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Induced currents and voltages along a horizontal wire above a lossy ground

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Abstract

The transmission line of a finite length above a lossy ground is analyzed applying the theory of wire antennas. The boundary element procedure is used for solving the electric field integral equation (EFIE) and the induced current along the line due to the incident plane wave excitation is obtained. Utilizing the weak formulation of the problem and conveniently performing the integration by parts a scattered voltage along the line is calculated by integrating the electric field from the imperfect ground plane to the conducting wire surface.

1 Introduction

The calculation of the currents and voltages induced on a horizontal wire above a real ground is very important in EMC applications such as surface wave propagation, oceanography, geosounding or transmission line modeling[1-5]. Some useful results pertaining to a loaded horizontal dipole antenna radiating above a dissipative half-space are published by Y. Rahmat-Samii et al[6] featuring the pulse basis functions and point matching for handling the corresponding integral equation. In addition, Sommerfeld integrals that appear in the integral equation kernel representing the effect of a lossy half-space, are replaced by certain asymptotic expansion and solved by the standard saddle-point technique. However the point-matching technique has been definitely proved to suffer from a rather poor convergence rate[7]. On the other hand, the calculation of the current induced along an long overhead wire in the presence of a lossy earth is usually

performed using the transmission line (TL) theory. This approach, though very useful if the longer lines are considered, fails if one deals with lines of a finite length. One of the main drawbacks is related to the predictions of resonances[3]. This problem can be handled using the linear antenna theory, though the restrictions of this approach are often related to the computational cost, the convergence rate and the stability of numerical results.
This paper uses the linear antenna model of the single wire transmission line of finite length and also proposes the BEM approach for treating the thin wire integral equation. The effect of the lower half-space is taken into account via the reflection coefficient (RC) appearing within the integral equation Green function. The principal advantage of the RC approach versus rigorous Sommerfeld integral approach is much less computational cost. Once obtaining the current distribution along the line the scattered electric field same as the induced voltage on the line can be calculated. The induced voltage on the line is obtained by integrating the vertical component of the scattered electric field from the ground to the electrode surface. The results obtained by BEM for the plane wave excitation are in good agreement with available experimental data.

2 Formulation

The horizontal wire antenna of length 2L and radius a, placed above a lossy medium at height h is of interest. In accordance to the theory of wire antennas, featuring the thin wire approximation and the reflection coefficient (RC) approximation, the tangential component of the electric field on the wire surface is given by[8,9]:

$$E_x = \frac{1}{j4\pi\omega\varepsilon}\int_{-L}^{L}\{[\frac{\partial^2}{\partial x^2}+k^2]g(x,x')]\}I(x')dx' \tag{1}$$

where I(x') is the equivalent current distribution in the antenna axis while g(x,x') denotes the integral equation kernel[10]:

$$g(x,x') = g_0(x,x') - R_{TM}g_i(x,x') \tag{2}$$

where R_{TM} is the reflection coefficient for the TM polarization:

$$R_{TM}=\frac{\varepsilon_{eff}\cos\theta-\sqrt{\varepsilon_{eff}-\sin^2\theta}}{\varepsilon_{eff}\cos\theta+\sqrt{\varepsilon_{eff}-\sin^2\theta}},\ \varepsilon_{eff}=\varepsilon_r-j\frac{\sigma}{\omega\varepsilon_0},\ \theta=\mathrm{arc}tg\frac{|x-x'|}{2h} \quad (3)$$

and g_0 and g_i are the Green functions of the free space for the source wire and its image, respectively:

$$g_0(x,x')=\frac{e^{-jkR}}{R},\quad g_i(x,x')=\frac{e^{-jkR^*}}{R^*} \quad (4)$$

where k is the free space propagation constant and R and R^* are the distances from the source wire and from its image to the observation point:

$$R=\sqrt{(x-x')^2+a^2},\quad R^*=\sqrt{(x-x')^2+4h^2} \quad (5)$$

The further details concerning the RC approximation can be found elsewhere.[10,11] The total tangential electric field can be written as the sum of the incident electric field E_x^{inc}, and the related scattered field on the wire surface E_x^{sct}:

$$E_x^{tot}=E_x^{inc}+E_x^{sct} \quad (6)$$

If the wire is assumed to be perfectly conducting the total tangential electric field equals zero:

$$E_x^{tot}=0 \quad (7)$$

and the electric field integral equation (EFIE) for the horizontal wire above lossy ground yields:

$$E_x^{inc}=-\frac{1}{j4\pi\omega\varepsilon}\int_{-L}^{L}\left[\frac{\partial^2}{\partial x^2}+k^2\right]g(x,x')I(x')dx' \quad (8)$$

Solving the integral equation (8) the equivalent current distribution is obtained and the scattered voltage along the line can be determined by integrating the vertical component of the scattered electric field from the lossy ground to the conducting surface of the wire[1]:

$$V^{sct}(x) = -\int_0^h E_z^{sct}(x,z)dz \tag{9}$$

where the normal component of the scattered electric field is given by[12]:

$$E_z^{sct}(x,z) = \frac{1}{j4\pi\omega\varepsilon}\int_{-L}^{L}\frac{\partial^2}{\partial x \partial z}I(x')g(x,x',z)dx' \tag{10}$$

Taking into account the property of the integral equation kernel the former equation can be rewritten in a following manner[12]:

$$E_z^{sct}(x,z) = -\frac{1}{j4\pi\omega\varepsilon}\frac{d}{dz}\int_{-L}^{L}\frac{\partial I(x')}{\partial x'}g(x,x',z)dx' \tag{11}$$

Substituting the relation (11) into relation (9) it yields:

$$V^{sct}(x) = \frac{1}{j4\pi\omega\varepsilon}\int_0^h \frac{d}{dz}\left(\int_{-L}^{L}\frac{\partial I(x')}{\partial x'}g(x,x',z)dx'\right)dz \tag{12}$$

and finally, it follows:

$$V^{sct}(x) = \frac{1}{j4\pi\omega\varepsilon}\int_{-L}^{L}\frac{\partial I(x')}{\partial x'}[g(x,x',h) - g(x,x',0)]dx' \tag{13}$$

As a matter of fact, by conveniently using the integration by parts, the integration over z-path is avoided. Equation (13) is the final expression for the scattered voltage along the wire, convenient for numerical evaluation.

3. The boundary element procedures

Integro-differential equation (8) is usually treated by point matching technique[5,6] which principal feature is simplicity. On the other hand, the point-matching approach suffers from relatively poor convergence rate[7] and the kernel quasisingularity[6,8,9] problem also arises.

In this paper, a variational boundary element scheme[13] is applied leading to the following expression:

$$\sum_{i=1}^{n} I_i [\frac{1}{j4\pi\omega\varepsilon} \{ -\int_{-L}^{L} \frac{df_j(x)}{dx} \int_{-L}^{L} \frac{df_i(x')}{dz'} g(x,x')dx'dx - \\ -k^2 \int_{-L}^{L} f_j(x) \int_{-L}^{L} f_i(x') g(x,x') dx' dx \} = \\ = - \int_{-L}^{L} E_x^{inc} f_j(x) dx, \quad j=1,2,...,n \tag{14}$$

where n denotes the total number of basis functions, I_i are the unknown coefficients of the solution, and f_i and f_j are the basis and test functions, respectively. The boundary conditions for current vanishing at free ends of wire (thin wire approximation) are incorporated subsequently into the global matrix. Performing the boundary element discretization of the domain of interest the linear equation system arising from (14) is then given in the form:

$$\sum_{i=1}^{M} [Z]_{ji} \{I\}_i = \{V\}_j, \quad j=1,2,...,M \tag{15}$$

where $[Z]_{ji}$ is the local matrix presenting the mutual impedance between the i-th source boundary element to the j-th observation boundary element:

$$[Z]_{ji} = -\frac{1}{j4\pi\omega\varepsilon} \{ \int_{\Delta l_j} \int_{\Delta l_i} \{D\}_j \{D'\}_i^T g(x,x') dx' dx + \\ +k^2 \int_{\Delta l_j} \int_{\Delta l_i} \{f\}_j \{f'\}_i^T g(x,x') dx' dx \} \tag{16}$$

Matrices {f} and {f'} contain shape functions $f_k(x)$ and $f_k(x')$, while {D} and {D'} contain their derivatives, where: M is the total number of boundary elements, Δl_i, Δl_j is the width of i-th and j-th boundary element, respectively. Functions $f_k(x)$ are the Lagrange's polynomials defined by:

$$L_k(x) = \prod_{i=1}^{m} \frac{x-x_i}{x_k-x_i}, \quad i \neq k \tag{17}$$

$\{V\}_j$ is the local voltage vector on the j-th observation boundary element along the wire:

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$$\{V\}_j = -\int_{\Delta l_j} E_x^{inc}\{f\}_j dx \tag{18}$$

In this paper, the linear approximation over boundary element is used since it was shown[8,9] that this choice ensures accurate results and satisfactory convergence rate.

The important feature of the method is the replacing of the second-order differential operator from the kernel thus avoiding the problem of quasi-singularity.

Once the current distribution is determined the scattered voltage along the line caused by the induced current (the reradiation effect) can be calculated using the following BEM formalism:

$$V^{sct}(x) = \frac{1}{j4\pi\omega\varepsilon}\sum_{i=1}^{M}\int_{\Delta l_i}\{D'\}_i^T[g(x,x',h)-g(x,x',0)]dx'\{I\}_i \tag{19}$$

4 Numerical results

Fig 1 shows the comparison of current magnitudes calculated via BEM with the available measured results[14] for the dipole antenna over lake water. The excitation function is the unit time-harmonic voltage. A good agreement between numerical and experimental results is noticed. Fig 2 and 3 show the real and imaginary part of the induced current and the scattered voltage along an overhead line, respectively. The line is characterized by a length L=16m, with conductor radius a=10mm, while the height above ground is h=0.5m. The excitation at the wire surface is a plane wave of the form[2]:

$$E_x^{inc} = E_0(1-e^{-j2kh}) \tag{20}$$

The relative dielectric constant of the ground is $\varepsilon_r=10$ and the ground conductivity is $\sigma=0.01$S/m. The magnitude of the plane wave is $E_0=35$kV/m and the frequency is f=200MHz.

The expression (20) is a usual form for the excitation in the case of the perfectly conducting half-space. The rigorous analysis for the case of imperferct ground requires the use of reflection coefficient for the representation of the total tangential field at the wire surface. This will be reported in future work.

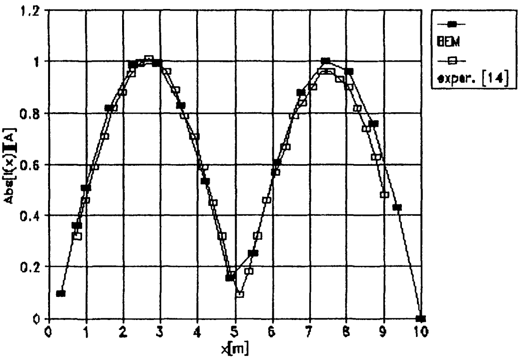

Figure 1: Magnitudes of normalized current distribution along horizontal wire above lake water with: L=λ, h/λ=0.00775, a/λ=0.0015, ε_r=82 and σ=0.092S/m

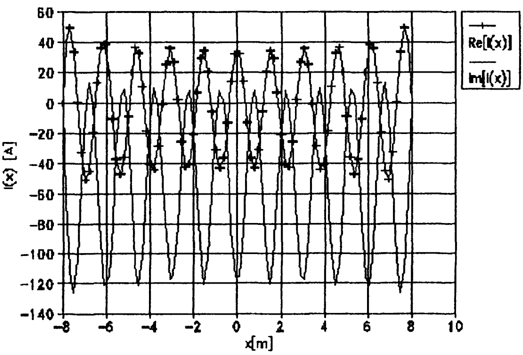

Figure 2: Induced current distribution along the line

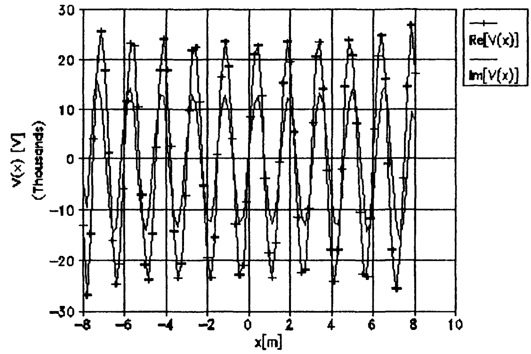

Figure 3: Scattered voltage along the line

5 Conclusions

A boundary element approach to the analysis of a single wire transmission line above a lossy ground is presented. The electric field integral equation (EFIE) for the thin wire embedded in a two media configuration is solved by the variational boundary element procedure. This BEM procedure also provides a simple and efficient algorithm for the scattered voltage evaluation.

The proposed method shows advantages over the commonly used point-matching techniques avoiding the quasi-singularity of the kernel and ensuring the satisfactory convergence rate. The procedure discussed so far can be readily applied to multiple wire transmission line problems.

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