Lumped parameter modelling of diastolic function

P. Verdonck, P. Segers, R. Verhoeven, D. De Wachter
Hydraulics Laboratory, University of Gent,
Sint-Pietersnieuwstraat 41, 9000 Gent, Belgium

Abstract

With the advent of transesophageal echocardiography, pulmonary venous flow is easily obtained in all patients by pulsed-wave Doppler echocardiography. Therefore it has become a current topic of clinical investigation as a part of an assessment of left ventricular filling dynamics. In order to understand the relation between left ventricular function and Doppler indices, a lumped parameter model of the flow through the left heart is developed. The model predicts the time course of pressure and flow in the pulmonary veins and the left heart chambers given pulmonary venous compliance, inertance and resistance, left atrial and ventricular stiffness and mitral valve impedance. A case study is presented.

1 Introduction

Normal systolic function is the ability of the heart to deliver adequately a stroke volume to meet the metabolic needs of the body. In this context normal diastolic function is defined as the ability of an adequate filling of the ventricle at low pressure [1]. Regardless of the systolic contractile state the heart can only pump the amount of blood it receives. Therefore diastolic filling of the left ventricle is a primary determinant of cardiac output. Diastolic dysfunction of the left ventricle is an important cause of cardiac morbidity [2]. It appears frequently in patients with arterial hypertension and normal systolic function.

Most attempts to examine diastolic behaviour focused on the relation between the change in ventricular pressure and volume. Unfortunately these measurements require invasive techniques (cardiac catheterization) which are, for ethical reasons, difficult to repeat and lack the potential to assess the effects of exercise. Therefore one has searched for non-invasive methods for assessing diastolic function like Doppler echocardiography.
One can use the mitral flow velocity curves as a conceptual framework to study diastolic function. Mitral flow velocity pattern (measured clinically at mitral valve leaflets tips) represents the relative change between left atrial and left ventricular pressure. In vivo derived indices are usually restricted to the E-velocity (maximum early filling velocity), deceleration time DT and A-velocity (maximum velocity during atrial contraction). Also the isovolumic relaxation time IVRT is determined as the time between aortic valve closure AVC and mitral valve opening MVO (figure 1).

![Diagram representing ECG, pulmonary venous flow velocity and transmitral flow velocity.](image)

The pulmonary veins conduct blood from the lungs to the left atrium. Normal pulmonary venous flow is composed of systolic S and diastolic D forward flow and a small reversal R of flow with atrial contraction (figure 1). Recently, Doppler echocardiographic assessment of pulmonary venous flow has contributed in characterizing left ventricular function in various diseases of the left heart. The clinical practice presently includes: quantification of mitral stenosis [3] and mitral regurgitation [4,5], diagnosis of left atrial pressure [6], further understanding of restrictive myocardial diseases, constrictive pericarditis [7] and hypertensive hearts [8]. Nevertheless the clinical observations are not always well understood due to both the impact of technical related parameters (TEE or TTE, left upper or right upper pulmonary vein, position of sample volume) and the influence of physiological changes like age [9], respiration [10], heart rate, loading conditions [11], right and left heart compliance, status of mitral valve [3-5]. Therefore a lumped parameter model of pulmonary venous and mitral valve flow is useful to unravel these clinical flow registrations.

### 2 Lumped parameter model

In order to study the effect of both isolated and complex changes in Doppler indices of pulmonary venous flow velocity Thomas’ fluid dynamics model of mitral valve flow during early filling is extended [12,13]. Pulmonary veins, left atrium and left ventricle are modelled as an elastic chamber with compliance C. The amount of blood flow in between is determined by a resistance R and an inertia L (figure 2).
Atrial contraction is modelled by introduction of a third order function (S-shaped curve) in the exponential pressure-volume relation.

\[
P_a = P_{ao} e^{\alpha_a V_a} \left[ 1 + 3K \left( \frac{t - t_{ac}}{T_c} \right)^2 - 2K \left( \frac{t - t_{ac}}{T_c} \right)^3 \right]
\]

where \(P_a\) = left atrial pressure [mmHg], \(P_{ao}\) = left atrial pressure at zero volume [mmHg], \(\alpha_a\) = left atrial stiffness [l/ml], \(V_a\) = left atrial volume [ml], \(t_{ac}\) = onset of atrial contraction [msec] and \(T_c\) = duration of atrial systole [msec]. This pressure-volume is only valid if \(t_{ac} \leq t \leq t_{ac} + T_c\).

For any other time equation (2) is valid:

\[
P_a = P_{ao} e^{\alpha_a V_a}
\]

1 + \(K\) is the ratio of maximal atrial pressure \((t_{ac} + T_v)\) to the pressure at \(t_{ac}\) in case of an isovolumic contraction. In this way \(K\) stands for the contractility of the left atrium to increase the pressure during atrial contraction.

To model active left ventricular relaxation an extra term is added to the exponential pressure-volume relation:

\[
P_v = P_{vo} (1 + \Gamma e^{-\tau}) e^{\alpha_v V_v}
\]

where \(P_v\) = left ventricular pressure [mmHg], \(P_{vo}\) = left ventricular pressure at zero volume [mmHg], \(\Gamma\) = ratio of ventricular pressure at onset of ventricular relaxation to the pressure when the left ventricle is fully relaxed for a constant volume, \(\tau\) = isovolumic relaxation constant [msec], \(\alpha_v\) = left ventricular stiffness [l/ml] and \(V_v\) = left ventricular volume [ml].

The mitral valve apparatus is modelled as an imaginary cylindric column of blood with length \(l\), area \(A\) and density \(\rho\). For known blood density \(\rho\), pulmonary veins characteristics \((R_{pv}, L_{pv}, C_{pv})\), mitral impedance \((l,A)\), compliance of left heart chambers \((C_a, C_v)\), atrial \((K, T_c, t_{ac})\) and ventricular \((T, \tau)\) pressure volume parameters and given initial pressure values, flow and pressure are computed by integrating five coupled differential equations. \(R_c\) and \(R_p\) represent respectively convective and unsteady flow resistance.
The numerical solution of this set of five equations, solved by a fourth-order Runge-Kutta procedure on a microcomputer, gives the time course of the five variables $P_v, P_s, P_v, Q_{pv}$ and $Q_{mv}$. Since both pulmonary venous and mitral valve cross-sectional area are constant, the time variation of flow coincides with the velocity.

### 3 Results

#### 3.1 Normal pattern

The left pannels in figures 3, 4 and 5 represent respectively the computed time course of left atrial ($P_a$) and ventricular pressure ($P_v$), transmitral velocity and pulmonary venous velocity given a reference set of parameters: left atrial stiffness (0.017 l/ml), left ventricular stiffness (0.015 l/ml), mitral impedance (constant valve area = 3 cm², mitral valve leaflet length = 2 cm), pulmonary venous compliance (3.5 ml/mmHg), resistance (0.015 mmHg, s/ml) and inertia (0.002 g/cm²), left atrial ($T_{ae} = 320$ ms, $t_{ae} = 40$ ms, $K = 0.6$) and ventricular ($T = 17$, $T_v = 30$ ms) pressure volume parameters for a start value of left atrial pressure of 8 mmHg. Both the morphology and values of the computed results are physiologically relevant.

#### 3.2 Preload dependency

The effect of an isolated preload change on both pulmonary venous and mitral flow velocities is studied by changing the start value of the left atrial pressure (8, 12, 16 and 20 mmHg) keeping all other parameters constant. Table 1 summarizes the computed Doppler indices: peak velocities [cm/s], time of peak velocity after onset of diastolic filling [msec] and the velocity integral [cm]. Also the duration of the ventricular relaxation from 100 mmHg to the onset of mitral valve flow $t_r$ [msec] and the duration of diastolic filling $t_{d}$ [msec] are given in table 1.
Some derived Doppler indices are given in table 2: peak E/A ratio, peak S/D ratio, velocity integral E/A ratio, velocity integral S/D ratio and velocity integral S/S+D ratio.

These results are in accordance with in vivo results. Keucherer et al. [6] correlated Doppler variables of pulmonary venous flow with simultaneously obtained mean left atrial pressure measured directly with catheters in patients undergoing cardiovascular surgery. They found that the systolic velocity-time integral expressed as a function of total systolic plus early diastolic velocity time integral correlated strongly with mean left atrial pressure (r = -0.88) confirmed by our computed correlation of -0.98 (p < 0.01). In vivo observations show that elevation of mean left atrial pressure results in a shift of higher pulmonary venous flow velocities towards early diastole: peak systolic velocities and systolic velocity-time integrals decrease while peak early diastolic velocities and early diastolic velocity-time integrals increase.
The reduction of the systolic peak is not found (table 1) due to the absence of a ventricular contraction in the model. During ventricular ejection there is a movement of mitral annulus toward the apex, decreasing left atrial pressure and thus increasing the systolic wave in the pulmonary veins.

3.3 Decrease in left ventricular compliance
Table 3 summarizes the set of parameters changed to simulate the combined effect of a stiffer left ventricle (higher $\alpha_v$), and a higher filling pressure $p_{ao}$. The impact on pressure and velocities is displayed in figures 3 - 5. The clinically derived indices are summarized in table 4.

**Figure 3**: Left atrial and left ventricular pressure for increased left ventricular stiffness.

**Figure 4**: Transmitral flow velocity patterns for increased left ventricular stiffness.

**Figure 5**: Pulmonary venous velocity patterns for increased left ventricular stiffness.

<table>
<thead>
<tr>
<th></th>
<th>(A)</th>
<th>(B)</th>
<th>(C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{ao}$ [mmHg]</td>
<td>8</td>
<td>12</td>
<td>16</td>
</tr>
<tr>
<td>$\alpha_v$ [1/ml]</td>
<td>0.015</td>
<td>0.020</td>
<td>0.025</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>17</td>
<td>20</td>
<td>23</td>
</tr>
<tr>
<td>$K$</td>
<td>0.6</td>
<td>0.4</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Table 3. Model parameters for combined increase in left ventricular stiffness and filling pressure.
Table 4. Doppler indices derived from computer results for increasing left ventricular stiffness.

As the left ventricle becomes stiffer a restrictive mitral flow velocity pattern is observed due to the restricted filling. An increase in E/A ratio, a decreased deceleration time DT (< 160 msec) and isovolumic relaxation time IVRT lower than 70 msec. As the diastolic forward flow in the pulmonary veins reflects the diastolic left ventricular filling also a higher D wave is observed reducing the S/D ratio (table 4, figure 5). Again the atrial reversal peak is increased due to both a reduced left ventricular compliance and a higher left atrial pressure (figure 5).

Figure 3 is an illustration of the effect of keeping stroke volume constant while increasing left ventricular stiffness. For any given volume the pressure in the more compliant left ventricle (panel A) is lower, similar as the mean left atrial pressure. The atrioventricular pressure gradient has also a slower frequency of oscillation in the more compliant left ventricle. This has its consequence in the transmitral velocity pattern (figure 4). The higher the stiffness of the left ventricle, the steeper the early filling wave velocity. The latter is observed in dogs with dilated cardiomyopathy [14].

Figure 6: Oscillographic record of left ventricular and left atrial pressure obtained from dog with congestive heart failure following three weeks of rapid ventricular pacing.

4 Conclusion

The lumped parameter model presented in this study provides a manageable, mathematical basis for the assessment of left ventricular diastolic function based on Doppler echocardiography derived indexes. It is shown that increasing ventricular stiffness and filling pressure result in an increased E/A ratio, a decreased deceleration time and isovolumic relaxation time and a higher pulmonary venous forward and reversed flow. These events
are also observed in dog and in vivo studies. In this way the model is a helpful tool to unravel the complex Doppler flow velocities recordings in search for a non-invasive procedure to assess diastolic filling.

5 Acknowledgement

This work is part of a concerted action program of the University of Gent supported by the Flemish Government : GOA 95003.

6 References