Application of BEM and cellular automata to determining optimal shapes of the sound-insulating wall

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1 Introduction

There are many optimizing procedures using sensitivity analysis or genetic algorithms, and they are successfully applied to a wide variety of practical problems. In general, however, an optimizing procedure using sensitivity analysis frequently fails to find the global minimum of an objective function, depending on the initial values of parameters. The present paper is concerned with application of cellular automata [1] [2] [3] [4] [5] combined with the boundary element method to finding optimal shapes of the sound-insulating wall. In application of the cellular automata, the domain of interest is discretized into a number of uniform cells. Some quantity defining the cell state is assigned each cell, and then modified by a transition rule under the condition that a local rule for cells is satisfied. An optimal solution can thus be found in an iterative procedure. No sensitivity analysis is required in the cellular automata, and it is reported that the optimal solution or a ‘satisfactory’ solution can be rather easily obtained under a simple local rule and also a simple transition rule.

The present paper aims at finding optimal or satisfactory shapes of the sound-insulating wall for auto-highways. A two-dimensional model of the auto-highway with wall, in which a sound-insulating wall stands perpendicularly to the infinite horizontal plane of the ground and a point source of noise is located at the central point on the road. Details of the cellular automata applied together with the
boundary element method are discussed in authors’ separate paper[6]. After the optimization procedure using BEM and cellular automata is briefly explained in this paper, it is applied to finding the optimal shapes of the sound insulating wall under several important situations. It will also be demonstrated that an optimal or satisfactory shape of the wall which can drastically reduce noise can be found if the cells change even in the diagonal directions.

2 Boundary element analysis of acoustic fields

Under the assumption of time-harmonic vibration with an infinitesimal amplitude, the acoustic fields are governed by the non-homogeneous Helmholtz equation as follows[7]:

\[ \nabla^2 p(x) + k^2 p(x) + f(x) = 0 \]  \hspace{1cm} (1)

where \( p \) is the sound pressure, \( f \) the source term, and \( k \) the wave number.

The boundary element method [8] for the acoustic fields governed by the above Helmholtz equation(1) is briefly explained. The integral expression of eqn(1) multiplied with the fundamental solution \( p^*(x, y) \) over the whole domain of interest is twice integrated by parts. The boundary integral equation can eventually be obtained in the regularized form if the uniform potential condition is taken into account, and it can be expressed as follows [8]:

\[
\left[ \int_\Gamma \{q^*(x, y) - Q^*(x, y)\} \, d\Gamma(x) \right] p(y) \\
+ \int_\Gamma q^*(x, y) \{p(x) - p(y)\} \, d\Gamma(x) \\
= i\omega \rho \int_\Gamma p^*(x, y) v(x) d\Gamma(x) + Ip^*(x^s, y) 
\]  \hspace{1cm} (2)

where \( Q^*(x, y) \) is the normal derivative of the fundamental solution for the Laplacian operator, and \( I \) the intensity of point sound source. It is interesting to note that the standard Gaussian numerical quadrature can be applied to numerical computation of discretized set of equations, because the regularized integral equation is employed in this study. A semi-infinite acoustic field of two dimensions is treated in this study, but eqn(2) can be used even for such problems without any difficulty, because the Sommerfeld radiation condition is satisfied at infinity by the fundamental solution of the Helmholtz equation.

The boundary integral equation (2) can yield a set of equations for the sound pressure \( p \) and the particle velocity \( v \), if the boundary \( \Gamma \) is discretized into boundary elements. Application of the boundary conditions provides the solution of the problem[8]. As has been well known, the solution of the external problem is
influenced by fictitious eigenvalues of the internal problem. To circumvent this difficulty, an appropriate number of the boundary integral equations with the source points located in the external domain are supplemented into the system of equations, and the solution is obtained by using the least-square method[8].

3 Application of cellular automata

In the cellular automaton (CA), the domain of interest is discretized into a number of uniform cells. Some quantity which represents the cell state is assigned to each cell, and then is modified by a transition rule under the condition that a local rule for cells is satisfied[3] [4]. An optimal solution can thus be found in an iterative manner. A concrete procedure of the CA applied to a two-dimensional problem will be explained in the following.

Triangular or rectangular cells can be used for the two dimensional problems. In this study, rectangular cells are employed and the domain of investigation is discretized into cells of a uniform size. The Moore neighbor cells are defined so that the target cell is surrounded by neighbor eight cells. Then, we introduce a local rule determining a mutual relation between the neighbor cells for the current state of iterative computation. In this study, a particular portion of the sound-insulating wall is altered to reduce the sound pressure level (SPL) of a mean sound pressure of several evaluation points in the acoustic field. Only the domain of investigation is discretized into uniform rectangular cells as shown in Fig. 1. The fictitious cells are introduced so that the local rule can be applied even if the target cell is located at the corners of the domain.

![Feasible area of wall](image)

Figure 1: Feasible region divided into square cells
3.1 Local rule

A local rule for the target cell is determined by considering some constraints on the wall shape. In this study, we shall impose the constraint that the wall should be a simply connected domain. It is assumed that the state of each cell can be ‘Alive’ or ‘Dead’. It is checked whether the Moore neighbor cells follow the local rule or not, by assuming that the target cell changes from the current state to its opposite state. If the neighbor cells follow the local rule, no change in the cell state is assumed and the next cell is examined. If the neighbor cells do not follow the local rule, the assumed state of cell is adopted and boundary element analysis is carried out under the modified shape of wall. In this study, we also assume that there can be cells which always remain, called ‘Remaining Cell’.

Type A : Upper and lower cells as well as right and left cells are all ‘Alive’ or ‘Dead’.

Type B : Cells on the diagonal line are ‘Alive’ and their surrounding cells are ‘Dead’. In addition, one cell among the neighbor cells is ‘Alive’.

Type C : Cells on the diagonal line and their surrounding cells on one side are all ‘Alive’. In addition, the surrounding cells on the other side are ‘Dead’.

Type D : Upper and lower cells are ‘Alive’, and right and left cells are ‘Dead’. Or, upper and lower cells are ‘Dead’ and right and left cells are ‘Alive’.

3.2 Method of evaluation

As an evaluation value of optimization, we shall take the mean value of absolute sound pressures obtained at several points of evaluation and its sound pressure level (SPL) in dB.

3.3 Transition rule

We shall employ a transition rule in which three cells are added in one direction for single BE analysis. In what follows, such a transition rule is explained.

Step 1 In ‘Procedure 1’ of Fig. 2, the target cell of cell 1 is assumed to be ‘Alive’ (See ‘Procedure 2’ in the figure.)

Step 2 Apply the local rule to the target cell of ‘Alive’ to check whether the neighbor cells follow the local rule. If the local rule holds for the neighbor of the target cell, the target cell should be ‘Dead’. Then, we proceed the target cell to the next one.
Step 3 If the local rule does not hold for the target cell of ‘Alive’, we add another cell of ‘Alive’ in the same direction. If the local rule holds for the added cell, the two cells of ‘Alive’ should be ‘Dead’. Then, we proceed the target cell to the next cell.

Step 4 When three cells in the same direction are ‘Alive’ as shown in ‘Procedure 4’, Step 3 is examined. This step can be examined even if the target cell is located at the boundary of domain under consideration, because fictitious cells are assumed along the boundary.

Step 5 If the local rule does not hold in the steps 2 to 4, boundary element analysis is carried out by adding three cells in one direction.

Step 6 The best evaluation value and the corresponding data of shape are stored as the new value for the next loop of computations from Step 2.

Step 7 If the evaluation value does not change anymore, it is assumed that the optimal shape is obtained.

4 Numerical results and discussions

To demonstrate the versatility of the proposed method for optimal shape design of the sound-insulating wall, we now consider a two-dimensional model shown in
Fig. 3 in which the sound source is located at point A. It is assumed that the infinite horizontal plane and the sound-insulating wall are subject to the rigid condition in which particle velocity $v = 0$. Furthermore, we assume that the intensity of point source A is $(2.0, 1.0) \text{[Pa]}$, the sound speed $C_0 = 340 \text{[m/s]}$, and density of mass $\rho = 1.2 \text{[kg/m}^3\text{]}$. It is noted that symmetry with respect to the axis $x_1$, the infinite horizontal plane, is taken into account, whereas symmetry with respect to the axis $x_2$ is not considered. Four evaluation points of sound pressure are located on the horizontal ground as shown in Fig. 3. Assuming that the shape of sound-insulation wall can change in the area hatched in the figure, this area is divided into rectangular cells of a uniform size $0.2 \times 0.2 \text{[m]}$. It is reported that a wall shape, the so-called reindeer-horn shape, can reduce drastically noise. We now assume this form as the initial wall shape and apply the present optimization procedure to find a more effective shape for reduction of noise. The initial shape is shown in Fig. 4, and its cell division in Fig. 5. The frequency is assumed to be $500 \text{[Hz]}$. The optimal shape of the wall found by the present procedure is shown in Fig. 6, and a convergence property is illustrated in Fig. 7 for evaluation of the mean SPL. Next, we shall show the results obtained for a simple noise model which includes two components of $400 \text{[Hz]}$ and $500 \text{[Hz]}$. The optimal shape is shown in Fig. 8, which has been found from the initial shape shown in Fig. 4. The convergence property is shown in Fig. 9.
Figure 4: Initial wall shape

Figure 5: Initial cell condition
Figure 6: Final wall shape for noise of 500 [Hz]

Figure 7: Change in sound pressure level for frequency of 500 [Hz]
Figure 8: Final wall shape for noise of 400 [Hz] and 500 [Hz]

Figure 9: Change in sound pressure level for each frequency of 400 [Hz] and 500 [Hz]
5 Conclusion

The boundary element method (BEM) has been combined with a cellular automaton (CA) to find optimal shapes of the sound-insulating wall. A new transition rule has been proposed for application of CA, and the computer code has been developed. A few examples of the sound-insulating wall were investigated, and the results obtained were discussed. It was demonstrated that the present optimization procedure could find a wide variety of optimal or most suitable shapes of the wall, which may imply importance in practical use.

As future work in this direction, we can recommend optimal shape design of the sound-insulating wall under various constraints for a wide range of frequency and real noise.

References