Application of elastodynamic BEM to ultrasonic NDE

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Abstract

The elastodynamic boundary element method is applied to the ultrasonic nondestructive evaluation with an emphasis on visualization of the scattered wave field around flaws. First, the unknown quantities on the flaw boundaries are determined by the boundary element method. Then near field calculations are performed by using the integral representation of the scattered wave field. The calculated near fields are visualized in the vicinity of flaws. The results of the near field visualization are useful to interpret the far field backscattered waveforms. The backscattered waveforms at the transducer position are obtained from the far field expression of the scattered wave. It is shown from the numerical examples that the combination of the far field backscattered waveform and the near field visualization makes it possible to characterize the geometrical property of flaws.

1 Introduction

In the maintenance of structures, making sound engineering judgment on an accept/reject decision is essential. For the decision, it is of prime importance to determine the geometrical characteristics of flaws in the structural component. In nondestructive evaluation, ultrasonics provides fundamental tools for the characterization of flaw geometry\[1\]. In this paper, an application of the elastodynamic BEM to the ultrasonic NDE is investigated with an emphasis on visualization of the scattered wave field around flaws. In the pulse-echo ultrasonic NDE\[2\], a transducer generates the elastic wave in a solid and the elastic wave propagates to the flaws. In the process of the interaction of the elastic waves and flaws, the scattered waves are generated and the scattered waves propagate back to the transducer. The transducer
is usually located far away from flaws, thus the far field integral representation is used to calculate the scattered wave field at the transducer position. In order to interpret the scattered waveform received at the transducer, it is important to understand the interaction process of the elastic waves and flaws. For this purpose, the visualization of the elastic wave fields around flaws is carried out by using the near field integral representation of scattered waves. An application to characterize the geometrical properties of flaws is shown from the calculated results of scattered waveforms.

Typical numerical methods to investigate the interaction process of the incident waves with flaws are the finite difference method[3], the finite element method[4], and the boundary element method[5]. Our present numerical treatment of the transient BEM is based on the Fourier transform method in Refs.[6, 7].

2 Crack characterization problem

We consider a crack $S_C$ emanating from the circular cavity $S$ as shown in Fig.1, where the crack exists shadow side on the cavity surface in view from the transducer position. The wave field generated by the transducer becomes an incident wave $u'$ for the cavity and crack. In the interaction process of the incident wave with the cavity and crack, the scattered wave $u^S$ is generated and it propagates back to the transducer. In this paper, the scattered wave is visualized in the vicinity of the cavity and crack, and then the scattered waveform is calculated at the transducer position in the far field. The final goal is to characterize the crack location and crack length from the backscattered waveforms.

![Figure 1: Crack $S_C$ emanating from cavity surface $S$.](image-url)
3 Near field calculations

The Fourier transform of the displacement field $u$ is defined as

$$\hat{u}(x, \omega) = \int_{-\infty}^{\infty} u(x, t)e^{i\omega t} dt$$

(1)

where $\omega$ is the angular frequency. After getting solutions in the frequency domain, the time domain solution is obtained by the inverse Fourier transform

$$u(x, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{u}(x, \omega)e^{-i\omega t} d\omega .$$

(2)

In what follows, we formulate the scattering problem in the frequency domain and $\hat{u}_i(x, \omega)$ is abbreviated to $\hat{u}_i(x)$ for simplicity. Now the scattered wave $\hat{u}^S(x)$ is defined as $\hat{u}(x) = \hat{u}^S(x) + \hat{u}^I(x)$, where $\hat{u}(x)$ is the total wave field and $\hat{u}^I(x)$ is the incident wave.

The traction free boundary condition is assumed on both surfaces of the cavity and crack in this study

$$\hat{t}(x) = \hat{t}^S(x) + \hat{t}^I(x) = 0, \quad x \in S \cup S_C$$

(3)

where $\hat{t}^I(x)$ is the traction by the incident wave. For the given incident wave, the boundary integral equation on the cavity surface $S$ reduces to the following form

$$C_{ij}(x)\hat{u}_j(x) = -\int_S T_{ij}(x, y)\hat{u}_j(y) dS_y + \int_{S_C} T_{ij}(x, y)\Delta\hat{u}_j(y) dS_y + \hat{u}^I_i(x), \quad x \in S .$$

(4)

On the crack surface $S_C$, the traction form of the integral representation leads to

$$-\hat{t}^I_j(x) = n_i(x)C_{ijkl} \int_S T_{km,l}(x, y)\hat{u}_m(y) dS_y + n_i(x)C_{ijkl} \int_{S_C} T_{km}(x, y)\Delta\hat{u}_m,l(y) dS_y, \quad x \in S_C$$

(5)

where the integration by parts is used and the differentiation is carried out at the source point $y$ as $(\bullet)_m = \partial(\bullet)/\partial y_m$. In Eqs.(4) and (5), $\Delta\hat{u}_j(y)$ is the crack opening displacement $\Delta\hat{u}_j(y) = \hat{u}_j^+(y) - \hat{u}_j^-(y)$, and $T_{ij}(x, y)$ is the traction kernel defined by

$$T_{ij}(x, y) = C_{jklm}U_{li,m}(x, y)n_k(y)$$

(6)

where the isotropic elastic tensor is expressed as $C_{ijkl} = \lambda \delta_{ij}\delta_{kl} + \mu (\delta_{il}\delta_{jk} + \delta_{ik}\delta_{jl})$, and the unit normal vector $n_k(y)$ is evaluated at the source point $y$. 
In the two-dimensional elastodynamics, the fundamental solution $U_{ij}(x, y)$ has the following form

$$U_{ij}(x, y) = \frac{i}{4\mu} H_0^{(1)}(k_Tr)\delta_{ij} + \frac{1}{k_T^2} \frac{\partial^2}{\partial x_i \partial x_j} \left[ H_0^{(1)}(k_Tr) - H_0^{(1)}(k_Lr) \right]$$

(7)

where $r = |x - y|$, $k_L$ and $k_T$ are longitudinal and transverse wave numbers, and $H_0^{(1)}$ is the zeroth order Hankel function of the first kind.

Boundary integral equations (4) and (5) can be solved for $\hat{u}_j$ on $S$ and $\Delta \hat{u}_j$ on $S_C$ by the boundary element methods. Several methods of treatment for the hypersingular kernel in the crack problem have been given by Zhang & Gross.[8] The scattered wave $\hat{u}_i^S(x)$ in the near field is calculated by the integral representation

$$\hat{u}_i^S(x) = - \int_S T_{ij}(x, y) \hat{u}_j(y) dS_y + \int_{S_C} T_{ij}(x, y) \Delta \hat{u}_j(y) dS_y, \quad x \in D.$$  

(8)

4 Far field calculations

The distance of the transducer from flaws is usually large enough compared with the flaw size. In this sense, the following far field approximation

$$r = |x - y| \simeq |x| - \hat{x} \cdot y$$

(9)

is introduced to calculate the waveform at the transducer position. In this equation, $\hat{x}$ is the unit vector pointing to the field point $x$ from the coordinate origin. Introduction of Eq.(9) to Eq.(7) leads to the far field expression of the fundamental solution

$$U_{ij}^{far}(x, y) \approx \sum_{\alpha=L,T} A_{ij}^\alpha(\hat{x}) \frac{e^{i(k_\alpha|x| - \pi/4)}}{\sqrt{k_\alpha|x|}} e^{-ik_\alpha \hat{x} \cdot y}$$

(10)

where $A_{ij}^L$ and $A_{ij}^T$ have the forms

$$A_{ij}^L(\hat{x}) = \frac{i}{4\rho c_L^2} \sqrt{\frac{2}{\pi}} \hat{x}_i \hat{x}_j, \quad A_{ij}^T(\hat{x}) = \frac{i}{4\rho c_T^2} \sqrt{\frac{2}{\pi}} (\delta_{ij} - \hat{x}_i \hat{x}_j).$$

(11)

From Eqs.(8) and (10), the scattered far field can be expressed as

$$\hat{u}_i^{S, far}(x) = - \int_S T_{ij}^{far}(x, y) \hat{u}_j(y) dS_y + \int_{S_C} T_{ij}^{far}(x, y) \Delta \hat{u}_j(y) dS_y, \quad x \in D$$

(12)

where $T_{ij}^{far}(x, y)$ is the far field expression of the traction kernel obtained from Eqs.(6) and (10).
5 Incident waves

Our present aim is to detect the crack in the shadow side of the circular cavity. For this purpose, it is useful to send the transverse wave as the incident wave. The transverse wave effectively generates the creeping waves that travel along the circumference of the cavity and arrives at the crack in the shadow zone from the transducer direction.

As a model of the amplitude profile of the transducer, the Gaussian amplitude profile is adopted in this study. A transverse wave with the Gaussian amplitude profile propagating to the \( p \) direction can be written as

\[
u^I(x, t) = F(r)u^0 f(t - \frac{p \cdot x}{c_T})
\]  

(13)

where \( u^0 = e_3 \wedge p \) is the polarization vector and \( c_T \) is the velocity of the transverse wave. In Eq.(13), \( F(r) \) is the Gaussian profile of the amplitude as shown in Fig.2. The \( F(r) \) is defined as

\[
F(r) = e^{-\frac{r^2}{(d/2)^2}}
\]  

(14)

where \( r \) is the distance from the center axis of the transducer. At \( r = d \), the amplitude reduces to

\[
F(d) = e^{-4} \approx 0.0183 .
\]  

(15)

For convenience, we call \( d \) as the beam radius of Gaussian amplitude profile in this paper.

![Figure 2: Gaussian amplitude profile of the incident wave.](image)

The Fourier transform of Eq.(13) with respect to the time \( t \) is

\[
\hat{u}^I(x, \omega) = F(r)u^0 e^{ik_T p \cdot x} \hat{f}(\omega).
\]  

(16)

In the frequency domain, the incident wave is the product of the wave component \( F(r)u^0 \exp(ik_T p \cdot x) \) and the Fourier transform of the incident
waveform \( \hat{f}(\omega) \). From the linearity of the field, the wave component

\[
\hat{u}^I(x) = F(r)u_0^0 e^{ikr} P(x)
\]

(17)

can be used as the incident wave in the stage to solve the boundary integral equations (4) and (5). After solving the system of integral equations in the frequency domain, the transient solution can be obtained by multiplying the function \( \hat{f}(\omega) \) and taking the inverse Fourier transform.

As the incident waveform \( f(t) \) in Eq. (13), the following Ricker wavelet[9] is adopted in this study

\[
f(t) = \frac{\sqrt{\pi}}{2} (\alpha - 0.5)e^{-\alpha}, \quad \alpha = \left( \frac{\pi(t - t_s)}{t_p} \right)^2.
\]

(18)

Figure 3: (a) Ricker wavelet in time domain, (b) Fourier spectrum of Ricker wavelet.

Fig.3 shows the time history of the Ricker wavelet and the Fourier spectrum of it. In this figure, \( a \) is the typical length of the scatterer and it is selected as the radius of the circular cavity in the following calculation.

6 Visualization of near fields and calculation of far field waveforms

Fig.4 shows the circular cavity with a crack. The radius of the cavity is \( a \) and the length of the crack is \( h \).

The scattered near fields can be calculated from Eq.(8). The calculated near fields are visualized in the upper part of Fig.5 for the crack with the length \( h = a \). In this calculation, the incident wave is the plane transverse wave with the beam radius \( d = \infty \) in Eq.(14) and it is propagating from the bottom in this figure. The time step \( \Delta t \) is chosen to be \( \Delta t = 0.0524(a/c_T) \). The backscattered far fields can be calculated from Eq.(12) and the backscattered waveforms are shown at the lower part of
Fig. 4: Circular cavity of radius $a$ with crack of length $h$.

The first wave (1) received at the transducer position in the far field is the reflected wave from the surface of the circular cavity and it propagates with the velocity $c_T$ of the transverse wave. The second wave (2) is the reflected wave from the crack face. This second wave is propagating approximately with the Rayleigh wave velocity $c_R$ along the surface of the cavity. Fig. 6 shows the results for the case of the incident transverse wave with the Gaussian amplitude profile of $d = a$ in Eq. (14). The center of the Gaussian beam is grazing the left end of the circular cavity. In this case, it should be noted that the third wave (3) is propagating back to the transducer direction. This third wave (3) is the tip diffracted wave from the top of the crack and it is also propagating approximately with the Rayleigh wave velocity along the crack surface.

### 7 Estimation of crack location and length

The crack location can be estimated from the time difference of the first reflected wave (1) from the circular cavity and the second reflected wave (2) from the crack face in Figs. 5 and 6. If we send the Gaussian beam and the diffracted wave from the crack tip is received as in the case of Fig. 6, the crack length in the shadow zone can be estimated.

#### 7.1 Crack location

The time difference $\Delta t_1$ between the first reflected wave (1) from the circular cavity and the second reflected wave (2) from the crack face can be written as

$$\Delta t_1 = \left( \frac{a}{c_T} + 2 \frac{a\theta}{c_R} + \frac{a}{c_T} \right) = \left( \frac{1}{c_T} + \frac{\theta}{c_R} \right) 2a$$

(19)
where $\theta$ is the angle measured from the perpendicular direction to the propagation direction of the incident wave as shown in Fig.4. From Eq.(19), the angle $\theta$ that estimates the crack location is expressed as

$$\theta = \left(\frac{\Delta t_1}{2a} - \frac{1}{c_T}\right) c_R .$$

(20)

In the result of Fig.5 for the backscattered waveform, the time difference $\Delta t_1$ is $\Delta t_1 = 5.445a/c_T$. In this case, the crack location is estimated from Eq.(20) as

$$\bar{\theta} = 1.723c_R/c_T = 1.58(rad) = 90.5^\circ$$

(21)

where the approximate value of $c_R/c_T = 0.918$ for the material with the Poisson ratio $\nu = 1/4$ is used. The estimated crack location $\bar{\theta} = 90.5^\circ$ approximates well the real location $\theta = 90^\circ$ of the crack.

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**Figure 5:** Visualization of the scattered near fields ((a)~(f)), and the backscattered waveform in the far field (g). The incident wave is the plane transverse wave with $d = \infty$ in Eq.(14).
Figure 6: Visualization of the scattered near fields ((a)–(f)), and the backscattered waveform in the far field (g). The incident wave is the transverse wave with Gaussian amplitude profile of \( d = a \) in Eq.(14).

In the result of Fig. 6 for the incidence of the Gaussian amplitude profile with \( d = a \), the time difference \( \Delta t_1 \) is \( \Delta t_1 = 5.440a/c_T \). In this case, the crack location is also estimated to be \( \theta = 90.5^\circ \) from Eq.(20).

### 7.2 Crack length

The crack length can be estimated from the time difference \( \Delta t_2 \) between the second wave \( 2 \) reflected from the crack face and the third wave \( 3 \) diffracted from the top of the crack. The time difference is expressed as \( \Delta t_2 = h/c_R + h/c_R = 2h/c_R \). The crack length is estimated from

\[
h = \frac{c_R \Delta t_2}{2}.
\]
In the results of Fig.6, the time difference $\Delta t_2$ is $\Delta t_2 = 2.152a/c_T$. From Eq.(22), the crack length is estimated to be

$$h = 1.076(c_R/c_T)a = 0.988a$$

(23)

where $c_R/c_T = 0.918$ is used and the estimated crack length is a good approximation of the real crack length of $h = a$.

8 Conclusion

The incident transverse wave with the Gaussian amplitude profile generates the crack tip diffraction and it propagates back to the transducer direction. If we can observe this tip diffraction, it may contribute to estimate the crack length in the shadow side of the cavity.

References


