Structural shape optimization using genetic algorithms

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Abstract

During the last years more and more research efforts have done for shape optimization due to its relevance in structural design. This method changes the boundary shape of the structure to find the optimal design by verifying the prescribed constraints.

The conventional approach is usually to replace the original implicit design problem by solving a sequence of explicit subproblems, each one constructed by using local approximation concepts involving function values an derivatives at the current design point. This is done because the objective function and constraints depend implicitly upon design variables.

The aim of this paper is to present a different approach to solve the optimization problem by using evolutionary techniques. Genetic Algorithms (GAs) probably is one of the most used evolutionary methods due to its robustness, easy implementation as well as flexibility. All these characteristic allows the application of this method for a wide variety of engineering non-conventional problems.

Finally, some numerical examples are solved to demonstrate the applicability of the proposed approach.

1 Introduction

In the past, many design processes were made by designer's intuition and experience instead of an intensive application of optimization theory. This fact has recently changed since optimal designs became relevant in order to reduce costs in manufacturing.

All this has been possible thanks to the development of new numerical methods such as the Finite Element Method (FEM) and the Boundary element Method (BEM) and new optimization techniques, along with more powerful computers.

The goal in optimum design is to obtain a solution for a given engineering problem which must satisfy all the limitations and constraints and under a previous selected criteria.

The optimization techniques have been developed in several ways, such as the determination of the cross-sectional areas of a structure, or by considering variation in the structure geometry, i.e., both joints position and members geometry are considered as design variables. Other problems are the optimization of the restraint conditions and the shape optimization, a type of problem which has recently received much effort.

The shape optimization problem can be solved by using several methods. One of them involves the fundamental equation written in discrete form. Then, this equation is derived in order to get the function gradients. Other approaches need the differentiation of the fundamental equation and, by using the definition of an adjunct problem, the function gradients are calculated as the variations of the domain occur, as shown in Braibant&Fleury [1]. It is usually necessary to use a numerical method, which allows the user to get the values of the fundamental variables and their derivatives with respect to the design variables.

Several works have been developed using the FEM to solve shape optimization problems. Zienkiewicz&Campbell [2] used such design variables (joint positions of the finite element mesh). They derived the stiffness matrix and loading vector with respect to the design variables and then they used a sequential linear programming method as the optimization technique.

Ramakrishnan&Francavilla [3] used a formulation based on finite elements, but they used the penalty method as the optimization technique. Francavilla et al [4] employed the same formulation in order to minimize the stress concentration around the concave zone of the union between two different thickness.

The main drawback of those approach to the shape optimization problems lies on the loss of accuracy when computing the function derivatives, which is an essential factor for using conventional optimization techniques. The search for a robust method, which presents a good balance between efficiency and efficacy, is of the most concern to get good results in many different environments. In this work, Genetic Algorithms (GAs) have been selected due to the fact that they are theoretically and empirically proven to provide robust search in complex space [5-7].

Evolution programs started in the sixties, when a group of biologists [8-10] used digital computers to simulate genetic evolutionary systems. But it was J. Holland [5], who in 1975 published the work entitled "Adaptation in Natural and Artificial Systems", laying down the two main principles for current GAs: the ability of simple representation (bit strings) to encode complicate structures and the power of simple transformations to improve such structures. In his schemata theory, this author explained how by using an appropriate control, a rapid improvement in the bit strings is obtained, as it is observed in animal populations. Their application to engineering problems is recent and it is due to Goldberg works[11-13].

2 Basic insight into Genetic Algorithms

Genetic Algorithms simulate the mechanics of natural genetics for artificial systems based on operations which are the counterparts of the natural ones.

When a population of biological species evolves over generations, characteristics that are useful for survival tend to be passed on to future generations, because individuals carrying them get better chances to survive. Individual characteristic in biological populations are stored in chromosomal strings, likewise in artificial systems, the basic characteristics which describe the performance of a structure in a particular environment can be coded in a string of bits. It is the so-called artificial chromosomes or the genotype and the characteristics itself are the phenotype of the artificial individual.

Genetic Algorithms operate on populations of strings (structures) and progressively (t= 0,1,2...) modifies their genotypes to get the better performance of their phenotype environment E.

The adaptation process is based on the mechanics of natural selection and natural genetics. They combine the survival of the fittest together with string structures, with a structured yet randomized information, which is exchanged to form a search algorithm. In each generation, a new set of artificial creatures (strings) is generated by using bits and pieces of the fittest of the previous generation. They efficiently exploit historical information to speculate on new search points with expected improved performance.

3 Shape optimization methodology using Genetic Algorithms

The outline for a basic genetic algorithm is as follows:
(1). Some parameters of interest in a system are represented by some sets of strings coded by binary or other codes which correspond to the chromosomes of the living things. These strings form the initial population.
(2). The characteristics of the system are then evaluated and the fitness (objective function) of the system is calculated by using these strings.
(3). Choose some strings probabilitically according to their fitness.
(4). Reproduce new strings by cross over in a given probabilistic rule which results in copying and exchanging a part of codes among the strings.
(5). The operation of mutation to give sudden changes of a part of codes in the strings is proceeded. This operation is to maintain a variety of species.
(6). Repeat from 1 to 5 and improve the system.

3.1 Chromosome representation

In the examples presented herein, 2D finite-element models of mechanical pieces, the nodal restraints, loading conditions and the topological configuration are known. Hence the design variables are

a) the increment value (Δ_h) of each joint coordinate (x_h) in finite elements along the moving boundary.

Thus, the structure characteristics or phenotypes can be represented as a chain of decimal parameters, as shown below

$$\text{Structure} = \left\langle \Delta_1 \Delta_2 \Lambda \ \Delta_h \Lambda \ \Delta_{n_m} \right\rangle \tag{1}$$

where

n_m: the number of mobile joint coordinates.

The genotype structure is then obtained by encoding each parameter in binary format, in the following way

$$\text{Individual} = \langle 1\,0\,0\,0\,1\,0\,1\,1\,0\,0\,1\,1\,\Lambda \ 0\,1\,1 \rangle \tag{2}$$

3.2 The objective functions and constraints

Once the population has been created, it is necessary to define the merit functions, which permit the measurement of the success or failure of the chromosomes of the problem to be solved.

In the case of 2D finite element shape optimization, the problem consists in founding the minimum area or weigh of the model without violating the stresses and displacements limit values. The objective function is then stated by

$$\text{Min}\,(W) = \sum_{i}^{Nel} V_i \rho_n \tag{3}$$

where

V_i , ρ_i : Volume and specific weight of element i.
Nel : Number of finite elements of the model.

subjected to

a) Stress restrictions: the Von-Mises stresses, calculated at Gauss points of the finite element, must not exceed the limit value σ_{adm}

$$\sigma_{von_i} - \sigma_{adm} \leq 0 \tag{4}$$

The Von-Mises stress is calculated as:

$$\sigma_{von} = \left[\left(\sigma_x - \sigma_y \right)^2 + 3\tau_{xy}^2 \right]^{\frac{1}{2}} \tag{5}$$

b) Nodal coordinates restrictions: nodal coordinates of some nodes of the mesh should not move beyond certain limit values in X and Y

directions in order to maintain the mesh topology.

c) Restrictions on the shape of the elements of the moving boundary, in order to avoid singular or negative jacobians.

In order to transform the constrained problem in an unconstrained problem and due to the fact that GAs do not depend on continuity and existence of the derivative, penalty methods have been used in this paper [6,13,14,15].Thus, the evaluation function is the minimization of the model weight, penalized with the constraints

$$g_i = W + \lambda |\Delta\sigma|^2 + \mu |\Delta u^1|^2 + \eta |\Delta x^1|^2 \tag{6}$$

being

g: penalized weight
i: structured index.
$|\Delta\sigma|$: increment over allowed stress.
$|\Delta u^l|$: increment over allowed displacement.
$|\Delta x^l|$: increment over allowed coordinate
(l= 1,2,3)

The parameters λ,μ and η are adjusted by trial and, in this paper, they have been evaluated in such a way that a 10% of increase in stress or displacement increases the structure weight by about 5% [14].

3.3 Fitness function

The duality of cost minimization and profit maximization is well-known. In normal operations, the transformation of a minimization problem into a maximization problem is simply carried out by multiplying the cost function by minus one.

In this paper, the minimization of the evaluation function has been transformed into a maximization search by using

$$\text{fitness(i)} = \frac{\text{Constant}}{g(\text{i})} \tag{7}$$

This new evaluation function is called *fitness function.* The relative rate between chromosomes is maintained as defined by the original evaluation function (eq. 6).

4 Illustrative examples

The following example illustrate the ability of the algorithm to optimize finite element model shapes, in order to get the minimum area and an uniform stress distribution.

4.1 Plate subjected to traction

As a demonstration of effectiveness of shape optimization using GAs, the optimization of the surface area of a plate is considered.

The figure 1 contains the geometry, dimensions and the moving boundary AB of the plate. The loading and boundary conditions are depicted in figure 2

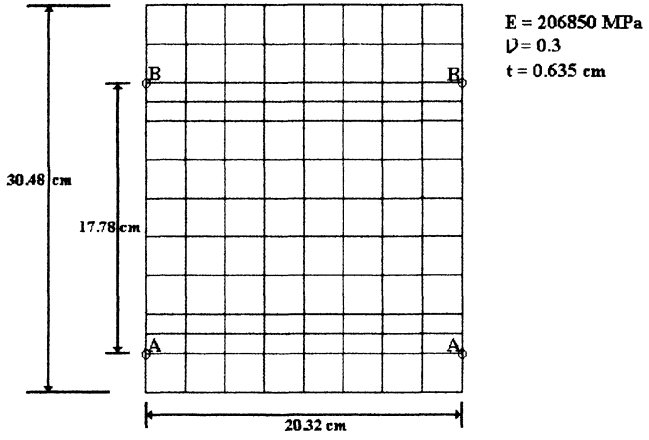


Figure 1. Plate with moving boundary (A-B), geometry and dimensions.

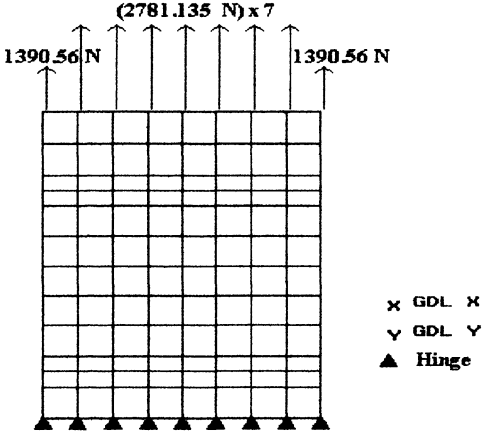


Figure 2. Loads and boundary conditions.

The objective function of the problem is the optimization of the surface area of the plate. The constraints are the Von-Mises stresses, which must not exceed 34.5 MPa. The initial area of the shape was 619.35 cm^2, as shown in figure 1.

The optimized shape is shown in figure 3. It can be noted how the plate area was reduced significantly. The final area obtained was 456.65 cm^2, which means a 26.27% reduction .

The comparison between the initial and final stresses is shown in figures 4 and 5. Note that the final shape is the result of the interaction between allowed stress and the elements stresses in the narrow zone of the final shape.

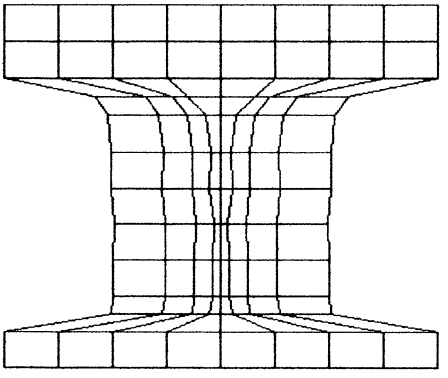


Figure 3. Final shape of the optimized piece.

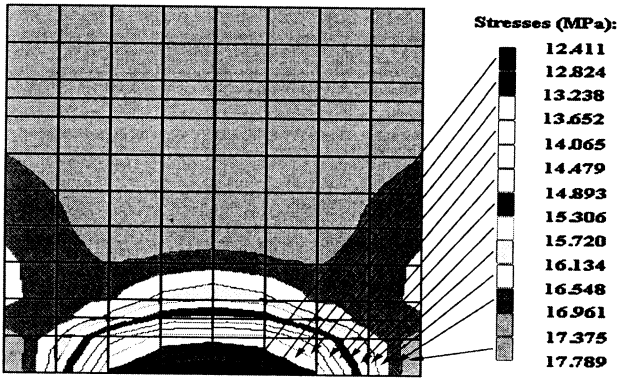


Figure 4. Initial stress distribution (plate subjected to traction).

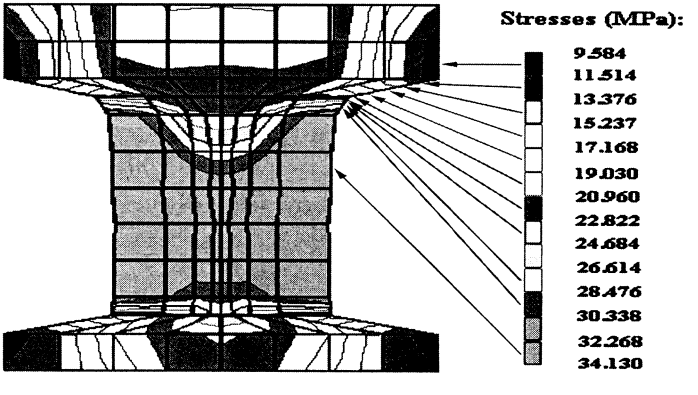


Figure 5 Final stress distribution (plate subjected to traction).

Finally, the figure 6 illustrates the evolution curves of the structure stresses and weight. It can be noted the slope decreasing in the adimensional weight factor as the generation process progresses. This factor is always below than one, which clearly shows that the algorithm is stable and will converge to the optimal solution in terms of minimum weight. Another subject dealing with the relation between both curves should be remarked herein. When the adimensional stress curve displays its minimum value (see figure 6, generation number 17), it is a clear indication that the area of the piece can be further reduced. However, beyond generation number 40 or so, the adimensional stress curve displays a nearly constant value close to one, which involves that the area is very close to its optimal value. This explain why the adimensional weight curve behaves in very stable manner after generation 40.

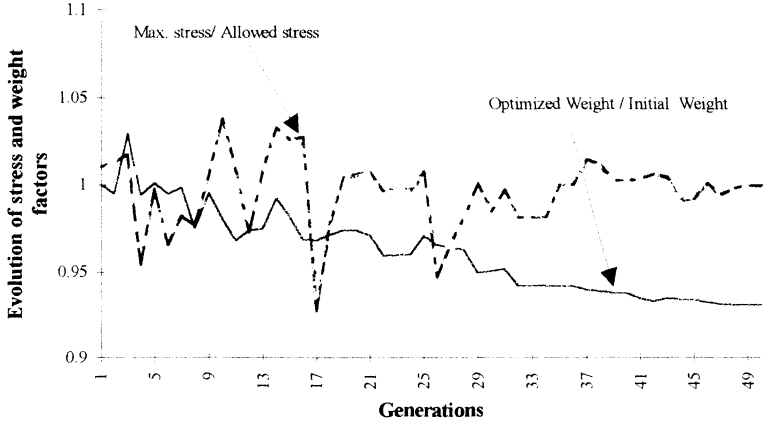


Figure 6. Genetic evolution of stresses and weight (plate subjected to traction).

5 Conclusions

The deterministic classic resolution in optimization methods starts from a pre-defined point and, from it the search is performed until the nearest minimum value is achieved. However, there is the possibility that such a minimum value is not the required point. When using genetic algorithms, the search starts from a set of points which form the initial population and from them the exploration of the design space of plausible solutions is started. It is carried out by the random application of crossover and mutation operators.

One of the greatest advantages of the method is that it is a discrete method. The power of the GA, based in the Darwin mechanics and having certain random behavior, is due to the exponential increment of the short and low degree "schemata" associated to good merit functions. Its discrete formulation and its ability in searching, suggest that this method is a powerful tool in engineering applications.

New algorithms to simulate the process artificially have been developed and applied to various types of engineering optimization problems. Studying the algorithms from the optimization point of view, they surpass the other existing optimization techniques in some points, and they require no calculation of sensitivities and comparatively to attain the global optimum point.

The 2D finite element models optimized herein have shown that the proposed approach is suitable to carry out this task. The results indicate a good performance even in the presence of stress concentrations caused by corners and geometry changes.

A promising field in structural optimization is currently under research. The application of GA methods to finite element models suggest the exciting possibility of optimizing complex 3D problems, such as the shape optimization of gravity dams.

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