A comparison of different radial basis functions in DRBEM solution of the Navier-Stokes equations

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Abstract

This paper describes the solution to transient incompressible two-dimensional Navier-Stokes equations in primitive variables by the dual reciprocity boundary element method. The coupled set of mass and momentum equations is structured by the fundamental solution of the Laplace equation. The boundary is discretized into simplest boundary elements with constant field and straight line geometry. Numerical examples include convergence and accuracy studies for the classical driven cavity problem at Re = 100. The performance of thirteen different global approximation functions, based on conicals, thin plate splines, multiquadrics and Gaussians is shown, assessed by comparison with the Ghia-Ghia-Shin finite difference reference solution.

1. Introduction

The primary goal of our current research is the BEM solution of the general one-phase mixture formulation [1] of coupled heat, mass, momentum, and species transfer in solid-liquid phase change systems. The heat transfer part of the problem has been worked out in [2, 3] by the DRBEM technique [4]. The complementary mass and momentum solution of the problem for one-phase flow has been proposed in [5] and its upgrade for melting or solidifying flows in [6]. This paper describes preliminary numerical tests with different radial basis functions for the solution of Navier–Stokes equations. This investigation would help to select the proper global approximation in perspective, computationally extremely involved, coupled solid-liquid phase change transport phenomena simulations.
2. Governing Equations

Consider a connected fixed domain $\Omega$ with boundary $\Gamma$ occupied by incompressible Newtonian fluid with constant viscosity $\mu$. The governing equations for mass and linear momentum transport in such media are

$$\nabla \cdot \mathbf{V} = 0, \quad \rho \frac{\partial}{\partial t} \mathbf{V} + \rho \mathbf{V} \cdot \nabla \mathbf{V} = -\nabla P + \mu \nabla^2 \mathbf{V} \tag{1}$$

where $t$, $\mathbf{V}$, and $P$ represent time, velocity, and pressure, respectively. The solution of the governing equations for the velocity and pressure field at final time $t = t_0 + \Delta t$ is sought where $t_0$ represents the initial time and $\Delta t$ the positive time increment. The solution is constructed by the initial and boundary conditions that follow. The initial conditions in position $p$ at time $t_0$ are

$$\mathbf{V}(p, t_0) = \mathbf{V}_0; \quad p \in \Omega \tag{2}$$

where $\mathbf{V}_0$ represents a known function. Condition $\nabla \cdot \mathbf{V}_0 = 0$ is assumed for a well posed problem. The boundary $\Gamma$ is divided into not necessarily connected parts $\Gamma^D$ and $\Gamma^N$: $\Gamma = \Gamma^D \cup \Gamma^N$, with Dirichlet and Neumann velocity boundary conditions respectively. These boundary conditions are at point $p$ and time $t_0 \leq t \leq t_0 + \Delta t$ defined through known functions $\mathbf{V}^D_\Gamma$, $\mathbf{V}^N_\Gamma$.

$$\mathbf{V}(p, t) = \mathbf{V}^D_\Gamma; \quad p \in \Gamma^D, \quad \frac{\partial \mathbf{V}}{\partial n_\Gamma}(p, t) = \mathbf{V}^N_\Gamma; \quad p \in \Gamma^N \tag{3}$$

3. Solution Procedure

Discretization of the momentum equation (1) over time is performed by fully implicit two-level finite differencing

$$\rho \frac{\mathbf{V} - \mathbf{V}_0}{\Delta t} + \rho \mathbf{V} \cdot \nabla \mathbf{V} = -\nabla P + \mu \nabla^2 \mathbf{V} \tag{4}$$

where index 0 denotes time $t_0$ and no index time $t_0 + \Delta t$, respectively. The solution is performed in an iterative way. The discussion of one iteration cycle that follows explains calculation of the velocity and pressure fields at iteration $m + 1$ from known velocity and pressure fields at iteration $m$, subject to initial and boundary conditions.

The iteration cycle starts by evaluation of the velocity field at the boundary and domain nodes. This is followed by correction of the pressure field, correction of the velocity field, and correction of the velocity gradient fields. The iterations over timestep are completed when the condition

$$| |^m \mathbf{V}^1 | - |^m \mathbf{V} | | < V_c \tag{5}$$

is satisfied with $V_c$ representing the velocity convergence criterion.
The differential form of the time discretized momentum equation is rewritten in boundary-domain integral form in order to be amenable for the DRBEM solution. For this purpose, the equation (4) is weighted over the domain $\Omega$ by the fundamental solution of the Laplace equation $T^*$. The present paper is limited to two-dimensional Cartesian coordinates, e.g.

$$ T^* = \frac{1}{2\pi} \log \frac{\bar{r}}{r} $$  \hspace{1cm} (6)

where $\bar{r}$ represents reference radius and $r$ equals

$$ r = r \cdot r; \quad r = r_x i_x + r_y i_y; \quad r_x = p_x - s_x, \quad r_y = p_y - s_y $$  \hspace{1cm} (7)

$p_x, p_y$ denote the Cartesian coordinates (base vectors $i_x, i_y$) of point $p$, and $s_x, s_y$ the Cartesian coordinates of the fundamental solution source point $s$ respectively. Green's second theorem for vectors allows us to write the following boundary-domain integral version of weighted equation (4)

$$ \int_{\Gamma} m+1 \frac{\partial \hat{V}}{\partial n_{\Gamma}} T^* d\Gamma - \int_{\Gamma} m+1 \hat{V} \frac{\partial T^*}{\partial n_{\Gamma}} d\Gamma - c^*_s m+1 \hat{V}_s = \int_{\Omega} m+1 \hat{S}_V T^* d\Omega $$  \hspace{1cm} (8)

where $c^*$ stands for the fundamental solution related factor. Index $s$ denotes evaluation in the fundamental solution source point position. The source term in the velocity Poisson equation (8) equals

$$ \hat{S}_V = \frac{1}{\mu} \left[ \rho \frac{m+1 \hat{V} - \hat{V}_0}{\Delta t} + \rho m+1 \hat{V} \cdot m \nabla \hat{V} - m \nabla P \right] $$  \hspace{1cm} (9)

Equation (8) is simultaneously solved for unknown velocity $m+1 \hat{V}$ in the domain and unknown velocity $m+1 \hat{V}$ and velocity gradient $(m+1 \partial \hat{V} / \partial n_{\Gamma})$ on the boundary. The hat over the velocity field denotes that it generally does not satisfy the mass conservation equation. A correction $m+1 \hat{V}$ to $m+1 \hat{V}$ is assumed after solving equation (8)

$$ m+1 \hat{V} = m+1 \hat{V} + m+1 \hat{V} $$  \hspace{1cm} (10)

in such a way to force the mass conservation

$$ \nabla \cdot (m+1 \hat{V} + m+1 \hat{V}) = 0 $$  \hspace{1cm} (11)

Consider that the velocity correction $m+1 \hat{V}$ occurs exclusively due to the action of the pressure gradient correction $\hat{P}$

$$ \xi \frac{\rho}{\Delta t} m+1 \hat{V} = -m+1 \nabla \hat{P} $$  \hspace{1cm} (12)

where $\xi$ stands for the dimensionless velocity-pressure correction relaxation factor. The pressure correction could thus be calculated from the pressure Poisson equation

$$ \nabla^2 m+1 \hat{P} = \xi \frac{\rho}{\Delta t} \nabla \cdot m+1 \hat{V} $$  \hspace{1cm} (13)
deduced from equations (11) and (12). The differential form of the pressure-Poisson equation (13) is rewritten in boundary-domain integral form in order to be amenable for the DRBEM solution. The boundary-domain integral expression for solving the pressure correction Poisson equation is

\[ \int_{\Gamma}^{m+1} \frac{\partial \tilde{P}}{\partial n_{\Gamma}} T^* \, d\Gamma - \int_{\Gamma}^{m+1} \tilde{P} \frac{\partial T^*}{\partial n_{\Gamma}} \, d\Gamma - e_s^{m+1} \tilde{P}_s = \int_{\Omega}^{m+1} S_P T^* \, d\Omega \] (14)

with the source term in the form

\[ m+1 S_P = \xi \frac{\partial}{\partial t} \nabla \cdot m+1 \hat{V} \] (15)

The pressure correction Poisson equation boundary conditions are

\[ m+1 \frac{\partial \tilde{P}}{\partial n_{\Gamma}} = m+1 \tilde{P}_N = 0; \quad p \in \Gamma^D, \quad m+1 \tilde{P} = m+1 \tilde{P}_D = 0; \quad p \in \Gamma^N \] (16)

The pressure correction, pressure gradient correction, pressure and pressure gradient fields are obtained in the following way. The pressure correction Poisson equation (14) is first used to solve for pressure correction unknowns \( m+1 \tilde{P} \). The same equation is subsequently used to explicitly solve the pressure correction unknowns \( m+1 \tilde{P} \) in the domain. The complete pressure field is updated as

\[ m+1 P = m P + m+1 \tilde{P} \] (17)

The calculated pressure field turns to be arbitrary up to an additive constant in the present incompressible fluid flow context. Pressure distribution over the boundary and in the domain is in our case shifted by an additive constant \( P_c \) in order to be distributed around 0 (for example for plotting purposes)

\[ m+1 P = m+1 P + m+1 P_c \] (18)

The constant \( P_c \) is determined from

\[ m+1 P_c = -\frac{1}{\gamma} \int_{\Gamma}^{m+1} P \, d\Gamma \] (19)

with \( \gamma \) standing for the total length of the boundary \( \Gamma \)

\[ \gamma = \int_{\Gamma} \, d\Gamma \] (20)

The pressure correction gradient \( m+1 \nabla \tilde{P} \) on the boundary and in the domain is explicitly calculated from the global DRBEM interpolation of the pressure field. The complete pressure gradient field is updated as

\[ m+1 \nabla P = m \nabla P + m+1 \nabla \tilde{P} \] (21)
The velocity field is updated through the pressure gradient corrections

\[ m^{+1}V = m^{+1}\dot{V} - \frac{\Delta t}{\xi} \nabla^{m+1}\vec{P} \]  

(22)

The velocity gradients on the boundary and in the domain are explicitly calculated from the global DRBEM interpolation of the velocity field.

After calculation of the velocity field, checking (5) is performed. In case of convergence, a move to a new timestep is made. In this case the initial values of the new timestep are taken as the final values of the current timestep. When the convergence is not achieved, the velocity and its gradients are relaxed as

\[ m^{+1}V = mV + \Xi (m^{+1}V - mV) \]  

(23)

\[ \nabla^{m+1}V = \nabla^mV + \Xi (\nabla^{m+1}V - \nabla^mV) \]  

(24)

with \( \Xi \) standing for the velocity relaxation factor, and the step by step explained iteration procedure makes another cycle. With this the discussion of the iteration procedure over timestep is complete.

The simplest discretization is used to approximate the boundary integrals in equations (8,14): boundary geometry is approximated by \( N_\Gamma \) straight line segments, and spatial variation of the fields on each of the boundary segments is represented by constant interpolation functions with gridpoints coinciding with the geometrical centers of the straight line segments. The Einstein summation convention is used in this text, i.e. any index which is repeated twice in a product is summed up. An underlined index is not summed up. For boundary integrals of a scalar valued function, or boundary integrals of a component of a vector valued function, both denoted with symbol \( F \), this gives

\[ \int_{\Gamma} \frac{\partial F}{\partial n_\Gamma} T^*_{li} d\Gamma - \int_{\Gamma} F \frac{\partial T^*_{li}}{\partial n_\Gamma} d\Gamma - c^*_l F_i \approx G_{lk} \delta_{ki} \frac{\partial F}{\partial n_\Gamma} - H_{lk} \delta_{ki} F_i - c^*_l \delta_{li} F_i, \]  

(25)

where \( k = 1, 2, \ldots N_\Gamma \) and \( i, l = 1, 2, \ldots N \). \( N = N_\Gamma + N_\Omega \) is the total number of points in which the solution is calculated. The first \( N_\Gamma \) points \( p_n \) coincide with the boundary gridpoints and the last \( N_\Omega \) points are arbitrarily distributed over the domain \( \Omega \). Index \( l \) stands for \( s_l = p_l \). \( \delta \) represents the Kronecker symbol. Matrix elements \( G_{lk} \) and \( H_{lk} \) are defined as follows

\[ G_{lk} = \int_{\Gamma_k} T^*_{li} d\Gamma, \quad H_{lk} = \int_{\Gamma_k} \frac{\partial T^*_{li}}{\partial n_\Gamma} d\Gamma, \]  

(26)

where \( \Gamma_k \) represents the \( k \)-th boundary segment, and \( c^*_l \) is equal to

\[ c^*_l = 0; \quad s_l \notin \Omega \cup \Gamma, \quad c^*_l = \frac{1}{2}; \quad s_l \in \Gamma, \quad c^*_l = 1; \quad s_l \in \Omega. \]  

(27)
The expressions (26) are evaluated analytically [7]. The domain integrals in equations (8,14) are transformed by considering the approximation of the spatial variation of the fields in $\Omega$ by the global interpolation functions of the form

$$F(p) \approx \psi_u(p) \zeta_u; \quad u = 1, 2, \cdots, N + N_{\text{aug}}$$

where $N_{\text{aug}}$ represents the number of augmented functions. The global approximation functions have been in this work selected and numerically implemented based on the proposals in [8]. The first chosen functions are the simple augmented conicals (CON) with $N_{\text{aug}} = 1$

$$\psi_n(p) = r_n; \quad n = 1, 2, \cdots, N, \quad \psi_{N+1} = 1$$

and

$$r_n^2 = (p - p_n) \cdot (p - p_n)$$

The second functions are the two-dimensional scaled augmented thin plate splines (TPS) with $N_{\text{aug}} = 3$

$$\psi_n(p) = r_n^2 \log r_n; \quad n = 1, 2, \cdots, N,$$

$$\psi_{N+1}(p) = p_x - p_x^0, \quad \psi_{N+2}(p) = p_y - p_y^0, \quad \psi_{N+3}(p) = 1$$

Scaling constants $p_x^0$ and $p_y^0$ stand for the mean coordinates of the domain $\Omega$. The third and the fourth functions are the multiquadrics (MQP) with $\beta = 1/2$, and the inverse multiquadrics (MQM) with $\beta = -1/2$, and with $N_{\text{aug}} = 1$, respectively

$$\psi_n(p) = (r_n^2 + c^2)^{\beta}; \quad n = 1, 2, \cdots, N, \quad \psi_{N+1}(p) = 1$$

with $c^2$ denoting an arbitrary constant. The fifth functions are the Gaussians (GAS) with $N_{\text{aug}} = 0$

$$\psi_n(p) = \exp\left(-\frac{r_n^2}{c^2}\right); \quad n = 1, 2, \cdots, N$$

Coefficients $\zeta_u$ are calculated by constructing a system of $N + N_{\text{aug}}$ algebraic equations

$$\Psi \zeta = F$$

The vectors are $\zeta = (\zeta_1, \cdots, \zeta_{N+N_{\text{aug}}})^T$ and $F = (F_1, \cdots, F_{N+N_{\text{aug}}})^T$, with $F_n = 0; n > N$. The first $v = 1, 2, \cdots, N$ rows of matrix $\Psi$ are of the form $(\psi_{1u}, \cdots, \psi_{Nu+N_{\text{aug}}})$, and the last $N_{\text{aug}}$ rows $v$ are of the form $(\psi_{1v}, \cdots, \psi_{Nu+N_{\text{aug}} v})$, with $\psi_{nu} = 0; n > N$. The notation has been shortened to $F_n \equiv F(p_n), \quad \psi_{nu} \equiv \psi_u(p_n)$. Coefficients $\zeta_u$ follow by inverting the system (34)

$$\zeta = \Psi^{-1} F$$
Consequently, the domain integral can be written in a compact dual reciprocity form \( k = 1, 2, \ldots, N_r, i, l = 1, 2, \ldots, N, u = 1, 2, \ldots, N + N_{\text{aug}} \)
\[
\int_{\Omega} \mathcal{F} T_l^* \, d\Omega \approx \int_{\Omega} \psi_u \Psi_{u_l}^{-1} \mathcal{F}_l T_l^* \, d\Omega = \Psi_{l_u} \Psi_{u_l}^{-1} \mathcal{F}_l,
\]
with
\[
\Psi_{l_u} \equiv \int_{\Gamma} \psi_u T_l^* \, d\Omega
\]
The integral \(\Psi_{l_u}\) is calculated by defining the harmonic functions \(\hat{\psi}_u\)
\[
\nabla^2 \hat{\psi}_u(p) = \psi_u(p)
\]
which allow us to write
\[
\Psi_{l_u} = \int_{\Gamma} \frac{\partial \hat{\psi}_u}{\partial n_{\Gamma}} T_l^* \, d\Gamma - \int_{\Gamma} \hat{\psi}_u \frac{\partial T_l^*}{\partial n_{\Gamma}} \, d\Gamma - c_s^* \hat{\psi}_u(s)
\]
The upper integrals are numerically evaluated by using the same discretization strategy that leads to expression (25)
\[
\int_{\Omega} \nabla^2 \hat{\psi}_u T_l^* \, d\Omega \approx G_{lk} \delta_{ki} \frac{\partial \hat{\psi}_{iu}}{\partial n_{\Gamma}} - H_{lk} \delta_{ki} \hat{\psi}_{iu} - c_s^* \delta_{li} \hat{\psi}_{iu}
\]
Other numerical method details can be found in [5].

4. Numerical Examples

Calculational domain \(\Omega\) is a square with coordinates \(p_x^- < p_x < p_x^+, p_y^- < p_y < p_y^+; p_{\pm} = \pm 0.5 \text{[m]}\). The boundary \(\Gamma\) is divided into south \(\Gamma_S\), north \(\Gamma_N\), east \(\Gamma_E\), and west \(\Gamma_W\) parts. \(\Gamma_S : p_y = p_y^-, p_x^- < p_x < p_x^+, \Gamma_N : p_x = p_x^+, p_y^- < p_y < p_y^+, \Gamma_E : p_x = p_x^-, p_y^- < p_y < p_y^+, \Gamma_W : p_x = p_x^-, p_y^+ < p_y < p_y^+\). The Dirichlet velocity boundary conditions are set to
\[
V_{\Gamma_x}^D = 0 \text{[m/s]}; \quad \mathbf{p} \in \Gamma_S \cup \Gamma_E \cup \Gamma_W, \quad V_{\Gamma_y}^D = 0 \text{[m/s]}; \quad \mathbf{p} \in \Gamma_S \cup \Gamma_E \cup \Gamma_W
\]
\[
V_{x}^D = 100 \text{[m/s]}; \quad \mathbf{p} \in \Gamma_N, \quad V_{y}^D = 0 \text{[m/s]}; \quad \mathbf{p} \in \Gamma_N
\]
Initial velocity boundary conditions are
\[
V_{0x} = 0 \text{[m/s]}; \quad \mathbf{p} \in \Omega + \Gamma, \quad V_{0y} = 0 \text{[m/s]}; \quad \mathbf{p} \in \Omega + \Gamma
\]
Initial pressure and pressure gradients are
\[
P = 0 \text{[N/m}^2\text{]; } \quad \mathbf{p} \in \Omega + \Gamma, \quad \frac{\partial}{\partial p_x} P = \frac{\partial}{\partial p_y} P = 0 \text{[N/m}^3\text{]; } \quad \mathbf{p} \in \Omega + \Gamma
\]
Material properties are both set to unit values $\rho = 1 \,[\text{kg/m}^3]$, $\mu = 1 \,[\text{kg/(m s)}]$ and the respective Reynolds number $Re = \frac{\rho V^2 (\Gamma_N) (p^+ - p^-)}{\mu}$ of the problem is thus 100.

The problem has been run on a uniform coarse mesh with $N_T = 40$, $N_N = 91$, $N = 141$. The relaxation coefficients $\xi$ and $\Xi$ have been both set to 1. The timestep used is $\Delta t = 0.001 \,[\text{s}]$. The velocity convergence criterion $V_\varepsilon$, evaluated in all gridpoints, has been set to 0.001 $[\text{m/s}]$. The steady-state was assumed to be achieved when the condition

$$|\mathbf{V} - \mathbf{V}_0| < V_{\text{steady}}$$

is satisfied in any of the gridpoints. The value for $V_{\text{steady}}$ was taken 0.01 $[\text{m/s}]$.

The globally interpolated components of the calculated velocity field are compared with the reference solution from [9] in non-uniformly spaced points. The $V_y$ is compared along horizontal line through geometric center of the cavity in seventeen positions. The $V_x$ is compared along vertical line through geometric center of the cavity in seventeen positions. The coordinates of points for comparison and values of reference solution are explicitly represented in [6]. The Table 1 shows the total number of timesteps to reach the steady state $\Delta t_N$, maximum number of timestep iterations $N_{\text{max}}$ and the total number of iterations to reach the steady state $N_{\text{tot}}$. The Table 2 shows accuracy of the solution. $Err_{\xi,\text{max}}$ represents the maximum absolute error between the reference and calculated solution $V_\xi$ in comparing points. $Err_{\xi,\text{mea}}$ represents the adjacent average error.

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Table 1: Convergence behavior of the solutions. Runs with $\infty$ diverge.
Table 2: Accuracy of the solutions in [m/s]/100. Runs with ∞ diverge.

5. Conclusions

The conclusions from the Table 1 are as follows. The optimum value of required total iterations for achieving the solution show functions MQP and MQM with the parameter \( c \) set to the typical mesh spacing. The solutions with an order of magnitude greater \( c \) diverge, and solutions with an order of one, two, three smaller magnitude of \( c \) systematically increase the number of required iterations. The CON results stand for the limiting MQP results with \( c^2 \to 0 \text{[m}^2\text{]} \). The MQM results with \( c^2 \to 0 \text{[m}^2\text{]} \) converge particularly slow. Solution with GAS converge only for \( c \) set to the typical mesh spacing.

The conclusions from the Table 2 are as follows. The most accurate results have been obtained by CON functions. The results with MQP improve and the results with MQM degrade with decreasing parameter \( c^2 \). The TPS results are somehow comparable with the GAS results.

It should be mentioned that the conclusions drawn from the described preliminary testing apply to relatively very coarse meshes. Our future work will complement the described testing for finer discretization. An open question remains also the performance of the different global approximation functions in alternative solution procedure which uses integral representation formulas [5] instead the direct use of global approximation for evaluation for the pressure and velocity field gradients.
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