Iterative coupling in fluid-structure interaction: a BEM-FEM based approach

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Abstract

An iterative coupling of finite element and boundary element methods for the investigation of coupled fluid-solid systems is presented. While finite elements are used to model the solid, the adjacent fluid is represented by boundary elements. In order to perform the coupling of the two numerical methods, a successive renewal of the variables on the interface between the two subdomains is performed through an iterative procedure until the final convergence is achieved.

Keywords: iterative coupling, solid-fluid interaction, BEM, FEM, adjustable time-steps, nonlinear analysis.

1 Introduction

Most of the BEM/FEM coupling algorithms [1-3] are formulated in a way that, first, a coupled system of equation is established, which afterwards has to be solved using a standard direct solution scheme. Such a procedure leads to several problems with respect to accuracy and efficiency. First, the coupled system of equation has a banded structure only in the FE part, while in the BE part it is fully populated. Consequently, for its solution the optimized solvers usually used in the FEM cannot be employed anymore, which leads to rather expensive calculations with respect to computer time. Second, the duration of a time step needs to be the same in all subsystems. In general, however, the velocities of the
propagating waves in the solid and the fluid are quite different, such that a unified time step may cause serious problems in the numerical solution algorithms (instabilities, lack of accuracy etc.). Third, in the case of taking into account some nonlinearity within the FE sub-region, the rather big coupled system of equations needs to be solved in each step of the iteration process, i.e., a few times within each time step. This is very computer time consuming.

2 BEM/FEM coupling

Considering the coupling conditions at each node \( i \) of the BEM/FEM interface (superscript \( I \)), the following equations must hold (\( \hat{\rho} \) is the fluid mass density):

\[
T(f_iF)_{I} = 0
\]

\[
N(f_iF)_{I} = - (\hat{\rho}p)_{i}
\]

\[
N(f_i\dot{U})_{I} = (1/\hat{\rho}) (f_iQ)_{i}
\]

where - in order to obtain consistency between the FE (subscript \( F \)) and the BE formulation (subscript \( B \)) - \( \hat{P} \) represents the resultant nodal hydrodynamic pressure force, which is obtained from the potential distributions \( P \). \( F \) denotes the FE nodal forces; \( Q \) is the fluid flux and \( \dot{U} \) is the solid acceleration. The functions \( N(.)_{i} \) and \( T(.)_{i} \) lead to the normal and the tangential component of their arguments, respectively.

2.1 Iterative coupling

In the iterative BEM/FEM coupling, first the FE problem is solved and the accelerations \( f_i\dot{U}_{(k+\alpha)} \) are obtained. Then a relaxation parameter \( \alpha \) is introduced, according to equation (4), in order to ensure and/or to speed up convergence:

\[
f_i\dot{U}_{(k+\alpha)}^{f} = \alpha f_i\dot{U}_{(k)}^{f} + (1-\alpha) f_i\dot{U}_{(k)}^{f}
\]

Once the FEM accelerations at the interface are computed, equation (3) can be used to obtain the BEM flux \( b_iQ_{(k+1)}^{f} \). In the present formulation different time-step durations in each subdomain can be taken into account by means of extrapolations and interpolations (with respect to time) of the variables at the interface. These interpolations and extrapolations are done according to the time interpolation functions adopted by the BE formulation, as indicated by equations (5-6) (piecewise constant interpolation for the flux and linear interpolation for the potential).
\[ \dot{\mathbf{Q}}^{(k+1)} = \dot{\mathbf{Q}}^{(k)} \]  

Once \( \dot{\mathbf{Q}}^{(k+1)} \) is obtained (equation (5)), the BE subdomain can be solved, having prescribed flux values at the interface. As the result, \( \dot{\mathbf{P}}^{(k+1)} \) is obtained and it needs to be interpolated as well, in order to be used by the FEM. By means of equations (1-2) and \( \dot{\mathbf{P}}^{(k+1)} \) (equation (6)) one finally obtains the new FEM nodal forces \( \dot{\mathbf{F}}^{(k+1)} \), which are needed to solve the FE problem once more. The iterative loop goes on until convergence is achieved. A sketch of the iterative coupling is shown in Figure 1.

2.2 Numerical example

In this example, a dam-reservoir system, as depicted in Figure 2, is analyzed. The structure is subjected to a sinusoidal, distributed vertical load on its crest, acting with an angular frequency \( \omega = 18 \text{ rad/s} \). The material properties of the dam are: Poisson’s ratio \( \nu = 0.25 \); Young’s modulus \( E = 3.437 \times 10^6 \text{ N/m}^2 \); mass density \( \rho = 2.00 \text{ Ns}^2/\text{m}^4 \). A perfectly plastic material obeying the Drucker-Prager yield criterion is assumed: cohesion \( c_0 = 0.15 \text{ N/m}^2 \); internal friction angle \( \phi =\)
20°. The adjacent water is characterized by a mass density $\rho = 1.00 \text{Ns}^2/\text{m}^4$ and a wave velocity $c = 1436 \text{m/s}$. The time-step duration adopted for the BEM and FEM are $\beta \Delta t = 0.00350 \text{s}$ and $\varepsilon \Delta t = 0.00175 \text{s}$, respectively. The transient behaviour of the vertical displacement at point A is shown in Figure 3(a). Results of linear as well as nonlinear analyses are given and two different water levels, namely $h = 50 \text{ m}$ and $h = 35 \text{ m}$, and their influence on the dam are investigated. In Figure 3(b) the transient hydrodynamic pressure at point B is depicted. A comparison of the results obtained with the iterative coupling procedure with those from the standard coupling scheme used in von Estorff and Antes [1] shows good agreement. The average number of iterations per time step (it was adopted $\alpha = 0.5$), for the current linear model, was 2.4 ($h = 50 \text{ m}$) and 2.2 ($h = 35 \text{ m}$). For the nonlinear analysis, the average number of iterations per time step was 2.6 ($h = 50 \text{ m}$) and 2.4 ($h = 35 \text{ m}$). As one can observe, the convergence is quite fast: the iterative coupling can be regarded as a very attractive tool to deal with high scale linear or (especially) nonlinear models. For more details on the iterative BEM/FEM coupling taking into account solid-solid interactions, one is referred to Soares Jr et al. [4].

![Figure 2: Dam with storage-lake.](image)

### 3 Conclusions

In the present paper, an iterative coupling scheme for the investigation of a continuous solid coupled to an adjacent fluid was presented. The major advantage of such a procedure can be seen in the fact, that the FE and BE subsystems can be solved separately using optimised solution algorithms according to the special features of the respective system of equations, which are – a further advantage – much smaller than the coupled matrices resulting from a
standard coupling approach. In addition, the iterative coupling offers two advantages: It is straightforward to use different time steps in each subdomain; and, moreover, to take into account nonlinearities (within the FE subdomain) in the same iteration loop that is needed for the coupling.

![Diagram](image)

Figure 3: Results for the standard [1] and the iterative FEM/BEM coupling: (a) vertical displacements at point A; (b) hydrodynamic pressure at point B.
References


