Poroelastic analyses near subsurface excavations and underneath a dam

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Abstract

The presence of a fluid in rocks can engender a coupling between the solid and fluid deformations. Biot theory of poroelasticity provides the correct coupling between the hydraulic and mechanical effects. In this paper two types of problems are investigated using a poroelastic boundary element formulation in the Laplace transform domain. The excavation induced pore pressure and stresses are treated first. Comparison of results for circular and elliptical boreholes shows that excavation geometry significantly modifies the stress concentration. In addition, by studying the interaction between two neighboring boreholes it is found that drilling the second hole in the direction of maximum stress is less likely to cause tensile failure. In the second type of problems, the evolution of stress below a dam has been analyzed. It is found that a rapid impoundment of the reservoir reduces the effective normal horizontal stress and may lead to failure in tension and/or slippage on pre-existing fractures.

1 Introduction

Geological formations are often saturated with a fluid. Stress analysis in such formations should take account of the interactions between the solid and the fluid. Biot theory of poroelasticity [1] is the theory that provides the correct coupling between the hydraulic and mechanical effects. It improves upon the conventional Terzaghi consolidation theory. It reveals phenomena which uncoupled theories could not predict. Various analytical and numerical methods have been developed for the solution of boundary value problem
in poroelasticity. The boundary element approach includes the direct and the indirect methods. A time domain direct BEM formulation has been presented in [2]. This paper reports on the application of a direct coupled BEM that was developed based on the reciprocal theorem in the Laplace transform domain [3]. The method has been applied herein to solve a number of problems of scientific and industrial interest. Specifically, two types of problems are investigated. In the first type (exterior domain), we examine the transient stresses induced by underground excavations such as wells drilled for production of petroleum resource. Particularly, the influence of well geometry and a neighboring opening on the stress concentration around the well and its stability is studied. In the second type of problems (interior domain), the stress evolution below a dam is analyzed. It is assumed that the reservoir is initially without water. The rapid impoundment of the reservoir induces a stress loading on the ground, together with a seepage flow. The induced stresses and pore pressures have been known to cause slippage on pre-existing fractures underneath dams [4].

2 Equations of poroelasticity

The theory of poroelasticity, developed by Biot, describes the linear deformation of saturated porous media. In this paper, only the main features of the theory will be briefly discussed. Theoretical details and mathematical formulations are beyond the scope of this paper and can be found elsewhere [5].

The main characteristic of saturated porous media deformation is that it is a transient process due to diffusion resulting from the presence of a freely moving fluid. The fundamental aspects of the deformation of fluid-saturated porous media described by Biot's theory are: the sensitivity of the volumetric response of the rock to the rate of loading; volumetric variations caused by changes in pore pressure; and variation of pore pressure due to changes in mean stress [5]. Under undrained conditions, the increase in pore pressure caused by an increase in confining pressure, $\Delta P$, is described by:

$$\Delta p = B \Delta P$$

where $B$ is Skempton’s pore pressure coefficient. A rock volumetric expansion, $\Delta V$, is induced by an increase in the pore pressure, $\Delta p$, which can be expressed as:

$$\frac{\Delta V}{V} = \frac{\alpha \Delta p}{K}$$

where $K$ is the rock bulk modulus, and $\alpha$ is Biot’s parameter, describing the ratio of volumetric deformation of the fluid to that of the bulk solid. By combining the constitutive equations, Darcy’s law, the continuity equations, and the equilibrium equations, one can obtain a set of coupled field equations:
where $u$ is the solid displacement, $G$ represents the shear modulus; $c$ is the diffusivity or the consolidation coefficient; $M$ and $\kappa$ are Biot's modulus and the permeability coefficient, respectively.

### 3 Boundary element formulation

The derivation of the boundary integral equations has been reported in [3,6]. The procedure is based on the Betti-Rayleigh reciprocal theorem, using the fundamental solution for an impulse point force. It yields integral equations for pore pressure and stresses:

$$G u_{i,jj} + \frac{G}{1 - 2\nu} u_{j,ji} - \alpha p_{,i} = 0$$  \hspace{1cm} (3)

$$\frac{\partial p}{\partial t} = cp_{,jj} - M\alpha \frac{\partial \varepsilon_{kk}}{\partial t}$$  \hspace{1cm} (4)

where $u$ is the solid displacement, $G$ represents the shear modulus; $c$ is the diffusivity or the consolidation coefficient; $M$ and $\kappa$ are Biot's modulus and the permeability coefficient, respectively.

### 4 Applications

We will now present and solve a few exterior and interior domain problems using the Laplace transform BEM outlined above. In all examples except that of Figure 2, it is assumed that $\nu = 0.2$, $\nu_u = 0.31$, $B = 0.55$, and $G = 8580$ MPa.

Production wells are drilled to reach the hydrocarbon bearing layers. Wells can cost millions of dollars to drill, therefore, the stability of these
excavation is of paramount importance. To check for possibility of failure, a stress analysis is necessary. Consider the problem of a vertical hole drilled in a saturated rock subjected to a uniform in situ anisotropic stress and a pore pressure. The problem can be analyzed in two dimensions (Figure 1) by assuming plane strain conditions. This is valid provided that the borehole axes is parallel to a principal stress direction, and the time required for drilling a length equal to five times the hole's radius is much smaller than the characteristic time \( \frac{a^2}{c} \), where \( a \) is the radius of the well. This is an exterior domain problem and is solved by dividing the boundary into 16 quadratic elements. The loading may be decomposed into three fundamental modes [7]: (1) a far-field isotropic stress \( \left( \frac{P}{S_0} = 5 \right) \), (2) a virgin pore pressure \( \left( \frac{P}{S_0} = 2 \right) \), and (3) a far-field deviatoric stress \( \left( \frac{S_0}{2MP_a} = 1 \right) \). \( S_0 \) and \( P \) represent the deviatoric and hydrostatic components of the in situ stress tensor, respectively, and \( p \) is the formation pore pressure. Mode 3 loading is the only one that produces coupled poroelastic effects. This is illustrated in Figure 2 which shows the radial variation of pore pressure near the borehole wall for three values of the transform parameter \( s^* \), where \( c^* = sa^2/c \). These results are in agreement with those reported in [6]. The induced stresses and pore pressure at \( t = 180 \) sec are shown in Figure 3.

4.1 Multiple openings

The presence of a nearby opening can significantly alter the stress and pore pressure fields around an underground excavation. This is an important design concern when drilling a multilateral well consisting of an existing (mother) wellbore from which a number of lateral wells branch out [8]. This situation is depicted in Figure 4 for a single lateral well. Generally, in order to study the stability of the junction, a three-dimensional analysis would be needed. However, when focusing on the region between the two wells a two-dimensional analysis is useful. The problem cross-section is shown in Figure 5. Two openings having the same radius are separated by a distance of 4 cm (\( \lambda = 1.2 \)). Figure 6 and 7 illustrate the induced stresses and pore pressures around the mother well when the lateral hole is drilled to its east (CR) and south (CL), respectively. It can be observed that drilling the secondary well in the direction of the minimum horizontal stress \( (S_x) \) results in higher compressive stresses at \( \theta = 0 \), and lower tangential stresses at \( \theta = 90 \). Therefore, the risk of compressive failure and tensile failure of the borehole wall is increased at those locations, respectively. This situation is not as severe if the second well is kicked off in the direction of the maximum horizontal stress \( (S_y) \).

4.2 Influence of excavation geometry and orientation

Next we will consider the influence of excavation geometry on wellbore stability. The motivation for this problem is the fact that many wellbores tend
to become elliptical, i.e., $C \neq 1$ (Figure 8) under the influence of the in-situ stresses [9]. The safe density of the drilling mud used in the operations is affected by the stress concentrations around the wellbore. Therefore, mud support calculations based on a circular ($C = \frac{a}{b} = 1$) geometry may cause problems. Furthermore, future fracturing operations are also affected by the change in hole geometry. As for the circular geometry, the problem is solved using 16 quadratic elements. Figure 10 shows the induced total stresses and pore pressure. It can be seen that hole ellipticity in the direction of $S_x$, reduces the induced total tangential compressive stress and the pore pressure. The net result is a reduction in the effective hoop stress (Figure 11). The implication is that an elliptical wellbore will be more likely to fail in tension due to the pressure of the mud column. Hence a mud with a lower density than that for a circular well need be used. The situation is reversed if the ellipticity were to occur in the opposite orientation.

4.3 Stress and pore pressure under a dam

As an example of the interior domain type problems, the rapid impoundment of a reservoir and the ensuing ground loading and seepage flow are considered. A critical case of sudden increase of the water level by 20 m is examined. Such sudden increases in water level have been reported in [9]. The dam is considered to have a height of 30 m and a length of 220 m (Figure 9). The rock under the dam in assumed to have the same properties as in previous examples, however, a permeability of $7.67 \times 10^{-6}$ darcy is assumed. Figures 12-13 illustrate the induced stresses and pore pressures under the reservoir and the dam. The rise in the water level in the reservoir leads to advancement of the fluid pressure which affects the stress underneath the dam. Figure 12 shows the pore pressure and stress values at two different times at a depth of 30 m. It can be observed that both pore pressure and horizontal stress increase with time.

![Figure 1. Circular hole in an infinite medium.](image-url)
Figure 2. Mode 3 response ($\nu=0.2, \nu_u=0.4$) Figure 3. The case of a circular well ($\theta = 0$).

Figure 4. A multilateral well.

Figure 5. Multilateral problem, 2D geometry.

Figure 6. Results for mother well ($\theta = 0$). Figure 7. Results for mother well ($\theta = -90$).
Although small in magnitude, because of the small change in the reservoir height, the induced stresses and pore pressures can have a significant effect. If at a depth of 50 m, the lithostatic in situ vertical stress is assumed to be 1.3 MPa, it would yield a horizontal stress of 0.43 MPa. Assuming a normal hydrostatic distribution, the in situ pore pressure at that depth is 0.52 MPa. Therefore, as can be observed in Figure 13, the sudden increase in water level increases the vertical stress and can cause the effective horizontal stress to become tensile. Hence, the rock may fail or slippage may occur along pre-existing fractures in the subsurface. Small scale failure may promote leakage and large scale rock failure may cause reservoir induced earthquakes [9].
Figure 12: Results for a given depth.

Figure 13: Results for various locations.
References


