

# **An inverse problem modelled by dual reciprocity boundary element methodology: identification of a degraded Hithite inscription in Hattusas (central Anatolia, Turkey)**

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## **Abstract**

The condition of a Hithite inscription in Hattusas (central Anatolia, Turkey) has deteriorated so much during its life time of approximately 3400 years that, until now, archaeologists could only read a very small number of all signs in this inscription. A dual reciprocity boundary element model was designed to model this inverse problem and identify other hieroglyphs not recognized yet. The model was calibrated by using the existing alphabet assembled from other sources, and the better preserved parts of the inscription that are still readable. The results gained so far indicate that more degraded hieroglyphs, which are hardly detectable by optical means, can be recovered to a certain extent and matched to signs in the existing alphabet.

## **1 Introduction**

The ruins at Hattusas or Boğazköy (the present Turkish name) about 150 km east of Ankara are the remains of the capital of the Hithites who reigned in central Anatolia around 1400 - 1300 BC. The place was rediscovered in 1834, and since then, excavations and research have been carried out to increase our historical knowledge about the Hithites. An important part of this work is the study and understanding of ancient writings using hieroglyphs. Such inscriptions were also found in Hattusas. One of them was chiseled out of limestone in the open air with a size of about  $2.5 \times 10 \text{ m}^2$  (Fig.1).



Figure 1: A section of the degraded Hittite inscription in Hattusas (central Anatolia, Turkey)

The crosses originate from a Réseau plate of a photogrammetric camera used to take all the pictures.

Each sign in this inscription has a size of about  $25 \times 25 \text{ cm}^2$  consecutively written in horizontal rows, as can be seen in Fig.1. Some hieroglyphs in the upper rows are still readable because they are under a small cover of rocks above them so that they are more protected from degradation than the other signs. Other parts of the inscription completely disappeared. The present shape of still existing hieroglyphs can be determined by either optical means using shading from the sun or artificial light, or by using human touching senses. Because the latter technique is less objective, only optical observation with sun shading has been applied here. The simplest optical method is naked eye recognition. Other more advanced approaches are photogrammetric and digital image recognition techniques, which are going to be included in this research project at later stages. All pictures of the inscription were already taken with a photogrammetric camera.

The goal of this paper is to model the degradation of unidentified hieroglyphs in Fig.1, which are hardly visible and detectable by optical means, in order to match them to signs in the existing alphabet. There are several hundred

hieroglyphs known today. A smaller collection is given in e.g. Werner<sup>1</sup> and shown in Fig.2. This picture was taken at a well protected site in Hattusas.



Figure 2: Better preserved Hithite hieroglyphs in Hattusas (central Anatolia, Turkey)

In theory, the described inverse problem can be solved in two different ways. (i) The original shape of hieroglyphs plus other starting and boundary conditions in the past are estimated so that, after a forward time integration, a best-fit to the present shape of a hieroglyph is made using an iteration scheme. (ii) The model could start with the present shape of hieroglyphs and apply a backward time integration to estimate the original state of a hieroglyph. It is obvious that the latter approach is more unreliable because of the inaccuracy in determining the present shape of degraded signs and their corresponding boundary conditions. And applying a backward time integration based on uncertain starting and boundary conditions leads to very unreliable results. Thus, the first method is used in this paper based on the geometry of known hieroglyphs in the Hithite alphabet and a forward time integration scheme.

## 2 The DRM degradation model

The degradation model for Hithite hieroglyphs was implemented by using the dual reciprocity boundary element method (DRM). The DRM was developed to overcome the restrictions of the classical BEM (Partridge et al.<sup>2</sup>), such as the requirement for a fundamental solution to the original partial differential equation in order to obtain an equivalent boundary integral equation. Another limitation of the classical BEM has been that non-homogeneous parts in the governing equations are incorporated by means of domain integrals so that the BEM loses its original attraction of a 'boundary-only' method. The DRM uses a fundamental solution to a simpler governing equation and takes into account the remaining non-homogeneous terms in the original equation by applying reciprocity principles and certain approximating functions. Its simplicity allows for implementation of numerical solutions for e.g. Laplace, Poisson, convection, convection-diffusion, and steady Navier-Stokes problems for moderate Reynolds numbers (Power & Wrobel<sup>3</sup>) without changing the basic solution procedure. These DRM advantages were utilized in the 'Object-oriented Boundary Element Program (OBEP) 1.0' for Windows<sup>TM</sup> used in this work (Friedrich<sup>4</sup>). This program offers a visual interactive user interface to allow simple and fast pre-

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processing, computations, and post-processing, which is of great importance for this research project considering the huge number of Hithite hieroglyphs and their very different forms (Werner<sup>1</sup>) that need to be examined.

The governing equation of the designed degradation model is based on the two-dimensional diffusion equation

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{K} \frac{\partial u}{\partial t}, \quad (1)$$

where  $u$  is the height of the upper surface of a hieroglyph, and  $K$  a material constant. This leads to a matrix equation (Partridge et al.<sup>2</sup>)

$$\mathbf{H} \mathbf{u} - \mathbf{G} \mathbf{q} = - \frac{1}{K} \mathbf{S} \dot{\mathbf{u}}, \quad (2)$$

with

$$\mathbf{S} = (\mathbf{H} \mathbf{U}^P - \mathbf{G} \mathbf{Q}^P) \mathbf{F}^{-1}. \quad (3)$$

which allows a direct implementation of a time-marching scheme. For example, a two-level time integration formula reads

$$\mathbf{S} \frac{\mathbf{u}(t_{n+1}) - \mathbf{u}(t_n)}{t_{n+1} - t_n} + \mathbf{H} \frac{\mathbf{u}(t_{n+1}) + \mathbf{u}(t_n)}{2} = \mathbf{G} \frac{\mathbf{q}(t_{n+1}) + \mathbf{q}(t_n)}{2}, \quad (4)$$

or better sorted

$$\begin{aligned} & (2 \mathbf{S} + \mathbf{H} \Delta t) \mathbf{u}(t_{n+1}) - \mathbf{G} \Delta t \mathbf{q}(t_{n+1}) \\ & = (2 \mathbf{S} - \mathbf{H} \Delta t) \mathbf{u}(t_n) + \mathbf{G} \Delta t \mathbf{q}(t_n), \end{aligned} \quad (5)$$

which can be integrated, if enough starting values  $\mathbf{u}(t_0)$ ,  $\mathbf{q}(t_0)$  are given. Eq.(1) reflects the simplest degradation model with just one parameter  $K$ . Taking into account all physical, chemical, and biological causes would result into a more refined model with two, three, or more parameters (Collepardi<sup>5</sup>), but computational tests have shown that more sophisticated models do not improve the overall suitability of the model required here, mainly because there is no adequate way to determine accurately extra parameters. Or in other words, the known data is not sufficient to establish a reliable multi-parameter model. So far, the best results have been still obtained with Eq.(1), whereby the decay rate of hieroglyphs is not important for their identification considered here, but only their geometrical surface degradation.

### 3 Benchmark test of the DRM degradation model

The created DRM degradation model was benchmarked by simulating the cooling process of a square plate with a material constant of  $K = 1.25$  and a side length of  $l = 3.0$  with coordinates ( $x_{\min} = y_{\min} = 0.0$ ), ( $x_{\max} = y_{\max} = 3.0$ ). It

is cooled from a starting temperature  $u_0 = 30.^\circ\text{C}$  down to zero ( $u = 0.^\circ\text{C}$ ). The analytical solution of this diffusion problem for a field point P at a position  $(x,y)$  is given with  $K_x = K_y = K$  and  $l_x = l_y = 1$  after a time period  $\Delta t$  by (Partridge et al.<sup>2</sup>)

$$u(\Delta t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{4u_0}{nm\pi^2} [(-1)^n - 1][(-1)^m - 1] * \sin \frac{n\pi x}{l_x} \sin \frac{m\pi y}{l_y} \exp \left[ -\Delta t \left( \frac{K_x n^2 \pi^2}{l_x^2} + \frac{K_y m^2 \pi^2}{l_y^2} \right) \right]. \quad (6)$$

The analytical temperature gradients can be simply obtained by building the x and y derivatives of the sine terms in Eq.(6). The DRM solution of this diffusion problem with OBEP gave the following results as they are displayed in Fig.3 after an integration time of  $\Delta t = 0.150$  s. They fit very well into the values gained from Eq.(6) (Partridge et al.<sup>2</sup>).

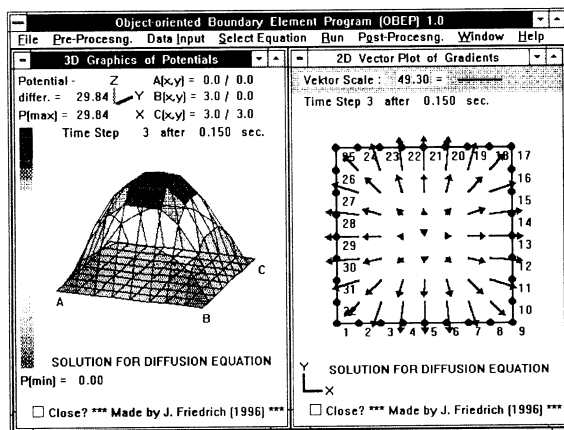


Figure 3: 3-D temperature plot and 2-D gradient plot of benchmark test

Thinking of hieroglyphic degradation and replacing the temperature by the surface height, the results of the degradation model for the same example are as follows. A square-shaped hieroglyph loses its round corners in time and degrades to a circular form because of the symmetric boundary geometry at the starting time of the integration. After a certain time period all material diffuses, leaving a completely flat surface with zero heights everywhere inside of the original boundary.

#### 4 Calibration test of the DRM degradation model

The designed DRM degradation model was calibrated using a still readable

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hieroglyph in the upper row of the inscription. It is number 238 in the sign list of Werner<sup>1</sup> called 'gate', also visible in the right bottom corner of Fig.2. The degraded gate hieroglyph is displayed in the following figure.



Figure 4: Degraded gate hieroglyph in Hattusas (central Anatolia, Turkey)

The degradation model for the gate sign with OBEP applies 50 boundary and 66 internal nodes as shown in the left-hand window of Fig.5a. The right-hand window of Fig.5a and Fig.5b display the results for two different integration times  $\Delta t = 0.05$  s and  $\Delta t = 0.10$  s, using the same parameters as before:  $K = 1.25$  and  $u_0 = 30.0$ .

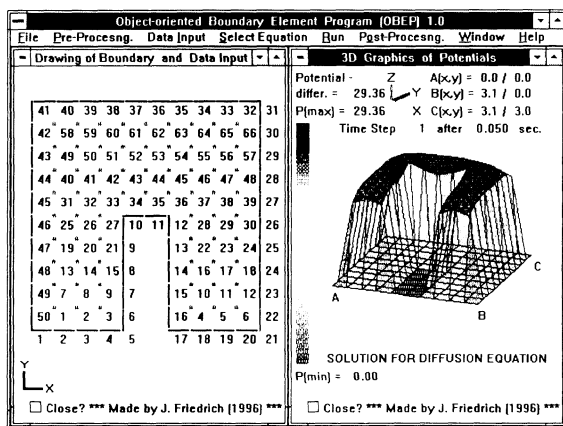


Figure 5a: Boundary discretization and 3-D surface plot of calibration test after  $\Delta t = 0.05$  s

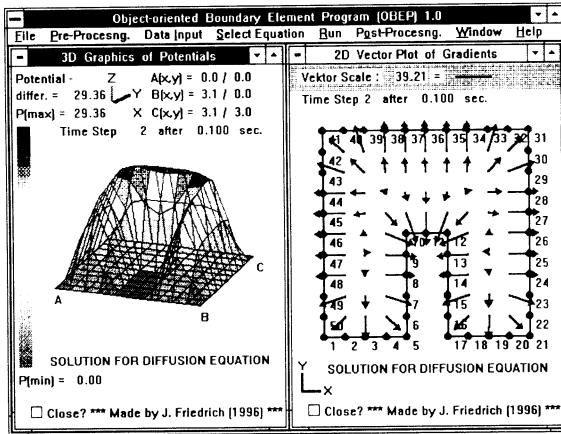


Figure 5b: 3-D surface plot and 2-D gradient plot of calibration test after  $\Delta t = 0.10$  s

There is good agreement between the shapes in Fig.4 and the left-hand window of Fig.5b, confirming the suitability of the chosen method. Examining Fig.5a and 5b in more detail, some characteristics of the degradation model already pointed at in the previous chapter become obvious. They are listed below.

- Sharp corners of hieroglyphs are rounded off.
- Lengthy parts of hieroglyphs become thinner.
- Thinner parts of hieroglyphs deteriorate faster than thicker parts.
- The average surface height of hieroglyphs decreases in time.

## 5 Identification of undetected hieroglyphs: an example

After benchmarking and calibrating the degradation model, the described method can be tried out with still undetected hieroglyphs. As example a readable hieroglyph from the third upper row of the inscription representing a lion or another predaceous animal is chosen (Fig.6).

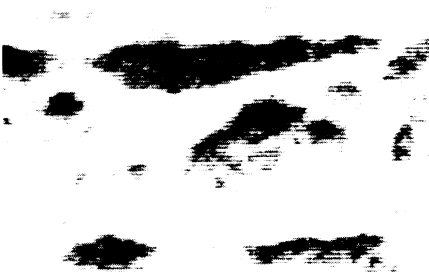


Figure 6: Degraded lion hieroglyph in Hattusas (central Anatolia, Turkey)

Such a geometrical shape was discretized by OBEP with 63 boundary and 65 internal nodes as shown in Fig.7.

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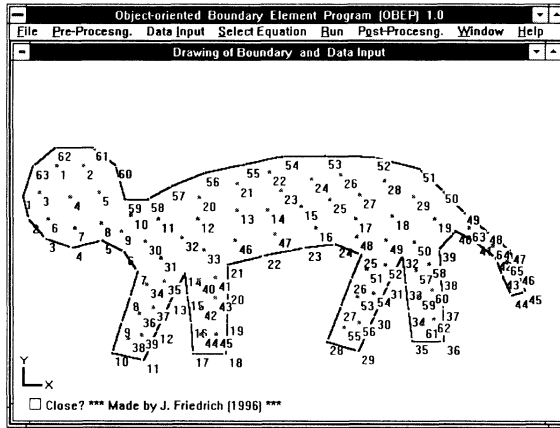
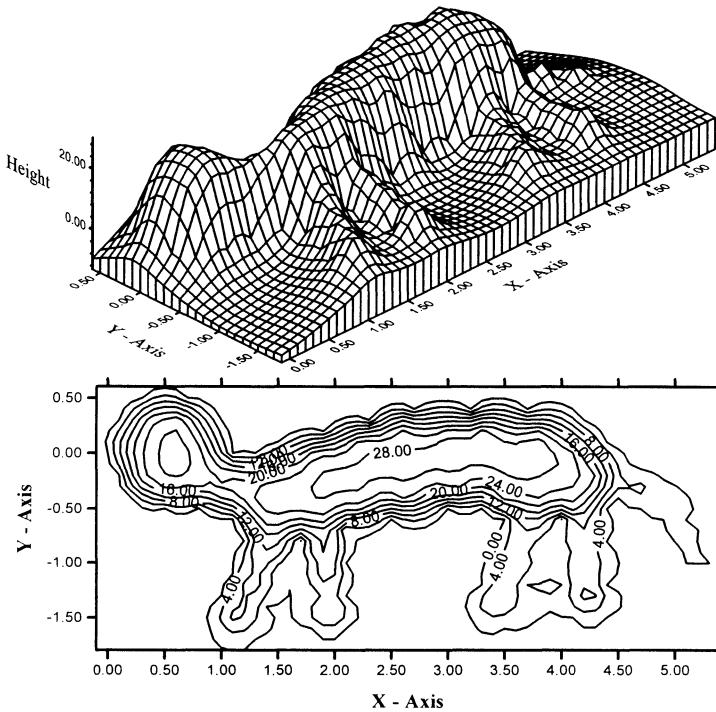


Figure 7: Boundary discretization of a lion hieroglyph

With the same parameters used before ( $K = 1.25$  and  $u_0 = 30.0$ ) the following results were gained for two different integration times  $\Delta t = 0.05$  s and  $\Delta t = 0.10$  s.

Figure 8a: 3-D surface plot of a lion hieroglyph after  $\Delta t = 0.05$  s

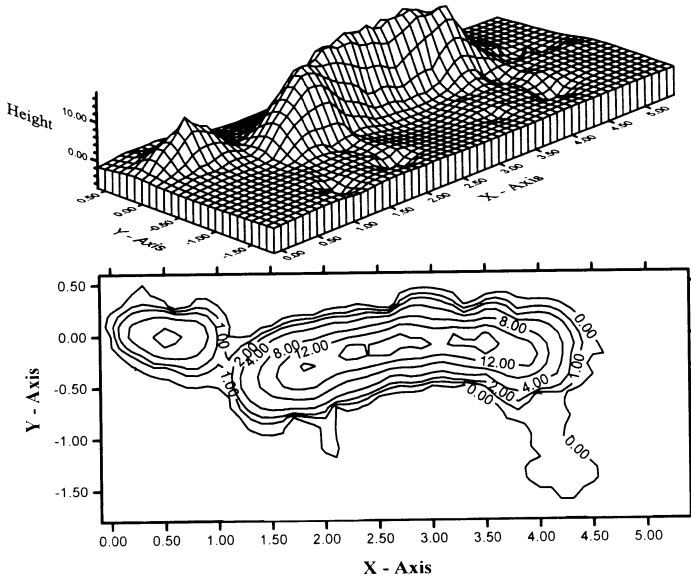


Figure 8b: 3-D surface plot of a lion hieroglyph after  $\Delta t = 0.10$  s

These figures illustrate the degradation process of the lion hieroglyph categorized in the following three periods.

- 1) In the beginning, legs and tail as thinner parts decay much faster than the rest of the animal.
- 2) After a certain time legs and tail disappear completely. Head and body of the animal deteriorate rapidly during the intermediate period (comp. Fig.6).
- 3) In the final stages, only head and body are left over disconnected from each other. Their remains are just a small closed area above the end of a single line.

After an intensive search such remains were found at several locations in the inscription, for example, in the right-hand section of the second upper row. This fitting image to Fig.8b with the head on the right-hand side is printed in Fig.9.



Figure 9: Remains of an animal hieroglyph in Hattusas (central Anatolia, Turkey)



## 6 Conclusions

The aim of this study was to introduce a degradation model for Hithite hieroglyphs using the dual reciprocity boundary element method. After benchmarking and calibrating the designed model, the suitability of the chosen approach was demonstrated by matching the remains of an animal hieroglyph (a small closed area above the end of a single line) to unidentified parts in the Hithite inscription. It is planned to extend the presented method along the following lines. (i) Naked eye observations should be improved and automatized by more advanced optical tools, such as photogrammetric and digital image recognition techniques. (ii) As the accuracy of measured surfaces of degraded hieroglyphs increases, the degradation model should also be further refined, incorporating other parameters and allowing to identify further hieroglyphs not recognized yet.

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