Estimation of cover resistance and protection electric current of a pipeline by means of the Boundary Element Method

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Abstract

We apply the Boundary Element Method to the Laplace equation for the electric potential in the region surrounding a pipeline. The resistivity effects are taken into account by means of the Kirchoff equation. The objective is the obtainment point by point of the pipeine cover resistance and the estimation of the electric current needed to meet the required protection criterion.

1 Introduction

The subject of cathodic protection on buried pipelines [1][2] has only recently begun to be studied by means of advanced numerical techniques [3].

In other corrosion problems, such as those of cathodic protection of submarine structures (i.e., vessels, platforms) [4][5] or tanks [6], the numerical technique largely accepted is the Boundary Element Method (BEM) [7]. The main difference between this type of problems and that one of buried pipelines rests on the order of magnitude of typical involved lengths. Meanwhile in submarine structures and in tanks, lengths are in meters or decameters, in buried pipelines lengths are in the kilometer order. This implies that for pipelines it is not possible to depreciate resistive effects, that are not taken into account in submarine structures.

In this work, the BEM is applied to the Laplace equation for the electric potential in the region surrounding the pipeline. In the pipeline, the resistivity effects are modelled by means of the Kirchoff equation applied to each element on which it is subdivided. The two equation systems are connected with equations relating the pipe potential with the potential of the soil adjacent to it. These equations involve the cover resistance and polarization [2], though the last effect is not taken into account in this first approach to the problem.

In opposition of other works [3][4][5][6], where the objective rests on the obtention of potential distribution and electric current density in the structure, in the present case, the objective is the obtainment point by point of the pipeline cover resistance and the later estimation of electric current need for the required protection criterion [1]. The cover resistance and the electric current need are the
quantities of concern for the engineer in charge of pipeline supervision, who usually obtains these values from measured potentials, by means of semi-empirical formulas.

The purpose of the present work is to improve this estimation, bearing in mind a more realistic model for the electric current in the pipeline and the electrolytic environment surrounding it. In addition, in semi-empirical estimation only mean values of cover resistance are possible to be obtained, meanwhile with the method herein presented, point by point resistance is obtained. It is important to underline that the cover resistance strongly depends upon soil conditions, which varies in time. Therefore, recent potential measurements are necessary.

With the values of cover resistance thus obtained, it is possible to study pipeline behavior in the presence of external perturbations (like sending electric current), through the obtainment of potential distribution, or else, to estimate the electric current need in order to ensure the protection, in accordance with the established criterion.

2 Mathematical model

In the soil, which is an electrolytical environment, it is satisfied Ohm’s law:

\[ j = -\sigma \nabla u \]  

(1)

where \( j \) is the electric current density vector, \( u \) is the electric potential and \( \sigma \) is the soil conductivity. The continuity equation is:

\[ \nabla \cdot j = 0 \]  

(2)

Therefore, taking the divergence in (1) and assuming that \( \sigma \) is constant, we obtain:

\[ \nabla^2 u = 0 \]  

(3)

which is the Laplace’s equation for the potential. We shall apply the BEM to the equation (3). The starting point in a potential-type problem as the present one, is Green’s equation. Since the potential \( u \) satisfies Laplace’s equation (3), then Green’s equation for a point \( r \) on the pipeline surface is:

\[ \frac{u(r)}{2} = \frac{1}{4\pi} \int u^*(r', r') \frac{\partial u}{\partial n}(r') d\Gamma' - \frac{1}{4\pi} \int u(r') \frac{\partial u^*}{\partial n}(r, r') d\Gamma' \]  

(4)

where integrals extend on the whole pipeline surface and a surface in infinite, \( n \) is the normal external to each one of these surfaces and

\[ u^* = \frac{1}{|r - r'|} \]  

(5)

is Green’s basic function. If we assume that the potential decreases with distance as:

\[ u(r) = - + O(\frac{1}{r}) \]  

(6)
then, the integral corresponding to the surface in infinite goes to zero in the limit $r \to \infty$ therefore integrals remain only on the pipe surface. The normal derivative of the potential can be related with the density of normal electric current to the pipe surface from (1) as:

$$\frac{\partial u}{\partial n} = -\rho j$$

where $\rho = \sigma^{-1}$ is the soil resistivity. Furthermore, we shall use the approximation that the potential on the pipe does not depend upon azimuthal angle, that is to say, it solely varies in the longitudinal direction. This is a good approximation when the disperser, that sends protection current to the pipe, is at a large distance. The distance in the longitudinal direction of any point of the pipe to the connection point of the electric current shall be called $z$. The next step is the discretization of Green’s equation, using cylindrical elements [7] on the pipe surface and approximating $u$ and $j$ values as constants $u_i$, $j_i$ on each element $i$:

$$\frac{u_i}{2} = \sum_{k=1}^{n} (-\rho j_k) \frac{1}{4\pi} \int_{\Gamma_k} u_i^* d\Gamma - \sum_{k=1}^{n} j_k \frac{1}{4\pi} \int_{\Gamma_k} \frac{\partial u_i^*}{\partial n} d\Gamma$$

for $i = 1, ..., n$, where $n$ is the number of elements. The system (8) can be written in a compact form as follows:

$$\sum_{k=1}^{n} H_{ik} u_k + \sum_{k=1}^{n} G_{ik} \rho j_k = 0$$

$$G_{ik} = \frac{1}{4\pi} \int_{\Gamma_k} u_i^* d\Gamma = \frac{1}{4\pi} \int_{\Gamma_k} \frac{1}{|\mathbf{r}_i - \mathbf{r}|} d\Gamma$$

$$H_{ik} = \frac{1}{4\pi} \int_{\Gamma_k} \frac{\partial u_i}{\partial n} d\Gamma + \frac{1}{2} \delta_{ik}$$

where $\delta_{ik}$ is Kronecker’s delta. By using cylindrical geometry, the element integrals (10) and (11) can be reduced to unidimensional integrals in the azimuthal angle, that are solved numerically.

The last equations are for the potential in the electrolytic environment and potentials $u_i$ refer to values immediately adjacent to the pipe. Now, we shall describe an equation system for potentials $v_i$ on the pipe. We start from Kirchhoff’s law for a generic element $i$ which states that the addition of all the incoming electric currents into a node $i$ ($i = 1, ..., n$) is zero:

$$I_{i-1} - I_{i+1} + j_i 2\pi R \Delta z = 0$$

where $I_{i-1}, I_{i+1}$ are the currents passing through elements $i - 1$ and $i + 1$ respectively, $j_i$ is the current from soil to element $i$, $R$ is the pipe radius and $\Delta z$ the length of the element.

Electric current on elements can be expressed in terms of the potential difference between nodes and longitudinal resistance of the pipe, by means of Ohm’s law; therefore, equation (12) can be written as follows:

$$\frac{v_{i-1} - v_i}{v_i - v_{i+1}} + j_i 2\pi R \Delta z = 0$$
where \( R_L \) is the resistance per unit length of the pipe. If there is a source connected to some node \( i_o \), corresponding to the connection point of the electric current \( I_e \), Kirchoff equation (12) is now:

\[
\frac{v_{i-1} - v_i}{R_L \Delta z} - \frac{v_i - v_{i+1}}{R_L \Delta z} + j_i 2\pi R \Delta z = I_e \delta_{i,i_o}
\]

The equation relating both systems (9) and (14) is the following:

\[
u_i = v_i + R_{C_i} j_i + f(j_i)
\]

where \( R_{C_i} \) is the cover resistance \[1\], measured in units of resistance per unit surface, and \( f(j_i) \) is the polarization term. Polarization is not linear in electric current density and it must be experimentally determined or at least, an empirical formula like Butler-Volmer's should be used \[2\]. In this work, as a first approximation, we shall ignore this term, which is in fact what is done in practice when cover resistance and current necessity are estimated by means of semi-empirical formulas, as indicated in the next section. Cover resistance depends, in general, upon the position; therefore in (15) it is expressed for the corresponding \( i \) node.

### 3 Practical determination of cover resistance

In practice, an electric current is sent and potential differences \( u_i - v_i \) are measured along the pipe ("on" potentials). Then, electric current is disconnected, a period of 20 seconds is awaited and potential differences are measured again ("off" potentials). The substruction of these differences is \( \Delta v_i = R_{C_i} j_i \) since it can be assumed that in both cases the term \( f(j_i) \) (equation (15)) is the same, since 20 seconds is not sufficient time for polarization to dissapear (this can take several days).

In corrosion engineering of pipelines it is commonly used the "telegraphic" formula \[1\], an approximation to potential behaviour in function of distance:

\[
\Delta v_i = \Delta v_{i_0} e^{-\alpha (z_i - z_o)}
\]

where \( \Delta v_{i_0} \) = on-off potential difference at reference point, \( z_o \) = reference point, \( z_i - z_o \) = distance of point \( i \) to reference point, \( \alpha \) = attenuation coefficient. The coefficient \( \alpha \) is calculated for each node \( i \) and then it is averaged. Mean cover resistance \( R_C \) is obtained from:

\[
\alpha_{mean} = \left( \frac{R_L 2\pi R}{R_C} \right)^{1/2}
\]

where \( R \) is the pipe radius and \( R_L \) is the resistance per unit length.

In this work, we compare cover resistance estimation with these semi-empirical formulas with the calculations from more fundamental equations such as Kirchoff's law and continuity equation as stated before. Moreover, this calculations allows us to obtain cover resistance in function of the position and not only as an average of the whole pipe.
4 Resolution of the equation system

We have seen that the application of the BEM to the continuity equation for the potential in the soil led us to the equation system (9), as well as the application of Kirchoff’s law on each node led us to the system (14). These two systems are linked by equation (15). Furthermore, if on-off potential differences are expressed as $\Delta v_i = R_{C_i} j_i$ then systems (9) and (14) can be written in the following way:

$$v_{i-1} - 2v_i + v_{i+1} + 2\pi R(\Delta z)^2 R_L j_i = I_e R_L \Delta z \delta_{i,i_o}$$  \hspace{1cm} (18)

$$\sum_{k=1}^{n} H_{ik} v_k + \rho \sum_{k=1}^{n} G_{ik} j_k = - \sum_{k=1}^{n} H_{ik} \Delta v_k$$  \hspace{1cm} (19)

for $i = 1, ..., n$. In this system, the quantities $\Delta v_k$ are data, obtained from measurements, as well as the transmitted electric current $I_e$. Longitudinal resistance $R_L$, soil resistivity $\rho$ and pipe radius $R$ are also known. Length of elements $\Delta z$ is conveniently chosen and coefficients $H_{ik}$ and $G_{ik}$ which are purely geometric are calculated as stated in (10) and (11). Also, two boundary conditions must be added, $v_o = v_1$ and $v_{n+1} = v_n$ that mean that electric current is null at pipe ends.

In this way, equations (18) and (19) constitutes a non-homogeneous linear system of $2n$ equations with $2n$ unknowns $v_1, ..., v_n, j_1, ..., j_n$. With the $j_i$, cover resistance $R_{C_i}$, which are the unknown quantities of interest, are obtained from $R_{C_i} = \Delta v_i / j_i$. Once $R_{C_i}$ are obtained, it is possible to estimate the electric current $I_e$ which should be necessary to obtain a potential difference $\Delta v_{i_1}$, at some specific node, greater than a particular value. A common criterion is to require $\Delta v_{i_1} = 300 \text{ mV}$ in order to ensure the protection, since, in general, corrosion voltage is $550 \text{ mV}$, measured with respect to a copper-copper sulphate electrode, and the protection criterion most used states a voltage of $850 \text{ mV}$ with respect to the copper-copper sulphate electrode [1][2]. In this case, and rewriting electric current densities as $j_i = \Delta v_i / R_{C_i}$, equations are as follows:

$$v_{i-1} - 2v_i + v_{i+1} + 2\pi R(\Delta z)^2 \frac{R_L}{R_{C_i}} \Delta v_i - R_L \Delta z \delta_{i,i_o} I_e = 0$$  \hspace{1cm} (20)

$$\sum_{k=1}^{n} H_{ik} v_k + \rho \sum_{k=1}^{n} \frac{G_{ik}}{R_{C_k}} \Delta v_k + \sum_{k=1}^{n} H_{ik} \Delta v_k = 0$$  \hspace{1cm} (21)

$$\Delta v_{i_1} = 0.3$$  \hspace{1cm} (22)

with the voltage in the last equation expressed in volts. The $I_e$ is the current need for the protection and is the unknown quantity of interest in practice.

5 Results

The case of determination of cover resistance in a gas line crossing a river, bearing the characteristic that the pipe changes wall thickness in the submerged area, was
studied aiming to compare the results of the method presented hereinbefore with semi-empirical estimations used commonly in the practice.

Table 1 shows in detail the results obtained from the experimental measurements, gas pipeline characteristics and estimation of average cover resistance and current need by means of the telegraphic formula (semi-empirical). Table 2 shows the results obtained with the numerical method herein presented.

6 Conclusions

The mathematical model of cathodic protection of a pipeline was presented. The formulation was based upon Laplace's equation for potential in the electrolytic environment and Kirchoff's equation for pipe electric currents. This last equation allows us to introduce resistivity effects of the pipeline.

The numerical boundary element method was applied to this problem. The equation system thus obtained allows us to achieve cover resistance in each point of the pipeline and current need for protection.

The numerical results of cover resistance and current need of a subfluvial pipeline were compared with the results obtained from semi-empirical estimations, achieving a good agreement between them.

In this case, the method presented hereinbefore is based upon a more realistic model of the electric current in the soil and the pipeline, and furthermore, it allows to obtain point by point values of cover resistance and not only average values. On the other hand, this method can be applied to more complex geometrical problems, such as the presence of more pipes, intersections, etc, usually conforming a distribution network.
### TABLE 1
DATA AND SEMI-EMPIRICAL ESTIMATIONS

**Gas pipeline: Subfluvial**

<table>
<thead>
<tr>
<th>Diameter (inches)</th>
<th>24.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>API 5L X 60</td>
</tr>
<tr>
<td>Wall thickness (mm) coastal area</td>
<td>3.74</td>
</tr>
<tr>
<td>Wall thickness (mm) submerged area</td>
<td>1.27</td>
</tr>
<tr>
<td>Length under study (km)</td>
<td>14.19</td>
</tr>
<tr>
<td>Total length (km)</td>
<td>31.48</td>
</tr>
<tr>
<td>Transmitted current (A)</td>
<td>8.1</td>
</tr>
<tr>
<td>Coating (liquid epoxy)</td>
<td>600 microns</td>
</tr>
<tr>
<td>Soil resistivity (Ohm x cm)</td>
<td>3000.0</td>
</tr>
</tbody>
</table>

**Data**

<table>
<thead>
<tr>
<th>km</th>
<th>Progressive</th>
<th>$V_{se}$ (volts)</th>
<th>$V_{sw}$ (volts)</th>
<th>$\Delta V$</th>
<th>Atten. coef. (km$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>conn. point</td>
<td>30.821</td>
<td>2.487</td>
<td>1.070</td>
<td>1.417</td>
<td>-</td>
</tr>
<tr>
<td>1.007</td>
<td>29.814</td>
<td>2.090</td>
<td>1.080</td>
<td>1.010</td>
<td>-</td>
</tr>
<tr>
<td>1.951</td>
<td>28.870</td>
<td>2.000</td>
<td>1.065</td>
<td>0.905</td>
<td>0.1163</td>
</tr>
<tr>
<td>3.967</td>
<td>26.854</td>
<td>1.846</td>
<td>1.097</td>
<td>0.749</td>
<td>0.1010</td>
</tr>
<tr>
<td>5.986</td>
<td>24.835</td>
<td>1.716</td>
<td>1.090</td>
<td>0.626</td>
<td>0.0961</td>
</tr>
<tr>
<td>9.152</td>
<td>21.569</td>
<td>1.516</td>
<td>1.106</td>
<td>0.410</td>
<td>0.1107</td>
</tr>
<tr>
<td>12.420</td>
<td>18.401</td>
<td>1.350</td>
<td>1.077</td>
<td>0.273</td>
<td>0.1146</td>
</tr>
<tr>
<td>14.190</td>
<td>16.631</td>
<td>1.298</td>
<td>1.067</td>
<td>0.231</td>
<td>0.1119</td>
</tr>
</tbody>
</table>

Average atenuation coeff. (1/km) coastal area — 0.1086 submerged area — 0.1083

Estimations by means of the telegraphic formula

- Longitudinal resistance (coastal) 0.0110 Ohm/km
- Longitudinal resistance (submerged) 0.0076 Ohm/km
- Cover resistance (coastal) 1787 Ohm m$^2$
- Cover resistance (submerged) 1237 Ohm m$^2$
- Current need (displac. 300 mV) 11.9 A

### TABLE 2
CALCULATION OF COVER RESISTANCE BY MEANS OF THE BEM

Pipe characteristics and measurements are included in Table 1

<table>
<thead>
<tr>
<th>Pos. (km) resp. to conn. point</th>
<th>Cover resistance (Ohm m$^2$)</th>
<th>Aver. cover resist.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.430</td>
<td>1266.83</td>
<td>1850.57</td>
</tr>
<tr>
<td>2.002</td>
<td>1213.34</td>
<td></td>
</tr>
<tr>
<td>2.574</td>
<td>1194.88</td>
<td></td>
</tr>
<tr>
<td>3.146</td>
<td>1568.58</td>
<td></td>
</tr>
<tr>
<td>3.718</td>
<td>1728.18</td>
<td></td>
</tr>
<tr>
<td>4.290</td>
<td>1945.54</td>
<td></td>
</tr>
<tr>
<td>4.862</td>
<td>2456.23</td>
<td></td>
</tr>
<tr>
<td>5.434</td>
<td>3010.65</td>
<td></td>
</tr>
<tr>
<td>6.006 (change thickness)</td>
<td>3574.85</td>
<td>1347.45</td>
</tr>
<tr>
<td>6.575</td>
<td>2929.35</td>
<td></td>
</tr>
<tr>
<td>7.150</td>
<td>2338.45</td>
<td></td>
</tr>
<tr>
<td>7.722</td>
<td>1805.56</td>
<td></td>
</tr>
<tr>
<td>8.294</td>
<td>1350.62</td>
<td></td>
</tr>
<tr>
<td>8.866</td>
<td>973.82</td>
<td></td>
</tr>
<tr>
<td>9.438</td>
<td>771.44</td>
<td></td>
</tr>
<tr>
<td>10.010</td>
<td>946.85</td>
<td></td>
</tr>
<tr>
<td>10.582</td>
<td>958.48</td>
<td></td>
</tr>
<tr>
<td>11.154</td>
<td>904.93</td>
<td></td>
</tr>
<tr>
<td>11.726</td>
<td>795.08</td>
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<tr>
<td>12.298</td>
<td>646.43</td>
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</tr>
<tr>
<td>12.870</td>
<td>676.36</td>
<td></td>
</tr>
<tr>
<td>13.442</td>
<td>723.54</td>
<td></td>
</tr>
<tr>
<td>14.014</td>
<td>824.14</td>
<td></td>
</tr>
</tbody>
</table>

Current need for $\Delta V$ (14 km) to be $300 \text{ mV} = 10.33 \text{ A}$
BIBLIOGRAPHY


