Transient study of induction motor starting using FEM-BEM hybrid method coupled with electric circuits and mechanical motion equations

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Abstract

In this paper, a two dimensional transient study of induction motor starting is presented. The analysis is based on the coupling between mechanical motion, electric circuits and magnetic field equations of the machine. The electromagnetic field discretization is obtained by a FEM-BEM hybrid method in order to account for movement and saturation effects. The mechanical motion is solved using a time-stepping procedure in which the electromagnetic torque computation is based on the numerical solution of the magnetic field of the machine. In order to study the transient performance of the machine, we have chosen some typical variations of the load torque. For these cases, we show the transients of the three phases armature currents, the rotation speed and torque. Finally, we discuss some numerical aspects: the advantage of the hybrid method, the use of the periodicity with the boundary element method and the reduction of computation time.

Introduction

The knowledge of transient phenomena that occur during an induction motor starting is of great interest for electrical machines designers. Such a knowledge allows to determine the starting performance of the machine as well as the requirements imposed on the power supply. However, the numerical modelling in this case should be able to include not only the field and circuits equations of the machine but also the mechanical motion equation. Indeed, in such an operating conditions, the electromagnetic behaviour of the machine is intimately lying on the mechanical motion due to the resultant torque exerted on the rotor.
and which depends on the magnetic saturation and the load conditions. Such an electro-magneto-mechanical coupled problem was the subject of many recent works, e.g. Ren & Razek,¹ Demenko,² Demenko & al.,³ Henrotte & al.⁴ and can be treated in two different ways.

The first approach, which is generally adopted, is the indirect coupling also called the "weak coupling". In this approach and for a given time-step, the magnetic field and electric circuits equations are firstly solved and the electromagnetic torque is computed. Using this value, the mechanical motion equation is then solved and the angular rotor position for the next time-step is determined. In this procedure, the differential equation of motion can be solved using an explicit or implicit integration scheme. In the implicit case, the determination of the rotor displacement requires the knowledge of the electromagnetic torque at the next time-step. Therefore, an iterative process is needed in the determination of the rotor position. The implicit method is unconditionally stable but it requires more calculation time since additional field solutions are required during the iterative procedure. In the explicit method, the next value of the rotor position is given directly in terms of the previous calculated torque and position. The method is however not unconditionally stable. In order to ensure the stability of the integration scheme, a suitable time-discretization is required. In the case of electrical machines and for smooth load torque variations, the explicit method can be used without problems by choosing a time-step short enough. If a significant change of the load torque is expected, one can use the implicit method in conjunction with the explicit one to take advantage of both methods.

The second approach is the direct coupling or "strong coupling". In this model, the mechanical equation is solved simultaneously with the electric circuits and the magnetic field equations. At each time-step, the solution of the global system of equations gives the electromagnetic quantities as well as the mechanical variable which can be the displacement or the velocity. An application of the method to an electromagnetic relay is provided by Ren & Razek.¹

In this paper, we use the indirect approach with an implicit integration scheme. The electromagnetic torque is however assumed to be constant over a time-step. In order to ensure the validity of this hypothesis and to reduce the computation time, a variable time-step was chosen in the simulation. The results presented in this paper were obtained by a step-by-step solver program developed in the laboratory and based on the FEM-BEM coupling method. In the studied machine, electric and magnetic parts are meshed with finite elements whereas boundary elements are used in the air-gap region. This hybrid meshing is well suited for transient simulation since any angular movement can be easily programmed. In order to reduce the calculation time and the memory storage, we have taken account of the periodicity with the BEM method. In this way, the simulation time becomes reasonable allowing a more practical use of the hybrid method.
Mathematical model

The field is supposed bidimensional and the end-region leakages are modelled by constant resistances and inductances introduced in the electric circuit equations of the machine. For non-linear magnetodynamic problems, the Maxwell’s theory leads to the governing equation (1) expressed in terms of the z-components of vector potential $A$ and current density $J$:

$$\frac{\partial}{\partial x} \left( \nu \frac{\partial A}{\partial x} \right) + \frac{\partial}{\partial y} \left( \nu \frac{\partial A}{\partial y} \right) = - J$$

where $\nu$ is the magnetic reluctivity. In a conducting region, the current density can be expressed as:

$$J = -\sigma \frac{\partial A}{\partial t} - \sigma \text{grad} \phi$$

where $\phi$ is the electric scalar potential and $\sigma$ the electric conductivity. In a 2-D problem, $\phi$ is constant over the cross-section area $S_c$ of a given conductor and varies linearly with the z-coordinate. Introducing the potential difference $\Delta U_c = -\int_{S_c} \sigma \frac{\partial A}{\partial t} dS_c$ between the conductor ends, the total current $I_c$ of the conductor is obtained by integrating equation (2) over $S_c$:

$$I_c = G_c \Delta U_c = \int_{S_c} \sigma \frac{\partial A}{\partial t} dS_c$$

where $G_c = \sigma S_c/l_m$ is the dc-conductance of the conductor and $l_m$ the axial length of the machine. Equation (3) represents Ohm’s law for massive conductors with induced currents like rotor bars. For thin conductors, like stator coil sides, the current density is supposed uniform and can be expressed from the armature current $I_e$ as:

$$J = \frac{N_s}{S_c} I_e$$

with $N_s$ and $S_c$ respectively the number of turns and the section of the coil. Using equation (3), the potential difference along the coil side, defined as the sum of the voltages of all filaments, can be expressed as$^6$:

$$\Delta U_c = R_c I_e + \int_{S_c} \frac{l_m N_s}{S_c} \frac{\partial A}{\partial t} dS_c$$

where $R_c = l_m N_s^2/\lambda \sigma S_c$ is the dc-resistance for thin conductor and $\lambda$ the filling factor.

Circuits equations

The circuits equations are given by the loop current method in which the variables are the loop currents and the potential differences of the conductors which can be seen as generalized circuits elements characterized by the
relations (3) and (5). Figure 1 represents the circuit plan for rotor cage. Every portion of the ring between two bars is represented by constant resistance \( r_a \) and inductance \( l_a \). With reference to fig. 1, Kirchhoff's laws for loop \( i \) lead to the following equations:

\[
\Delta U_{i} - \Delta U_{i+1} = 2r_a J_{i+1} + 2l_a \frac{dJ_{i+1}}{dt}
\]

Writing these equations for each loop \( i \), we obtain the set of equations defining the rotor cage circuit.

![Fig. 1: Electric circuit of the rotor cage (h: periodicity factor)](image)

The arrangement of the stator windings (fig. 2) can be considered as an electric circuit in which a set of \( n_l \) loops and arbitrary loop current directions can be established.

![Fig. 2: Arrangement of stator winding (three phase, double layer)](image)

In the case of a three phase windings with a star connection, only two loops and two independent currents exist. The vector \( I_c \) of currents in the conductors can be expressed in terms of the vector \( i_m \) of the loop currents by \( I_c = D i_m \) where \( D \) is a rectangular matrix depending on the winding arrangement of which nonzero elements are +1 or -1. Applying Kirchhoff's laws for all circuit loops, the voltage equations of the stator are derived:

\[
- D^T \Delta U_c + r_{ext} i_m + l_{ext} \frac{di_m}{dt} = V_{ext}
\]  

where \( r_{ext} \) and \( l_{ext} \) are matrices of external resistances and inductances and \( V_{ext} \) is a column matrix of voltage sources in the loops.
FEM-BEM coupling method

In the air-gap region, the vector potential satisfies the Laplace’s equation. Applying the Green’s identity to this equation leads to the following BEM formulation expressed in terms of A and its normal derivative $\partial A/\partial n$:

$$c_i A_i + \int_{\Gamma} A \frac{\partial G}{\partial n} \, d\Gamma = \int_{\Gamma} G \frac{\partial A}{\partial n} \, d\Gamma$$

where $G$ is the Green function of the 2-D Laplace operator, $\Gamma$ is the boundary of the air-gap subdomain and $c_i$ is a constant depending on the local curvature of $\Gamma$ ($c_i = 1/2$ on a smooth boundary).

In conducting and saturable regions, an approximate solution of Eq. (1) for a subdomain $\Omega$ of boundary $\Gamma$ is obtained using the following FEM formulation based on the Galerkin’s method:

$$\int_{\Omega} \left\{ v \text{grad} A \text{grad} w - J w \right\} d\Omega - \int_{\Gamma} v \frac{\partial A}{\partial n} d\Gamma = 0$$

where $w$ is the weighting function, $J$ the current density and $v$ the magnetic reluctivity. The boundary term is used at the interface between the FEM and the BEM domains to couple the methods. The current density $J$ in (9) is replaced using the relation (4) or (2) depending on the case of thin conductor or massive conductor.

Use of periodicity

In order to reduce the computation time and the memory storage, the numerical method should take into account any periodicity appearing in the system. If this is relatively easy with the finite element method, it becomes complicated with the boundary element method. If we consider a machine with $n$ periodic sectors, the algebraic system for the whole machine obtained after discretization of equation (8) can be written as:

$$\begin{bmatrix} [H_{11}] & \cdots & [H_{1n}] \\ \vdots & \ddots & \vdots \\ [H_{n1}] & \cdots & [H_{nn}] \end{bmatrix} \begin{bmatrix} [A_1] \\ \vdots \\ [A_n] \end{bmatrix} = \begin{bmatrix} [G_{11}] & \cdots & [G_{1n}] \\ \vdots & \ddots & \vdots \\ [G_{n1}] & \cdots & [G_{nn}] \end{bmatrix} \begin{bmatrix} \frac{\partial A}{\partial n}_{11} \\ \vdots \\ \frac{\partial A}{\partial n}_{nn} \end{bmatrix}$$

where the column matrices $[A_i]$ and $[\partial A/\partial n_i]$ correspond to the nodal values of vector potential and its normal derivative for sector $i$. With periodicity conditions, we can reduce the above system to the form corresponding to sector $i$ under interest:

$$\begin{bmatrix} f_{i1}[H_{11}] + f_{i1}[H_{i1}] + \cdots + f_{in}[H_{in}] \end{bmatrix} [A_i] = \begin{bmatrix} f_{i1}[G_{11}] + f_{i1}[G_{i1}] + \cdots + f_{in}[G_{in}] \end{bmatrix} \begin{bmatrix} \frac{\partial A}{\partial n}_{11} \\ \vdots \\ \frac{\partial A}{\partial n}_{nn} \end{bmatrix}$$

where $f_{gi}$ are periodicity factors equal to +1 or -1. This is the form we use for the studied sector.
Global system of equations

The global system of equations is obtained by adding the following equations:

* the field equations derived from the discretization of equations (8) and (9).
* Ohm’s laws given by equations (3) and (5) with total currents expressed in terms of loop currents.
* circuits equations (6) and (7) for the stator and the rotor.

We perform the calculation using the Euler’s implicit method for time-discretization and the Newton-Raphson iterative procedure for non-linearity treatment.

Mechanical motion equations

The differential equations for movement are given by:

\[ J \frac{d \omega_r}{dt} = T_{em} - T_r, \quad \omega_r = \frac{d \theta}{dt} \]

where \( T_{em} \) is the electromagnetic torque produced by the motor, \( T_r \) the load torque, \( \omega_r \) the angular velocity of the rotor, \( \theta \) the angular displacement of the rotor and \( J \) the moment of inertia. The time-discretization of the above equations using the backward Euler’s method leads to the following relations:

\[ \omega_r(t + \Delta t) = \omega_r(t) + \frac{\Delta t}{J} T_{em}(t) \]

\[ \theta(t + \Delta t) = \theta(t) + \omega_r(t) \Delta t + \frac{\Delta t^2}{J} T_{em}(t) \]

where \( \theta(t + \Delta t) \) is the rotor angular position used in the next field calculation. In these relations, the electromagnetic torque is assumed to be constant over a time-step. Using the concept of the Maxwell stress-tensor, the electromagnetic torque \( T_{em} \) is given by:

\[ T_{em} = \frac{l_m}{\mu_0} \int \left[ (\vec{r} \times \vec{B})(\vec{B} \cdot \vec{n}) - \frac{1}{2} B^2 (\vec{r} \times \vec{n}) \right] d\Gamma \]

where \( \Gamma_R \) is the rotor contour. The calculation of \( T_{em} \) requires the knowledge of the tangential and normal components of the flux density along \( \Gamma_R \) or in other words the potential and its normal derivative. The advantage of the hybrid method is evident since these values are directly obtained from the solution of the problem.

Numerical results

Calculations have been performed on a four poles induction motor with 24 stator slots and 16 unskewed rotor slots. The stator is star-connected without neutral connection and realized with double-layer windings with a shortened pitch equal to 5. For symmetry reasons, the solution region has been restricted
to the quarter of the cross-section area. In this paper, three cases are presented in order to study the starting performance of the machine.

The first case corresponds to the starting of the motor in no-load conditions. At time $t=0$, a sinusoidal voltage is applied to the stator winding. Figure 3.a shows the flux distribution in the cross section of the machine at the beginning of the simulation. It can be seen how the rotor cage restricts the penetration of the flux. Figures 3.c, 3.d and 3.e show respectively the transients of the three phase currents, the torque acting on the rotor and the rotation speed. The motor reaches its synchronous speed in about 5 periods of the line frequency. Figure 3.b shows the flux distribution at the end of the simulation corresponding to the no-load conditions.

![Fig 3.a: Flux distribution at t=30ms](image1)

![Fig 3.b: Flux distribution at t=300ms](image2)

**Figure 3:** Starting of the motor without a load torque
In the second simulation, the machine is started with a load torque proportional to the square of the speed. Such a load conditions are obtained in practice when the motor is driving a pump. The steady state obtained corresponds to a speed equal to 1420 r.p.m and the torque produced by the machine is equal to 660mN. The figure 4.a displays the time-variation of the armature currents and the figure 4.b shows the evolution of the electromagnetic torque and the load torque. The fluctuations in the torque are due to the relative movement of the rotor with respect to the stator. These slots harmonics could be reduced if a skewed slots model was used, e.g Boualem & Piriou*. 

Figure 4 : Starting of the motor with a load torque proportional to the square of the speed

In the following simulation, an opposite torque, varying linearly from 0mN to 600mN, is applied on the shaft when the motor is running at synchronous speed. This case is obtained in practice when the motor is first started without load and then coupled to a load system using a mechanical gear. In this example, an opposite torque linear variation is assumed but other variations can also be modelled as well. The aim of this simulation is to show the electromechanical behaviour of the machine in these conditions. Figures 5.a, 5.b & 5.c show
respectively the evolution of current, torque and rotation speed. In final steady state, the amplitude of the absorbed current is equal to 70A and the rotation speed is equal to 1440r.p.m.

The above results show the flexibility of the step-by-step method; however, the main limitation factor is the great computation time needed for the simulation. For this reason, any possibility to reduce the computation time must be taken into account. In this study, a significant reduction of the computation time has been obtained by implementation of the periodicity in the simulation program. On the other hand, there are other possibilities for shortening the computation time. The part of the global matrix corresponding to the BEM region remains unchanged during the Newton-Raphson iterations. It's therefore economic to compute this matrix only once and store it. Furthermore, to solve the global system, iterative solution was preferred, both for the minimal storage it requires and for the speed of the solution process that results when using the solution in the previous time-step as initial guess for the solution in the next.
Conclusion

In this paper, a transient study of induction motor starting in different load conditions has been presented. The method is based on the coupling between mechanical motion, electric circuits and magnetic field equations of the machine. The angular movement modelling has been easily obtained using the FEM-BEM hybrid method. The circuit may have any arbitrary topology and the exciting voltage any evolution form. The presented results show the flexibility of the step-by-step method and practical operating conditions have been studied.

References