied to compute the capacitance of capacitors in an infinite space, and it is shown that the application of the BEM is particularly effective in an infinite space. By the mathematical analogy between the electrostatic field and the steady current field, the BEM in the electrostatic field can be in itself applied also to the BEM in the steady current field. However the boundary condition for the calculation of capacitances is different from that of conductances. Because of the differnce a device is introduced for the computation of constriction resistances.

The constriction resistance R_c of a circular spot with radius a is presented by Holm [12] as

$$R_c = 0.5\,\rho/a = 0.886\,\rho/A^{1/2}, \tag{1}$$

where ρ is the resistivity of material and $A(=\pi a^2)$ is the area of the spot. The constriction resistance of a square spot with length L is computed by the FEM [1] as

$$R_c = 0.5\,\rho/L = 0.5\,\rho/A^{1/2}. \tag{2}$$

The constriction resistance is in inverse proportion not to the area but to the square root of the area. Equation (2) is a very rough estimation, and

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here a far more accurate value is computed by the BEM.

The first computation by the BEM is the constriction resistance of a circular spot to examine the calculational accuracy of the method. Next the constriction resistance of a triangular, square, hexagonal and octagonal spot is computed.

BOUNDARY ELEMENT METHOD FOR CONSTRICTION RESISTANCE OF CONDUCTING SPOTS

Equation (1) is formulated in the steady current field shown in Fig. 2. The field equation and boundary conditions of the field of Fig. 2 are, respectively,

$$\nabla^2 U = 0, \tag{3}$$

$$U = 1 \quad \text{at} \quad (r^2+z^2)^{1/2} \rightarrow \infty \ (z > 0) \tag{4}$$

and

$$U = 0 \quad \text{at} \quad (r^2+z^2)^{1/2} \rightarrow \infty \ (z < 0), \tag{5}$$

where U is the potential.

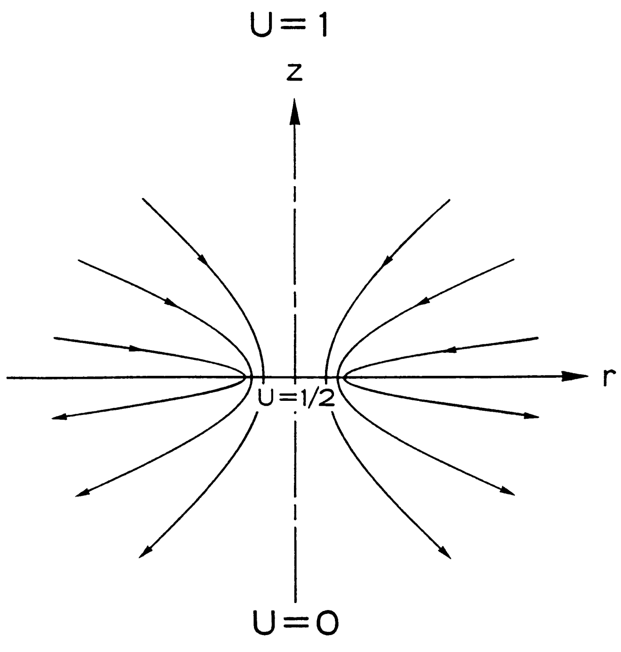

Figure 2: Steady current field where equation (1) is formulated.

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In the numerical methods such as FEM and finite difference method, boundary conditions (4) and (5) are only approximately considered by replacing the infinite space with an enough wide finite space. The application of BEM to the field of Fig. 2 is not efficient. Here a steady current field as shown Fig. 3 is introduced for the calculation by the BEM. The boundary conditions become

$$U = 1 \qquad \text{at } z=0 \ (r \leqq a) \tag{6}$$

and

$$U = 0 \qquad \text{at } (r^2+z^2)^{1/2} \rightarrow \infty. \tag{7}$$

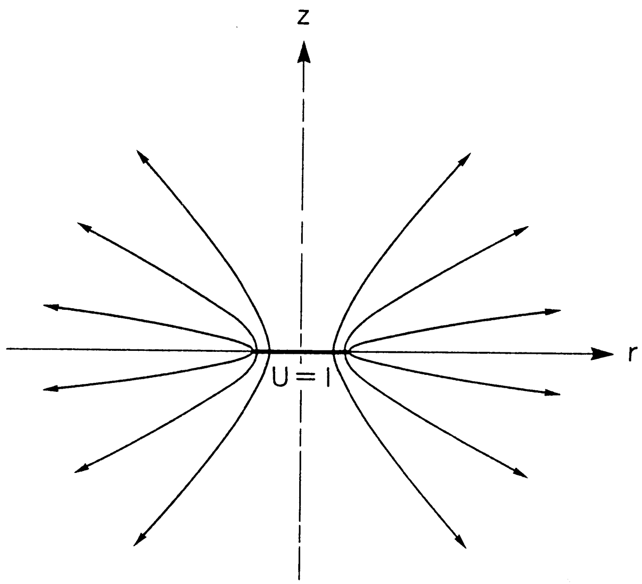

Figure 3: Steady current field for BEM.

In the BEM, boundary condition (7) is satisfied by applying the Green function G

$$G(\mathbf{r},\mathbf{r}')=1/4\pi\sigma\,|\mathbf{r}-\mathbf{r}'|, \tag{8}$$

where σ is the conductivity of the material. The boundary conditions of Fig. 3 are the same with those of the electrostatic field for the computation of capacitances [8–11]. Figure 2 is the field to obtain the actual constriction resistance, while figure 3 is the field for the calculation of the

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BEM. To obtain the constriction resistance of Fig. 2 from the computed constriction resistance of Fig. 3, the equivalent circuit of Figs. 2 and 3 is illustrated, respectively, in Figs. 4 and 5.

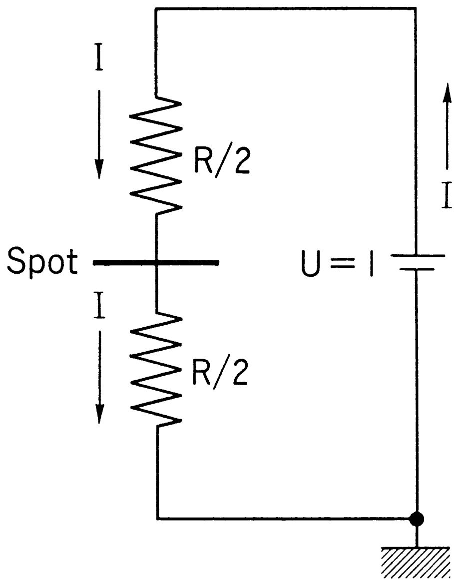

Figure 4: Equivalent circuit of Figure 2.

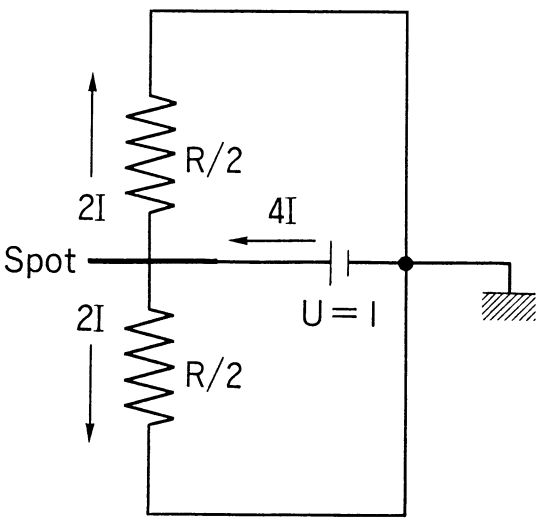

Figure 5: Equivalent circuit of Figure 3.

In Fig. 4 two resistances $R/2$ are connected in series between the potential difference, the resultant resistance is R and it is equal to the constriction R_c. In Fig. 5 two resistances $R/2$ are connected in parallel between the potential difference, the resultant resistance R_s is equal to $R/4$. Therefore the relation between the constriction resistances of Figs. 4 and 5 is

$$R_c = R = 4R_s. \tag{9}$$

By Eq. (9) the constriction resistance R_s computed by the BEM in the field of Fig. 3 is converted into the constriction resistance R_c in the field of Fig. 2.

NUMERICAL RESULT OF CONSTRICTION RESISTANCE OF CIRCULAR SPOT

The BEM for the computation of capacitances is presented [8–11]. The BEM can be applied to the computation of the resistance by replacing the capacitance and permeability with the reciprocal of resistance and conductivity, respectively, because of the mathematical analogy between the electrostatic field and steady current field.

For simplicity, the computation is carried out when a=1 (or $A=\pi$) and ρ=1 (or σ=1). The resistances when a and ρ are other values are obtained by multiplying the computed values by ρ/a. The exact solution for a=1 and ρ=1 of a circular spot is

$$R_c = 0.5. \tag{10}$$

Numerical values by the BEM for (10) and their relative errors are listed in Table I against the total number N of ring-shaped boundary elements.

As indicated in Table I the accuracy of calculated values is very high, and it is improved with the increase of element number, but the speed of improvement is very slow. Applying an extrapolation method [9–11] by which a computed value is estimated when $N\rightarrow\infty$, the computed values are plotted in Fig. 6 against $1/N$ ($N\geqq 200$). The straight line in Fig. 6 is fitted by the least-square method. As the result R_c and e become

$$R_c = 0.499992, \tag{11}$$

and

$$e = 0.00162\%. \tag{12}$$

A value with very high accuracy is computed.

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Table I. Numerical result by the BEM for constriction resistance R_c of circular spot and relative error e against number of elements N

Number of Elements N	Computed Value of R_c	Relative Error $e(\%)$
100	0.500625	0.125
200	0.500301	0.060
300	0.500196	0.039
400	0.500145	0.029
500	0.500114	0.022
600	0.500094	0.018
700	0.500080	0.016
800	0.500070	0.014
900	0.500061	0.012
1000	0.500055	0.011

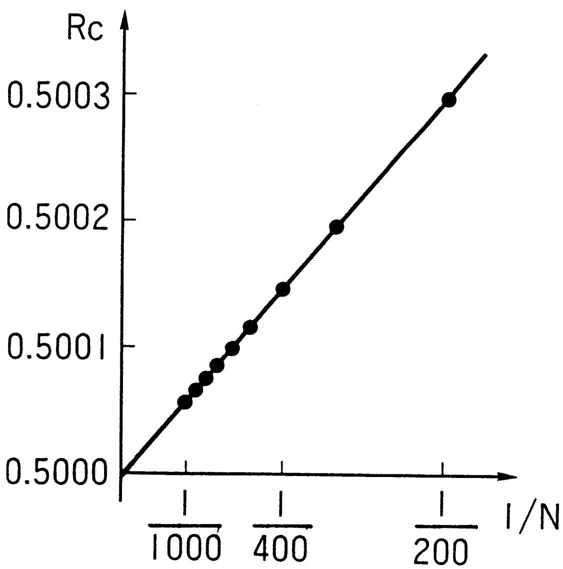

Figure 6: Estimation of constriction resistance R_c of circular spot when $N \to \infty$.

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The correlation coefficient γ of the data points in Fig. 6 is

$$\gamma = 0.9996. \tag{13}$$

The straight line of Fig. 6 is fitted to nine data points, and γ is very close to unity. It means that the fitness of the straight line is very good. When the extrapolation is applied to a few data points where the number of boundary elements is small, the computation can be carried out even on a computer without high performance and a great deal of memory.

CONSTRICTION RESISTANCE OF TRIANGULAR, SQUARE, HEXAGONAL AND OCTAGONAL SPOT

As shown in Fig. 7, four spots are inscribed in a circle with radius 1. The constriction resistances of the four spots are computed by the BEM and extrapolation method. The numerical results are listed in the second line of Table II, and compared with the circular spot.

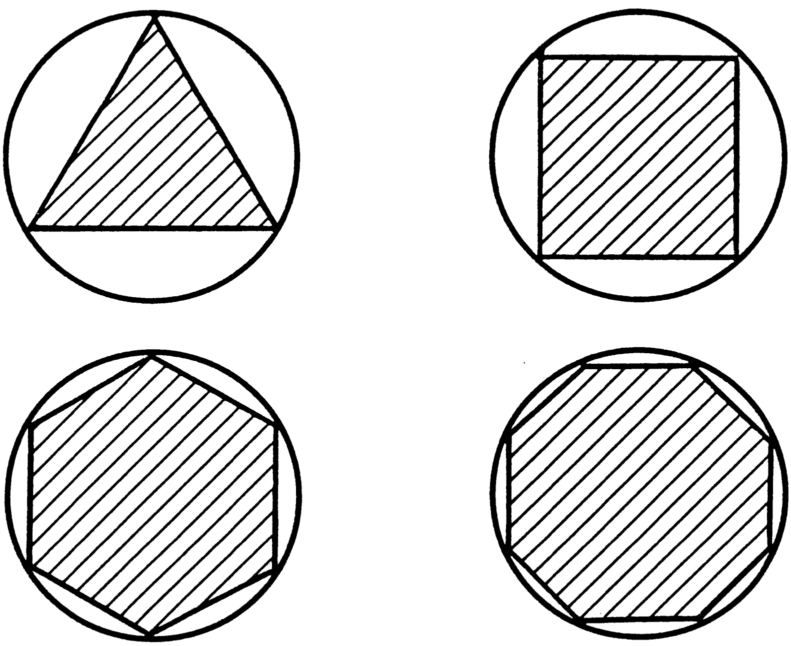

Figure 7: Triangular, square, hexagonal and octagonal spot inscribed in circle with radius 1.

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Table II. Constriction resistance by the BEM

Form of spot	Triangle	Square	Hexagon	Octagon	Circle
R_c	0.731	0.614	0.546	0.525	0.5
Area A	1.30	2.00	2.60	2.83	3.14
$R_c \times A^{1/2}$	0.833	0.868	0.880	0.883	0.886
Ratio	0.940	0.980	0.993	0.997	1

As expected, with the increase of the number of sides the constriction resistances become close to that of the circular spot. The constriction resistance is in inverse proportion to the square root of the area ($A^{1/2}$) or the circumference not to the area because of the strong current constriction effect. In the fourth line of Table II the constriction resistances of unit area of each spot are presented, and in the fifth line the ratios to the unit circle are listed. The result shows that the triangular spot is the most conductive and the circular spot is the most resistive when the areas are equal, but the difference is very small.

For a square spot, the value 0.5 of Eq. (2) computed by the FEM [1] is smaller than the value 0.868 by the BEM. The resistances computed by the FEM are always less than the exact resistance, but the FEM applied to obtain Eq.(2) is suitable to study the global property of contact conduction, and not suitable to compute the contact resistance of individual spots. To calculate a more accurate value by the FEM, the application of the FEM [5] suitable to obtain the local property of contact conduction is required. However the application of BEM to the computation of constriction resistance is more efficient and accurate, because in the BEM boundary condition (7) at the infinite distance is naturally satisfied and no computation of a great number of nodal potential values in a wide space is required.

CONCLUSION

The study of constriction resistance is important in the problem of electric contacts. In this paper the BEM is introduced to the computation of the constriction resistance of conducting spots in an electric contact surface. The advantage of the BEM is particularly demonstrated in the computation, and it is made sure that a very accurate value can be obtained by the BEM and an extrapolation method in the calculation of the constriction resistance of circular spot. Accurate values are computed for the constriction resistance of a triangular, square, hexagonal and octagonal spot. The result shows that the difference among the resistances is small when the areas of spots are equal.

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