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Unsteady free surface flow around hydrofoils

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Abstract

The unsteady two dimensional free surface motion produced by a submerged body in a uniform stream is studied in the potential flow approximation. The Laplace equation governing the irrotational flow of an incompressible fluid is solved by using a boundary element representation for the velocity potential. A Neumann condition is assigned on the body contour and a Dirichlet condition is used on the free surface. Results obtained for several geometrical configurations are in good agreement with available theoretical, numerical, and experimental data.

1 Introduction

The free surface flow generated by moving submerged bodies has been largely studied in the past either from theoretical [1, 2], numerical [3, 4], and experimental point of view [5, 6]. Most of the numerical studies have been performed only for steady flows, while current unsteady numerical simulations are mainly performed assuming a single valued function for the free surface elevation.

The aim of the present paper is the development of a numerical tool which is able to describe the free surface dynamics in some applications of relevant interest in naval hydrodynamics such as the slamming process and the onset of breaking of steep waves. To describe the multivalued wave elevation occurring in these cases, the free surface motion is followed in a Lagrangian way. The flow is assumed irrotational and the fluid inviscid so that the disturbance

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velocity field due to the body can be expressed in terms of a velocity potential ϕ. For fluid having constant density, the velocity potential satisfies a Laplace equation which is solved by using a boundary element formulation: the normal derivative of ϕ is assigned on the body contour while on the free surface the actual value of the velocity potential is updated according to the unsteady Bernoulli's equation.

To validate the numerical procedure in the long time integration, the simulation of the free surface flow around several geometries starting from rest is carried out until a quasi-steady solution is achieved. As benchmark solutions, the flow past a circular cylinder and two different hydrofoils at zero and nonzero incidence are computed and the results are compared with available theoretical, numerical and experimental data.

2 Numerical model

In this section some aspects of the mathematical model and of the numerical scheme are discussed. All quantities are expressed in non dimensional form by using the length L and the velocity U as characteristic values. The irrotational flow of an inviscid fluid can be expressed in terms of the velocity potential ϕ, so that the velocity field in a frame of reference attached to the body is given by

$$\vec{u} = -\vec{U}_b + \vec{\nabla}\phi \ , \tag{1}$$

$\vec{U}_b$ being the actual velocity of the body with respect to an inertial frame of reference. For fluid having constant density, the velocity potential is determined by solving a Laplace equation in the fluid domain with suitable boundary conditions on the body contour and on the free surface. The solution of the Laplace equation is obtained through a boundary element representation for the velocity potential so that in any point $\vec{x}$ inside the fluid domain. This results in the following:

$$\phi(\vec{x}) = \int_S \left[\frac{\partial \phi}{\partial \nu}\left(\vec{\xi}\right) G\left(\vec{x}-\vec{\xi}\right) - \phi\left(\vec{\xi}\right) \frac{\partial G}{\partial \nu}\left(\vec{x}-\vec{\xi}\right)\right] \, dS\left(\vec{\xi}\right) \tag{2}$$

where G is the Green's function of the Laplace operator in two dimensions, S is the boundary of the fluid domain and $\vec{\nu}$ is the unit normal vector directed inwards. Equation (2) allows us to obtain the velocity potential once both the velocity potential itself and its normal derivative are known all along the boundary. At each time step, the normal derivative of ϕ is assigned on the body contour according to the impermeability condition, while the velocity potential on the free surface is provided by the unsteady Bernoulli's equation. The velocity potential on the body contour and its normal derivative on the free surface are determined by solving the integral equation obtained by applying equation (2) on the boundary of the fluid domain.

From the numerical point of view, S is discretized by segments having constant values of the velocity potential and of its normal derivative. The solution of the integral equation provides the velocity potential on the body contour and its normal derivative on the free surface. A second order Runge-Kutta scheme is employed for the time integration of the free surface evolution and of the associated velocity potential. The latter is updated by using the unsteady Bernoulli's equation to obtain

$$\frac{D\phi}{Dt} = -\frac{1}{Fr^2}z + \frac{|\vec{\nabla}\phi|^2}{2} \quad . \tag{3}$$

where z is the wave elevation and $Fr = U/\sqrt{gL}$ is the Froude number. The free surface motion is integrated by moving the midpoint of each panel and then using a cubic spline interpolation to reconstruct the vertices distribution. In the time integration it is required that, for each panel, the product of the velocity amplitude by the time step must be smaller than a fraction f of the panel length.

Once the velocity potential on the body surface is known, the pressure can be computed by the following expression

$$p = -\frac{1}{Fr^2}z + \vec{U}_b \cdot \vec{\nabla}\phi - \frac{|\vec{\nabla}\phi|^2}{2} - \frac{\partial\phi}{\partial t} \quad . \tag{4}$$

Due to the finite extension of the computational domain, a wave absorbing beach model is employed to damp out waves that propagate outside the truncated domain by modifying the vertical component of the velocity as suggested in [7]

$$\frac{\partial\phi}{\partial z} = \frac{\partial\phi}{\partial z} - 2\mu z + Fr^2\mu^2\phi \quad , \tag{5}$$

where μ is a damping parameter which is maximum at the boundary of the computational domain and decreases with a cubic law inward. Both the damping coefficient and the beach extension must be properly chosen to absorb the wave energy avoiding wave reflection. Usually the damping coefficient is $O(1)$, while the extension of the absorbing beach is equal to 4λ, where $\lambda = 2\pi Fr^2$ is the characteristic wavelength provided by the linear theory.

3 Applications

The free surface flow generated past a body by a uniform stream is simulated at different Froude numbers and submergences. The stream is progressively accelerated with a sinusoidal law up to the desired value U. In all cases the final speed is reached at the non dimensional time $t = 50$ and numerical simulations proceed until a quasi-steady solution is obtained. The length of the transient stage and the law employed to accelerate the stream are chosen in order to reduce the long wave formation occurring in this phase.

Unless explicitely stated, results presented below have been obtained by using $f = 0.25$, and about 20 panels per wavelength.

3.1 Circular cylinder

As a first application, the free surface flow generated past a circular cylinder is investigated. The diameter is assumed as characteristic length, the center of the cylinder is at $(0, -2.5)$ and the Froude number is $Fr = 0.6$.

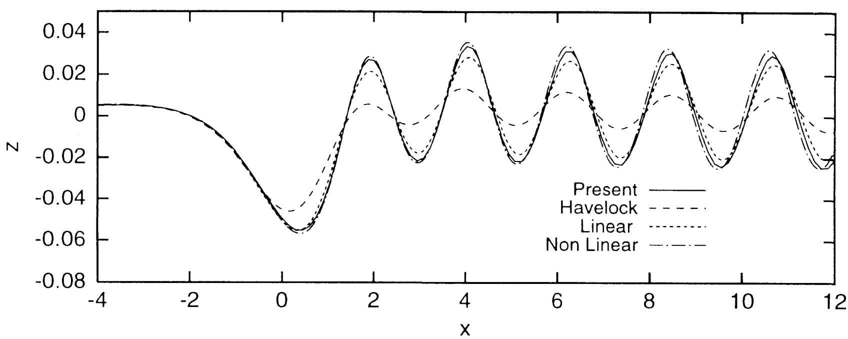

Figure 1: Free surface profile generated behind a circular cylinder.

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In Fig. 1 the wave pattern obtained by the present simulation is compared with the theoretical prediction provided by Havelock [1] and with the numerical results obtained with a different panel method for steady flows [4] that gives both the fully non linear solution and a linear wave profile obtained by linearizing about the double body flow. The wavelength is essentially the same for all cases but solutions accounting for non linear effects predict larger wave amplitude.

A brief analysis concerning the effects of the time integration accuracy is reported in Fig. 2 where the time histories of the longitudinal force computed by using different values of f are depicted for $t > 50$ and compared with the value predicted by the non linear steady simulation.

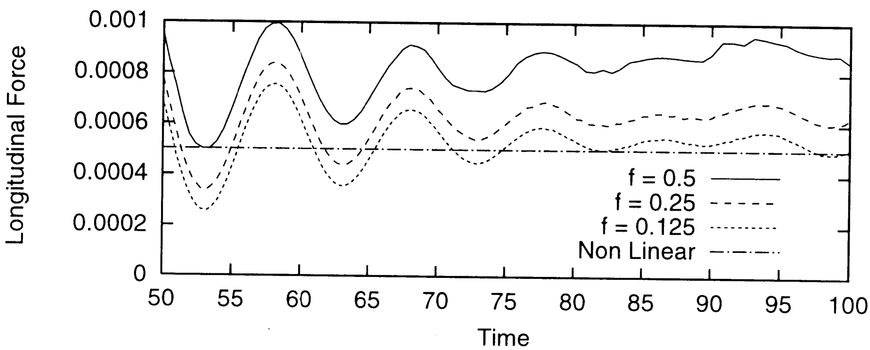

Figure 2: Time history of the longitudinal force in the post transient phase.

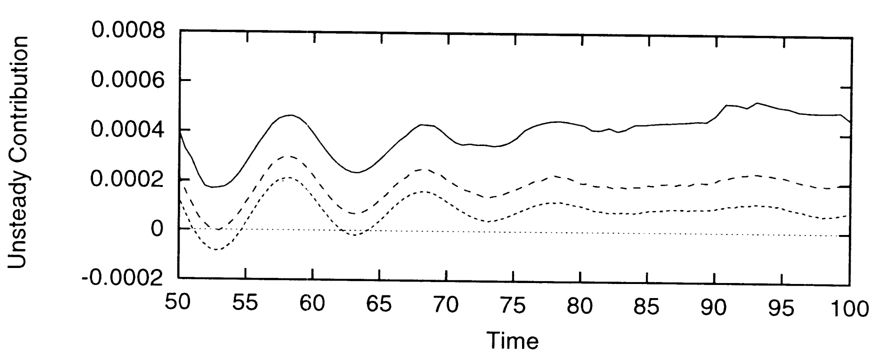

Figure 3: Contribution of the last term of eq. (4) to the force.

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Differences are essentially due to the unsteady part of the pressure, namely the last term in equation (4), as it is shown in Fig. 3. This is confirmed by the fact that wave profiles (not shown) are only weakly affected by f. For this reason, since the present work is mainly devoted to the analysis of the wave profile, in the following $f = 0.25$ is used.

3.2 Hydrofoil at zero incidence

The free surface flow produced past an hydrofoil at zero incidence is studied. Hydrofoil data are reported in Salvesen [5] and not repeated here. The chord is used as the characteristic length and the leading edge is located at $(0, -1.14)$.

In Fig. 4 the longitudinal force is plotted versus time. Since the force is reported in non dimensional form, no differences occur in the initial phase. For low Froude numbers the final wave resistance is smaller than the longitudinal force occurring in the transient phase due to the added mass effect.

In Fig. 5 the wave profiles are shown along with the experimental data and with results obtained through the steady non linear simulation. This latter is computed only up to $Fr = 0.59$ since a stabilizing viscous correction [4] is needed to achieve the solution at higher Froude numbers.

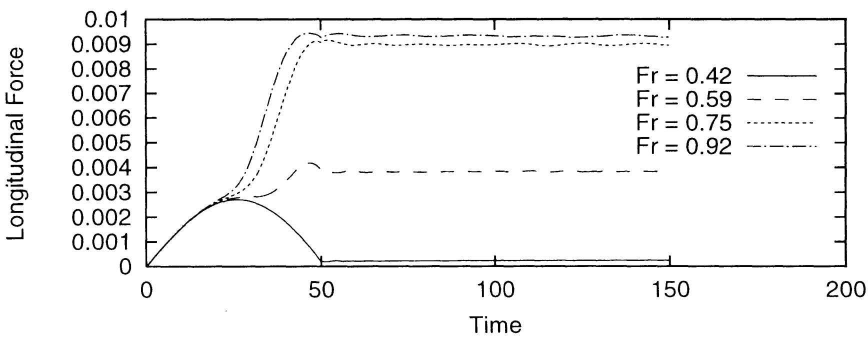

Figure 4: Time histories of the non dimensional longitudinal force computed at several Froude numbers.

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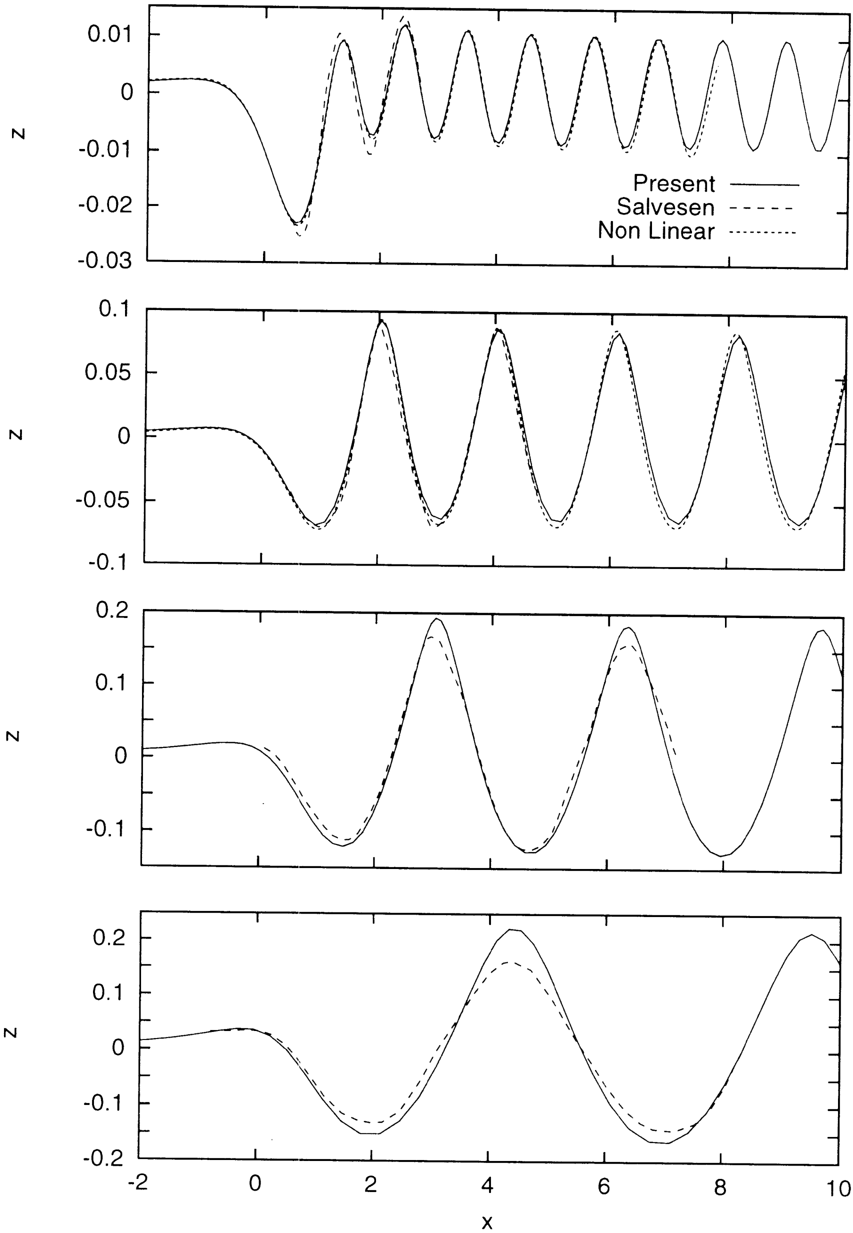

Figure 5: Free surface profile generated behind an hydrofoil at several Froude numbers. From the top: $Fr = 0.42$, $Fr = 0.59$, $Fr = 0.75$, $Fr = 0.92$.

Present numerical simulations are in good agreement with experi-

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mental data even though at the highest Froude number a larger wave amplitude is predicted.

3.3 Inclined hydrofoil

In order to validate the capability of the present code to predict the onset of wave breaking, the free surface flow about an inclined hydrofoil is studied and results are compared with experimental data obtained by Duncan [6].

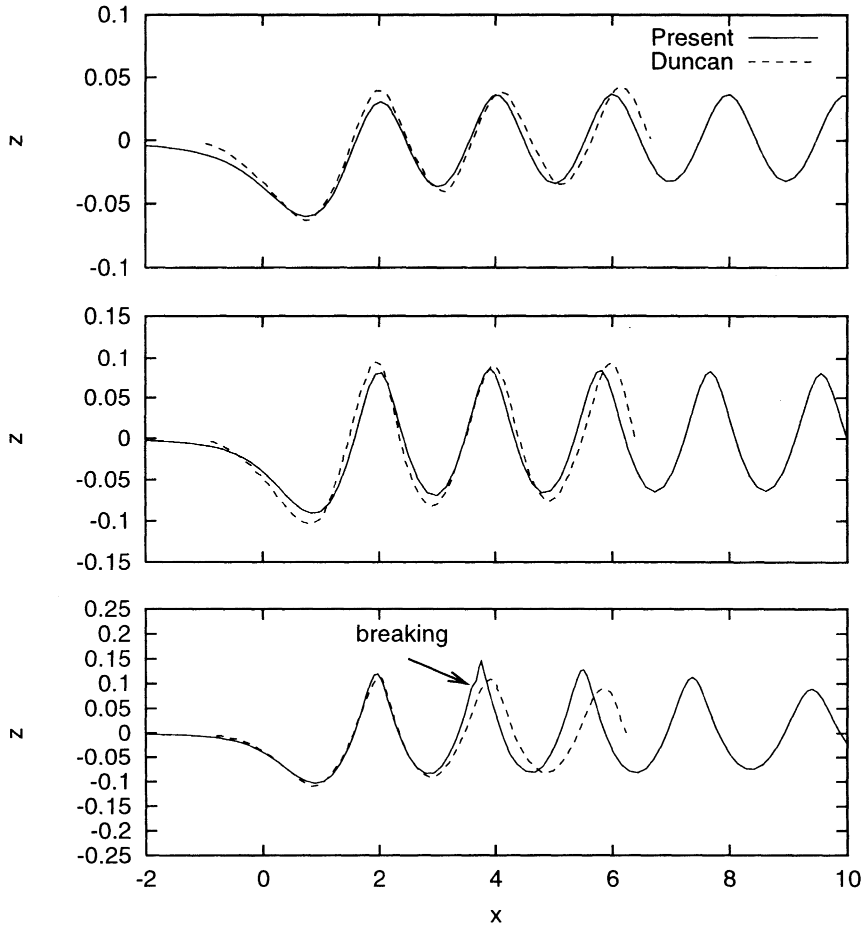

Figure 6: Comparisons of wave profile obtained at several submergences. The leading edge is at $(0, -h)$. From the top: $h = 1.285$, $h = 1.034$, $h = 0.955$.

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In the experimental setup the hydrofoil, a NACA 0012 profile at angle of attack of 5°, is towed in a channel having a finite depth while the present numerical simulation is performed for infinite depth. As stated by Duncan, this may cause weak differences on the onset point of breaking while no differences should occur in the wave profile since, at least for the Froude number employed ($Fr = 0.567$), the depth is greater than half of a characteristic wavelength.

The numerical procedure has been slightly modified to account for the vortex shedding. In particular a vortex panel is introduced on the chord of the profile and its circulation is computed at each step by enforcing the fluid velocity to be directed along the bisector at the trailing edge. In Fig. 6 results are compared with the experimental data: the agreement is satisfactory, even though a smaller wave amplitude is predicted for $h = 1.285$ and $h = 1.034$. Since circulation plays a relevant role in the wave profile, authors ascribe discrepancies to the model employed to impose the Kutta condition at the trailing edge.

For $h = 0.955$, numerical simulation predicts an instability of the wave profile at the time $t = 66$, when a steady solution is not yet reached. At the same depth, Duncan observes that both a breaking and a non breaking profile may exists (in Fig. 6 only the non breaking solution is shown). Since no model has been introduced to treat the wave breaking, the present numerical simulation is not able to go futher on.

4 Conclusions

In the present work the two dimensional unsteady free surface flow induced by a body in a uniform stream has been analyzed. The simulations have been carried out until a quasi-steady solution is reached. Results are satisfactory and assess the capability of the numerical scheme to describe the free surface motion in the long time evolution. Furthermore, particular attention has been posed on the initial transient stage in order to capture the onset of breaking of steep waves. The simulation correctly predict the growth of a free surface instability leading to a breaking solution. Futher improvements of the numerical model should be included in order to study the mechanics of quasi-steady breaking waves. In particular a model accounting for the wave breaking is needed to discern between transient instabilities or effective wave breaking.

Acknoledgement

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