Computation of maximal electric field value generated by a power substation

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Abstract

A procedure for a computation of the maximal value of extremely low frequency (ELF) electric field from a power substation is proposed. The present technique is based on a multiquadric approximation of the electric field. The approximation is obtained using discrete field values calculated by the Source Element Method (SEM) representing a variant of Indirect Boundary Element Method (IBEM). The approximation sufficiently handles multidimensional multieextreme functions by interpolating their discrete values accurately. Subsequently, the maximal electric field value is evaluated by minimizing the negative multiquadric approximation via the stochastic optimization method – differential evolution. Therefore, the procedure provides the maximal field value assessment on the basis of a limited number of computed discrete values, thus reducing the computational effort.

1 Introduction

A computation of the maximal value of extremely low frequency (ELF) electric field from a power substation has been of great interest in last decades. The reason for this is the increasing public concern regarding the possible health risk for human exposure to ELF electric field and magnetic flux density. It is to be noticed that the present piece of work has been focused only toward the technical aspects of electric field assessment. The ELF electric field from substations has been determined via the computations or/and measurements in [1–6], presenting a number of three-dimensional as well as contour plots. Nevertheless, they provided a useful description of the field characteristics, no technique incorporating certain optimization method for assessing the maximal electric field value has been applied in these papers. The spatial electric field function
may have a number of local extremes arising from the fact that substation represents a concentration of electric field sources.

Consequently, an application of a stochastic optimization method – differential evolution [7–9], which is in the scope of this work, could be a useful tool for the maximal field value assessment. The stochastic optimization method shows the well-known advantages over the gradient-based methods, namely: (i) the parallel global minimum search technique, (ii) the simplified set-up of the optimization task and (iii) the ability to find the global minimum. The optimization procedure is carried out using a multiquadric approximation [10, 11] of electric field, since the computed values are distributed in a set of discrete points through the volume of interest. The approximation based on the radial basis functions is capable to handle multidimensional multiminima functions by interpolating their discrete data accurately.

2 Electric field computation in a set of discrete points

The spatial distribution of electric field can be assessed using the source integration procedure featuring the Charge Simulation Method (CSM) or the Source Element Method (SEM). The approach has been already utilized for the electric field computation in the power substation environment [1–3] as well as in the modeling of a metallic post protection zone [12]. It can be referred to as a variant of the Indirect Boundary Element Method (IBEM).

Electric potential of a point \( P(x,z) \) due to a charged straight wire considered as a segment of a conductor (electrode), Figure 1, is given by [12]:

\[
\phi(x,z) = \frac{1}{4\pi\varepsilon} \int_{-L}^{L} \frac{\rho_l(x')dx'}{R},
\]

where:
- \( \rho_l \) denotes the line charge density,
- \( 2L \) stands for the wire length,
- \( R \) is the distance between a point on the straight wire and an arbitrary point \( P \).

![Figure 1: Straight wire geometry.](image-url)
If the potential function $\phi_i$ along the straight wire is known, the integral equation (1) can be written as:

$$\phi_i = \frac{1}{4\pi\varepsilon} \int_{-L}^{+L} \frac{\rho_i(x') dx'}{|x-x'|}. \tag{2}$$

Obviously, there is a singularity for $x=x'$. The problem can be overcome by assuming a finite wire radius $a$, i.e. it follows:

$$\phi_i = \frac{1}{4\pi\varepsilon} \int_{-L}^{+L} \frac{\rho_i(x') dx'}{(x-x')^2 + a^2}. \tag{3}$$

According to the standard Source Element Method (SEM) procedure, the conductors are divided into a number of segments, i.e. boundary elements [1]-[3]. Having performed a discretization of the substation conductors a system of equations for unknown charges along each segment is obtained. Therefore, the integral equation (3) transforms into a corresponding matrix equation [1]:

$$\begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\ P_{21} & P_{22} & \cdots & P_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ P_{n1} & P_{n2} & \cdots & P_{nn} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{bmatrix} = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_n \end{bmatrix}, \tag{4}$$

where: $\phi_1, \phi_2, \ldots, \phi_n$ stand for the boundary element potentials, $q_1,q_2,\ldots,q_n$ denote the boundary element charges, $P_{ij}, P_{12},\ldots,P_{nn}$ are Maxwell coefficients. The unknown charges can be calculated by inverting the Maxwell coefficient matrix. The details regarding the Maxwell coefficients can be found in [1].

The electric field components in Cartesian system at an arbitrary point $(x,y,z)$, produced by $i$-th boundary element, Figure 2, can be computed by means of the equations related to the finite length wire [1]:

$$E_x = -\frac{\partial \phi}{\partial x} = \frac{q_i}{4\pi\varepsilon_0 L_i} \left[ \frac{1}{\sqrt{(L_i-x)^2+W^2}} - \frac{1}{\sqrt{x^2+W^2}} \right], \tag{5a}$$

$$E_y = -\frac{\partial \phi}{\partial y} = \frac{q_i}{4\pi\varepsilon_0 L_i} \frac{y}{W^2} \left[ \frac{L_i-x}{\sqrt{(L_i-x)^2+W^2}} + \frac{x}{\sqrt{x^2+W^2}} \right], \tag{5b}$$
\[ E_z = -\frac{\partial \varphi}{\partial z} = \frac{q_i}{4\pi\varepsilon_0 L_i} \frac{z}{W^2} \left[ \frac{L_i - x}{\sqrt{(L_i - x)^2 + W^2}} + \frac{x}{\sqrt{x^2 + W^2}} \right] \, , \]  

where:  
\( q_i \) denotes the charge of \( i \)-th segment,  
\( L_i \) stands for the length of \( i \)-th segment, and  
\( W^2 = y^2 + z^2 \).  

(6)

The total field components are assembled from each boundary element.

\[ E_z \]
\[ E_y \]
\[ E_x \]

Figure 2: Electric field components produced by \( i \)-th segment.

3 Maximal field value assessment

The procedure for assessing the maximal electric field value generated by a power substation is outlined in a few steps: (A) multiquadric approximation of electric field, (B) minimization of objective function.

3.1 Multiquadric approximation

The vector of independent parameters, represented by the space co-ordinates, can be written by:

\[ \mathbf{P} = [x \ y \ z]^T. \]  

(7)

The parameters are generally subjected to both inequality and equality constraints:

\[ g_m(\mathbf{P}) \geq 0, \ m = 1, 2, \ldots, n_{i1} , \]  

(8)

\[ h_m(\mathbf{P}) = 0, \ m = n_{i1} + 1, n_{i1} + 2, \ldots, n_{i1} + n_{i2} , \]  

(9)
where \( n_{t1}, n_{t2} \) are the total numbers of inequality and equality constraints, respectively.

Applying the procedure, a vector related with the maximal electric field value is assessed, fulfilling the imposed constraints. The computation of discrete field values would require rather high effort, if applied to a large number of parameter vectors. Hence, the procedure of the maximum assessment is carried out using a multiquadric approximation [10, 11] of electric field. The approximation is determined by an expression:

\[
E_{MQ}(P) = \sum_{j=1}^{M_P} c_j r(P_j),
\]

where: \( M_P \) is the total number of sample points \( P_j \) having the computed values \( E_j(P_j) \),

\( c_j \) stands for the approximation coefficients, while

\[
r(P_j) = \sqrt{\|P - P_j\|^2 + h_z}, \quad j = 1, 2, ..., M_P
\]

denotes radial basis functions, where \( \|P - P_j\| \) is the Euclidean norm and \( h_z \) is the shift parameter.

The value of \( M_P \) should be a compromise between opposite requirements: the higher accuracy of the multiquadric approximation and the lower computational effort. The shift parameter \( h_z \) can be determined by means of a few additional samples used for the error estimation.

The unknown coefficients \( c_j \) are calculated from the matrix equation:

\[
c = r^{-1} \cdot E,
\]

where:

\[
r_{ij} = \sqrt{\|P - P_j\|^2 + h_z}, \quad i, j = 1, 2, ..., M_P,
\]

\[
E = \begin{bmatrix} E_1(P_1) & E_2(P_2) & \cdots & E_{M_P}(P_{M_P}) \end{bmatrix}.
\]

Hence, the discrete electric field data, obtained by the SEM computation, are related with \( M_P \) sample points only, thus reducing the required computational effort of the optimization procedure.

### 3.2 Minimization of objective function

Minimization of the objective function \( -E_{MQ}(P) \), resulting in the maximal electric field value assessed, is formulated by:

\[
\text{min}(-E_{MQ}(P)),
\]

\[\]
In order to perform the minimization procedure, i.e. to find the global minimum, a stochastic optimization method – differential evolution (DE) is applied [7]-[9].

A constant number \( n_p \) of parameter vectors, representing members of a population, is used in each generation \( G \):

$$\mathbf{P}_{i,G}, i = 0,1,...,n_p - 1.$$  \(\text{(16)}\)

The population of the first generation is chosen randomly via \( \text{rand} \) function. There are several variants of DE algorithms. The one, using the best population vector to generate a new population member, is selected as a part of the proposed procedure. Hence, for each \( \mathbf{P}_{i,G} \) there is the corresponding vector:

$$\mathbf{V}_{i,G+1} = \mathbf{P}_{\text{best},G} + F(\mathbf{P}_{r_1,G} - \mathbf{P}_{r_2,G}),$$  \(\text{(17)}\)

where: \( \mathbf{P}_{\text{best},G} \) is the member having the lowest objective function value of the generation \( G \),

\( r_1, r_2 \) denote randomly adopted integers from \([0,n_p-1]\),

\( F \) is a real factor controlling the amplification of a weighted difference.

To enhance a new population diversity, the crossover of the vectors \( \mathbf{V}_{i,G+1} \) and \( \mathbf{P}_{i,G} \) is introduced, by which the new parameter vector:

$$\mathbf{U}_{i,G+1} = [u_{0i,G+1} \ u_{1i,G+1} \ ... \ u_{(n-1)i,G+1}]^t$$  \(\text{(18)}\)

is obtained as:

$$\mathbf{U}_{i,G+1} = \begin{cases} \mathbf{V}_{j_i,G+1} & \text{for } j = \langle k_n \rangle_n, \langle k_n + 1 \rangle_n, ..., \langle k_n + L_n - 1 \rangle_n, \\ \mathbf{P}_{j_i,G} & \text{for all other } j \in [0, n-1] \end{cases},$$  \(\text{(19)}\)

where: \( n \) is the total number of independent parameters,

\(\langle \rangle_n \) stands for the modulo function with modulus \( n \),

\( k_n \) denotes the randomly chosen integer from \([0,n-1]\),

\( L_n \) is the number of exchanged parameters from \([1,n]\).

The evaluation algorithm of \( L_n \) is given by the following pseudo-code lines:

\[ L_n = 0; \]
\[ \text{do } \]
\[ \quad L_n = L_n + 1 \]
\[ \text{while } (\text{rand }() < \text{prob}_C) \text{ and } (L_n < n); \]

where \( \text{prob}_C \) denotes the control variable.

If the resulting vector \( \mathbf{U}_{i,G+1} \) gives the objective function value lower than the one corresponding to the vector \( \mathbf{P}_{i,G} \) and if \( \mathbf{U}_{i,G+1} \) fulfills the constraints, as well, it replaces \( \mathbf{P}_{i,G} \) being a population member of the generation \( G+1 \): \( \mathbf{P}_{i,G+1} = \mathbf{U}_{i,G+1} \).

Otherwise, \( \mathbf{P}_{i,G} \) is retained as a member of the generation \( G+1 \).

The vector, related to the maximal value, equals the best vector assessed upon the prescribed number of generations:
where $G_f$ is the number of the last generation.

The corresponding maximal electric field value from a power substation is then given by:

$$E_{MQ}^{\text{max}}(P_{\text{max}}) = \sum_{j=1}^{M_s} c_j r(P_{\text{max}})_j.$$  

(21)

4 Computational results

The procedure presented so far is illustrated by an example of 110/10 kV/kV transmission substation of GIS (Gas-Insulated Substation) type in Split, Croatia. All the electric field levels ($f = 50$ Hz) are computed at height $z = 1$ m above ground. There is no real possibility of a public exposure within the transmission substation, while a professional exposure is strictly limited to duration. Consequently, the computations are performed outside the fence of the substation, only. A simplified two-dimensional layout of the substation is shown in Figure 3.

It is to be underlined that the equipment having grounded shields (power cables), sheaths (GIS buses), or metallic casings (transformers, switchgears) can be neglected due to the shielding effect [5]. Namely, the metallic enclosures are connected to ground, thus producing negligible electric field strength. Consequently, the significant electric field sources regarding the substation considered are overhead transmission lines, as well as the unshielded conductors of both the high and middle voltage level. The corresponding domains, where the greater electric field intensities or magnetic flux density values are expected, can be seen in Figure 3. The areas assigned as 3 and 4 are the important ones for the magnetic flux density computation only, which is not within the scope of this work. The multiquadric approximations of the electric field distributions concerning the domains assigned as 1 and 2 are shown in Figures 4 and 5, respectively. The dependencies refer to ABC-CBA phase arrangement resulting in the greatest electric field values.

The field intensity increases by approaching the overhead line route, as shown in Figure 4. Moreover, there is a local intensity increase in the vicinity of the transformer due to the unshielded conductors connecting the transformer to the GIS buses. However, for the shortness of unshielded conductors, as well as their considerable distances from the substation fence, the main electric field sources are the overhead lines. This statement is confirmed by the field levels in domain 2, Figure 5. Minimizing the negative multiquadric approximation, the vector $P_{\text{opt}} = \begin{bmatrix} 55.35 \ -7.84 \ 1 \end{bmatrix}$, resulting in the maximal electric field intensity $E_{\text{max}} = 380.70$ V/m, is obtained. The maximum point is located just below the overhead line route. The field distribution over the area No. 5 is not presented, since no field value exceeds 32 V/m. The low field values are obtained as the main field sources (overhead lines) are located rather far away from the domain No. 5.
5 Concluding remarks

An efficient technique for the assessment of maximal ELF electric field value from a power substation is presented in this work. The procedure is based on the multiquadric approximation of electric field which is obtained using discrete field values. The values are calculated by the Source Element Method (SEM) representing a variant of Indirect Boundary Element Method (IBEM). The approximation is capable to handle multidimensional multiextreme functions by accurate interpolating their discrete values. Subsequently, the maximal value is assessed by minimizing the negative multiquadric approximation via the stochastic optimization method – differential evolution. Therefore, the presented procedure provides the maximal field value assessment on the basis of a limited number of calculated discrete values, thus reducing the computational cost.
Figure 4: Multiquadric approximation of electric field in the domain No. 1.

Figure 5: Multiquadric approximation of electric field in the domain No. 2.

References


