Implicit scheme time domain BEM analysis of transient electromagnetic fields
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Abstract

This paper discusses boundary element (BE) analysis of the transient electromagnetic fields scattering phenomena. Especially, an implicit scheme formulation is introduced to achieve stable calculations. The formulation is done based on the inhomogeneous wave equation to the scalar and vector potentials of electromagnetic fields. The discussion shows us the boundary integral equations which appear in the formulation have clear physical meaning, although potentials are used there. After formulation, some numerical simulation examples are also shown to confirm the validity of the formulation.

1 Introduction

In electromagnetic fields calculations, one of benefit of use of boundary integral equation is easy treatment of open boundary problems or scattering phenomena. In the present status, most of these problems (open boundary problems for static electro / magnetic field or the scattering phenomena in frequency domain) are well treated by the boundary element (BE) scheme. Then, one of still remained work in the BEM is the treatment of transient (electromagnetic) scattering fields. Difficulties of the application of BEM to the transient fields are mainly caused by numerical instabilities.

Taking these situations into account, this paper discusses BE analysis of the transient scattering electromagnetic phenomena. Especially, an implicit scheme formulation is introduced to achieve the stable calculation.
2 Boundary Element Formulation

In description of the formulation, the covariant form\(^3\) notations of the electromagnetic fields values are used to simply express equations. And the following formulation is done for the transient electromagnetic fields in vacuum, which are bounded by perfect conductors.

We know that such system can be described by Maxwell’s equations and the perfect conductor boundary conditions,

- Maxwell’s equations
  \[
  \partial^\mu F^{\nu\lambda} + \partial^\lambda F^{\mu\nu} + \partial^\nu F^{\lambda\mu} = 0
  \] (1)

- perfect conductor boundary conditions (for electric and magnetic fields)
  \[
  F F_n = 0
  \] (2)

where \(\varepsilon\) is the velocity of light, \(\varepsilon_0\) is the dielectric constant in vacuum, \(j^\mu = (c \rho, j)\) is four currents, \(n^\mu = (0, n)\) is 4D unit normal vector to the boundary, \(\epsilon^{\mu\nu\alpha\beta}\) is the Levi-Civita pseudotensor and \(F^{\mu\nu}\) is the electromagnetic field tensor. In term of scalar and vector potentials, Eqs.(1)-(3) are rewritten by using the following master equation and boundary conditions,

- master equations
  \[
  \partial^\nu \partial^\lambda A^\mu - \partial^\mu A^\lambda = \frac{j^\mu}{\varepsilon_0 c^2}
  \] (4)

- boundary conditions
  \[
  A_\mu = 0
  \] (5)

where \(A^\mu = (\phi /c, A)\) and \(x^\mu = (ct, x)\), are the four potentials and four coordinates. Especially, it should be noted that the ordinary Lorentz condition,

\[
\partial^\nu A^\nu = 0
\] (6)

is not used here.

The boundary element formulation for such time dependent phenomena starts with the following the Green theorem on the 4D space-time,
In this case, the “boundary” consists of two types (3D) super-surface, \( \partial V \times [-\infty, \infty] \) and \( V \) at \( t = -\infty, +\infty \) (see Fig.1).

To substitute fundamental solution to the wave equation and the four potentials to the functions \( \Phi \) and \( \Psi \), the resultant boundary integral equation become as follows,

\[
A^{\mu}(ct, x) = \frac{1}{4\pi\epsilon_0 c^2} \int_{V} j^{\mu}(t - \frac{|x - x'|}{c}, x') dV' + \frac{1}{4\pi} \int_{S} \frac{\partial \Lambda(t - \frac{|x - x'|}{c}, x')}{\partial n} dS' - \frac{1}{4\pi} \int_{S} n_{\nu} \frac{\partial A^{\nu}(t - \frac{|x - x'|}{c}, x')}{\partial n} dS'
\]

where \( \partial / \partial n \) denotes the normal derivative on the boundary. We can clearly understand the physical meaning of Eq.(9) by using the following 3D vector expression,

\[
\phi(ct, x) = \frac{1}{4\pi\epsilon_0} \int_{V} \rho(t - \frac{|x - x'|}{c}, x') dV' + \frac{1}{4\pi} \int_{S} \frac{\partial G}{\partial t} \frac{\partial \phi(t - \frac{|x - x'|}{c}, x')}{\partial n} dS'
\]

\(
A^{\mu}(ct, x) = \frac{1}{4\pi\epsilon_0 c^2} \int_{V} j(t - \frac{|x - x'|}{c}, x') dV' - \nabla G + \frac{1}{4\pi} \int_{S} \frac{\partial A^{\nu}(t - \frac{|x - x'|}{c}, x')}{\partial n} dS'
\)

where

\[
G(ct, x) = \frac{1}{4\pi} \int_{V} \Lambda(t - \frac{|x - x'|}{c}, x') \frac{dV'}{|x - x'|}
\]

That is to say, one can readily find that the values \( \partial \phi / \partial n \) and \( \partial A / \partial n \) in the integrands of the third terms of Eq.(10) are directly connected with induced surface charge and currents,

\[
E = -\frac{\partial \phi}{\partial n} = -(E \cdot n) n
\]

\[
= -\frac{\sigma}{\epsilon_0} n
\]
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\[
B = - \frac{\partial A}{\partial n} \times n = -(B \times n) \times n
\]

\[
= - \frac{K}{\varepsilon_0 c^2} \times n
\]

(13)

where \(\sigma\) and \(K\) denote surface charge and current densities. This means that Eq.(10) can be interpreted as the Coulomb & Biot-Savert laws for the boundary value problems. In the practical calculation, terms which include the gauge function \(G\), are not necessary to be calculated because these terms do not contribute to electromagnetic fields. Accordingly, one of most simple way to evaluate Eq.(10) is to eliminate the gauge term by combining two equations of Eq.(10) (this operation just corresponds to derivation of the electric field from the potentials),

\[
E(\mathbf{r}, \mathbf{x}) = E_{ext}(\mathbf{r}, \mathbf{x}) - \frac{1}{4\pi} \int_{S} \frac{dS}{|\mathbf{x} - \mathbf{x}'|} \left[ \frac{\partial A_{z}(t - \frac{\mathbf{x} - \mathbf{x}'}{c}, \mathbf{x}')}{\partial n} + \nabla \cdot \frac{\partial \phi(t - \frac{\mathbf{x} - \mathbf{x}'}{c}, \mathbf{x}')}{\partial n} \right] + \frac{1}{4\pi} \int_{S} \frac{dS}{|\mathbf{x} - \mathbf{x}'|} \left[ \frac{n \cdot (\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3} + \frac{n \cdot (\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3} \frac{\partial \phi(t - \frac{\mathbf{x} - \mathbf{x}'}{c}, \mathbf{x}')}{\partial n} \right] n
\]

(14)

where "ext." in the second term denotes source term.

To apply BE scheme to Eq.(14), we can numerically simulate the transient electromagnetic fields phenomena. In the transient phenomena, the unknown valuables \(\partial \phi / \partial n\) and \(\partial A / \partial n\) are independent of those at different time, therefore these values have to be evaluated on both of discretized space and time. The resultant matrix equation for the discretized model takes the form of Fig.2(a), for any time instance. The notation "i" indicates use of the future variable and this part contributes its implicit property. The implicit scheme is essential for obtaining of stable solutions in BEM because numerical integrals of the boundary values may contain much numerical noise. And then, the matrix equation for the whole time is constructed by iterative use of Fig.2(a) and takes form of something like Fig.2(b).

3 Numerical Calculation Examples

Charged particles, which is running in a particle accelerator tube with nearly equal to light velocity, produce transient electromagnetic fields as a result of interaction with structures inside the tube (see Fig.3). This is also one of typical transient electromagnetic field phenomena. In this paper, we apply the above BEM formulation to this phenomena. Then, to simplify the problem, only single disk are numerically treated here. (In such short time range, we can regard that main contribution of the induced fields to the particle dynamics is done by the nearest part of accelerator tube.)

In Fig.4, surface current (a) and charge (b) densities, which are induced
by the charged particles (electrons) are shown in time domain. Since the problem has axis symmetry in this case, one dimensional boundary of the cross section of the disk are treated in BEM calculation. Then, the dot \( \bullet \) in Fig.4 denotes cutting point (compare the cross section and figures (a) and (b) ). One can find reasonable behaviors of the induced current and charge densities.

4 Summary

This paper has presented BE analysis of the transient electromagnetic fields scattering phenomena. Especially, an implicit scheme formulation has been introduced to achieve the stable calculation. The numerical simulation results based on this formulation give us reasonable results. Still remained problems are too long calculation time and fugue storage memory requirements. Authors believe that some more improvements for such problems of memory and calculation cost are possible taking sparseness of the BE matrices into account.

References

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\[ t \quad + \Delta t \]

\[ t - \Delta t \quad t - 2 \Delta t \quad t - 3 \Delta t \]

Fig. 2 (a) Matrix equation profile for anytime instance

Fig. 2 (b) Matrix equation structure for whole time
Fig. 3  Particle accelerator model and its single disk model
Fig. 4  Induced surface current (a) and charge density (b) (calculated)