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Measuring the fractal dimension of EEG signals: selection and adaptation of method for real-time analysis

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ABSTRACT

In this paper we present our results in real-time EEG data analysis. We developed a real-time method for serial fractal dimension estimation of consecutive overlapping small epochs of the EEG signal; it is a modification of the Higuchi method [10] which gives a good approximation to the fractal dimension for a small number of points. Additional analysis for establishing the optimal epoch and overlapping segment sizes was also performed.

1. INTRODUCTION

Fractals are mathematical sets with a high degree of geometrical complexity that can model many natural phenomena (Mandelbrot [14]). Fractal images are the best known class of fractals, but there are also numerous natural processes described by time-series measurements that are fractals (Mandelbrot [14], Voss [15]); e.g., noises with power spectrum $1/\omega^{\beta}$, econometric and demographic data, pitch variations in music signals, EEG and ECG signals.

A very important characteristic of fractals, useful for their description and classification, is their fractal dimension. Intuitively, it measures the degree of their boundary fragmentation or irregularity over multiple scales. The dimension can be computed from several different measures: the capacity dimension, the correlation dimension, the information dimension and Lyapunov exponents (Farmer, Ott and Yorke [16]). Each of these measures is applicable only to an enormous number of points and takes a long time for calculation.

The EEG is an objective measure of brain activity that provides a global description of brain dynamics. Recently several studies have shown that

EEG signals may exhibit chaotic behaviour with complex dynamics (Babloyantz [1], Layne et al. [2], Babloyantz and Destexhe [3], Pijn et al. [4], Pijn and Lopes [5], Jinghua and Xiang-bao [6]). The fact that the attractor of the EEG signal has a not very big fractal dimension (4-8 for different normal brain states and 2-4 for pathological states) (Babloyantz [1], Babloyantz and Destexhe [3], Jinghua and Xiang-bao [6]) confirms that the EEG signal can be modelled by a deterministic dynamical system with a finite number of variables. We reported in our previous publications that successful modelling of epileptiform EEG can be accomplished with simple non-linear expressions. Measuring the fractal dimensions of consecutive 1.5 or 4 second epochs, we have observed a high-dimension period beforehand with a fall at or just before the onset of a continued epileptic discharge (Rudolf and Mahmood [7,8,9]). This implies that prediction of epileptic discharges may be possible.

For further studies, we developed a computer program for serial fractal dimension estimation of consecutive overlapping small epochs of the EEG signal (Wirth [11], Fox [12], Tischer [13]). This provides fine temporal resolution of changes in the fractal dimension and a possibility for simple comparison of fractal dimensions from two different EEG states (with and without epileptic discharges). In this paper we solve the problems: which method for fractal dimension estimation, what size of epochs and how big an overlapping segment between two epochs to choose?

2. METHODS AND MATERIAL

The fractal dimension of a set in metric space , such as a geometric object or the phase space trajectory of a dynamical system, can be computed from several different measures. The most popular are: capacity and correlation dimensions, applicable to EEG data sets, but not very useful in the case of real-time EEG data analysis when synchronization between the EEG recording machine and the method for fractal dimension estimation is essential. These two methods are valid for an enormous number of points and they take a long time for calculation. In the case of real-time EEG data analysis it is necessary to use a method for fractal dimension estimation which gives valid estimates from a small number of points.

The Higuchi method gives a good approximation to the fractal dimension, using the length of the irregular curve, from a small number of points (Higuchi, [10]). This method is applicable to EEG data sets and it may be a better solution for fractal dimension estimation in the case of real-time EEG data analysis, when the small number of points is the main limitation.

We have applied the modification of the box-counting method given by Liebovitch and Toth [17] to estimate the capacity dimension, the algorithm described by Grassberger and Procaccia [18] to estimate the correlation dimension and the Higuchi [10] method for graph dimension estimation, to several fractal sets with known dimensions. Our final aim is to choose the method for fractal dimension estimation appropriate for real-time EEG data analysis.

The methods were tested on two different attractors with fractal structures and one synthetic fractal signal. Each of these fractal sets is described in sequence.

The Henon map:

$$x_{i+1} = 1 + y_i - 1.4x_i^2 \qquad (1)$$
$$y_{i+1} = 0.3x_i$$

with fractal dimension 1.26, known from estimates based on Lyapunov numbers, capacity dimension and correlation dimension.

Modified Sierpinski triangle map:

$$x_{i+1} = y_i /2 + a_k \qquad k = 1, 2, 3 \qquad (2)$$
$$y_{i+1} = x_i/2 + b_k$$

where $a_1=a_2=b_1=1$ and $a_3=b_2=b_3=125$, applied with equal probability. The fractal dimension of this set is 1.58, and it is evaluated analytically. This fractal set was constructed using the Collage theorem given by Barnsley [19].

A synthetic fractal signal depending on a few parameters that uniquely determine the fractal dimension: this can be constructed by the deterministic Weierstrass cosine function:

$$W_H(t) = \sum_{k=0}^{\infty} \gamma^{-kH} \cos(2\pi\gamma^k t) \quad ,0<H<1, \quad \gamma>1$$

with fractal dimension D=2-H. In our experiment we synthesized a discrete-time signal by sampling $t\in[0,1]$ at N+1 equidistant points, using $\gamma=5$ and truncating the infinite series so that summation is done only for $0<k<k_{max}$. We synthesized a fractal signal with fractal dimension 1.85.

For the Henon map and Modified Sierpinski triangle map we have produced two different time series: $\{x_i, i=1,N\}$ and $\{y_i, i=1,N\}$ obtained using the formulas (1) and (2), respectively. The attractors can be reconstructed either from the first or second series, by presenting the (x_i, x_{i+1}) or (y_i, y_{i+1}) points on a plane coordinate system (Packard et al. [20]).

3. RESULTS

We produced three different fractal sets with length of 5000 points obtained respectively from the Henon map (HM), Modified Sierpinski triangle map (MSTM) and Weierstrass cosine function (WCF). Changing the different methods for fractal dimension estimation and the size of the epoch in the computer program mentioned previously, we obtained the results presented in tables 1, 2 and 3. For different methods and epoch sizes, in each table, are presented the mean fractal dimension and corresponding standard deviation. The size of the overlapping segment was 10 points.

Epoch size ; N	100 ; N=50	500; N=10	1000; N=5
Graph dim.	1.28±0.01	1.31±0.02	1.30±0.02
Capacity dim.	<1	1.39±0.01	1.34±0.04
Correlation dim.	<1	1.35±0.01	1.17±0.03

Table 1. *Henon map (fractal dim. = 1.26)*

Epoch size ; N	100 ; N=50	500; N=10	1000; N=5
Graph dim.	1.56±0.01	1.54±0.02	1.60±0.04
Capacity dim.	<1	1.67±0.01	1.64±0.03
Correlation dim.	<1	1.65±0.02	1.49±0.03

Table 2. *Modified Sierpinski triangle map (fractal dim.= 1.58)*

Epoch size ; N	100 ; N=50	500; N=10	1000; N=5
Graph dim.	1.81±0.01	1.90±0.01	1.93±0.03
Capacity dim.	<1	1.67±0.01	1.80±0.02
Correlation dim.	<1	1.95±0.02	1.71±0.03

Table 3. *Weierstrass cosine function (fractal dim.= 1.85)*

In figures 1, 2 and 3 are presented estimated fractal dimensions by the Higuchi method for different epoch sizes, respectively for the Henon map, Modified Sierpinski triangle map and Weierstrass cosine function. Figure 4 presents the dependence of the difference between fractal dimension at the beginning of the epileptic discharge and the fractal dimension at the end of the epileptic discharge, on the overlapping segment size. For this reason we have constructed a signal with normal beginning, epileptic continuation and normal end from a patient with petit mal. The EEG signal was recorded on an Oxford Medilog recorder.

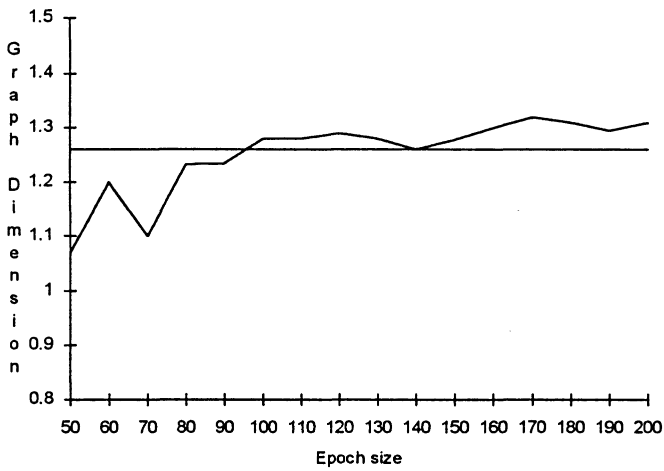

Fig. 1. *Estimated fractal dimensions by Higuchi method for Henon map.*

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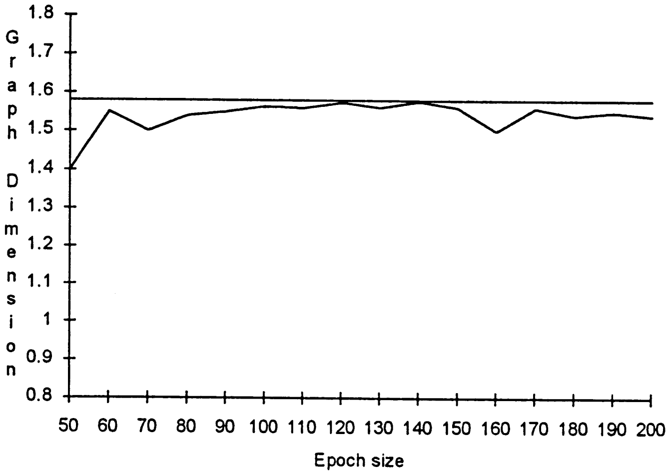

Fig. 2. *Estimated fractal dimensions by Higuchi method for Modified Sierpinski triangle.*

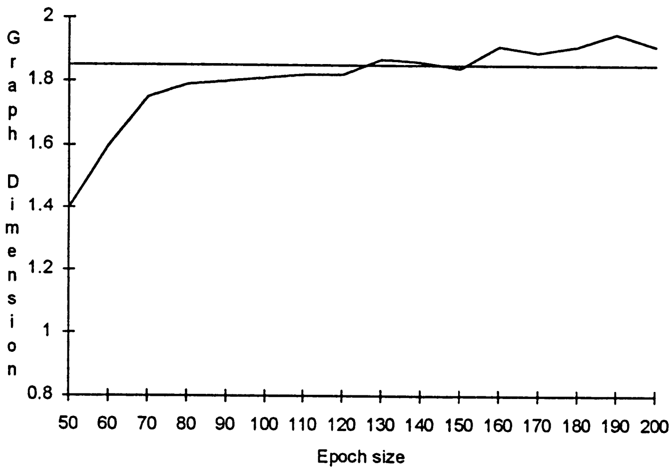

Fig. 3. *Estimated fractal dimensions by Higuchi method for Weierstrass cosine function.*

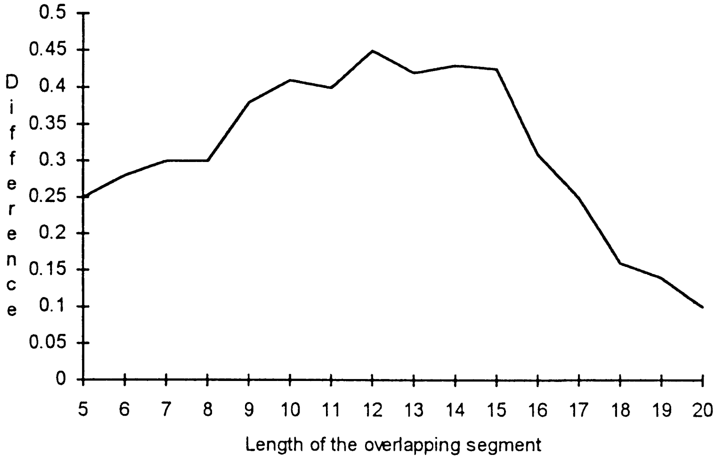

Fig. 4. *Difference between the fractal dimensions for two different EEG states (with and without epileptic discharges) influenced by the size of the overlapping segment.*

4. DISCUSSION

Real-time analysis implies a special approach to EEG data analysis. First of all, it is necessary to choose an appropriate method for fractal dimension estimation which will give accurate estimation for a very small data set. Analyzing tables 1, 2 and 3 it is more than apparent that the Higuchi graph method is the best one, as it gives the best fits for the smallest epoch size of 100 points. For this epoch size the other two methods give very unpredictable results.

After choosing the method, two problems are of special interest: what size of the epoch and what size of the overlapping segment to choose? Figures 1, 2 and 3 show that the best choice for the epoch size is between 100 and 150 points, where the estimated fractal dimension values are stabilized near the actual fractal dimension.

Figure 4 shows how the length of the overlapping segment between two succesive epochs influences the differentiation degree of two fractal dimensions for two different EEG states (with and without epileptic discharges). The bigger difference means the better differentiation, so the maximal value of the curve will give the optimal size of the overlapping segment. The estimated optimal overlap size is between 10 and 15 data points; it gives the finest differentiation of the fractal dimension values for two different states of the EEG signal.

5. CONCLUSION

The computerized detection and prediction of epileptic discharges from EEG data is a problem whose solution may lead to the prediction of epileptic seizures and planning of treatment. The recently confirmed fact that the EEG has a fractal nature enables a new approach to analysis of epilepsy. Measuring the fractal dimensions of 1.5 or 4 second epochs, we have observed a high-dimension period beforehand with a fall at or just before the onset of a continued discharge [8,9]. This implies that prediction of epileptic discharges may be possible.

In this paper we presented our further results in the real-time EEG data analysis. We developed a real-time method for serial fractal dimension estimation of consecutive overlapping short epochs of the EEG signal. It is a modification of the Higuchi method which gives a good approximation to the fractal dimension for a small number of points. This method was chosen in comparison with two other methods: the box-counting method and Grassberger and Procaccia algorithm. The Higuchi method gave the best approximation of the fractal dimension for small epoch sizes.

Analyzing the optimal epoch and overlapping segment size we can conclude that the optimal epoch size is between 100 and 150 points and the optimal overlapping segment size is between 10 and 15 points. For this size of the epoch the estimated fractal dimension is close to the actual one, and with an overlapping segment size of 10 to 15 points we accomplished the clearest differentiation of different EEG states (with and without epileptic discharges).

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