Dynamic response analysis of 3-D elastic systems by an improved frequency domain BEM

S.E. Kattis\textsuperscript{a}, D. Polyzos\textsuperscript{a} & D.E. Beskos\textsuperscript{b}
\textsuperscript{a}Department of Mechanical Engineering, University of Patras, Patras GR-26500, Greece
\textsuperscript{b}Department of Civil Engineering, University of Patras, Patras GR-26500, Greece

ABSTRACT

An improved boundary element methodology for the determination of the dynamic response of 3-D elastic systems is presented. The method works in the frequency domain and employs continuous as well as discontinuous quadratic quadrilateral 9-node elements in order to accurately treat corners, edges, region interfaces and discontinuities in boundary conditions. Advanced integration techniques are used for the computation of singular, nearly singular and hypersingular integrals for increased accuracy. Presently only problems associated with harmonic dynamic disturbances can be treated by the method. Numerical examples are presented dealing with wave diffraction in the infinite space and soil-structure interaction on the half-space.

INTRODUCTION

It is well known that the boundary element method (BEM) possesses a number of advantages over other numerical methods for a wide class of problems in applied mechanics. The main advantages of the BEM, namely the reduction of the spatial dimensions of the problem by one, the automatical accounting of radiation conditions and the high accuracy, are more pronounced when the domain of interest is infinite or semi-infinite. Obviously, these advantages become more important in three spatial dimensions. Thus, problems in linear elastodynamic analysis dealing with the structural system response to external dynamic loads or the wave propagation in infinite or
finite elastic solids are ideally treated by the BEM. Comprehensive reviews on the BEM as applied to elastodynamic problems can be found in Beskos [1-3], while a more complete treatment of the subject in the books by Manolis and Beskos [4] and Dominguez [5].

Elastodynamic problems are analysed by the BEM either in the time domain or the frequency domain. The frequency domain BEM is conceptually and computationally simpler than the time domain BEM in the framework of linear elastodynamics. In addition, the frequency domain BEM can very easily accommodate linear viscoelastic material behavior, through the correspondence principle.

This paper describes a computationally advanced frequency domain BEM for the accurate treatment of three-dimensional (3-D) elastodynamic problems. The associated computer program is presently restricted to harmonic loads or harmonic wave disturbances. However, there is work underway for making it capable of treating transient disturbances as well with the aid of numerical Fourier or Laplace transforms.

Most of the existing frequency domain BEM codes have various limitations. Thus, e.g., they cannot properly deal with problems of corners, edges, interfaces and discontinuities in boundary conditions and/or are not capable of accurately computing singular, nearly singular and hypersingular integrals. To be sure, one can find in the recent literature various methods for overcoming these problems. However no one has as yet employed all these methods together in a single general 3-D code. This is done in the present paper, where problems of corners, edges, interfaces and discontinuities in boundary conditions are easily handled by employing a combination of continuous and partially / full discontinuous quadratic elements and problems of accurately computing singular, nearly singular and hypersingular integrals are resolved by employing direct and highly accurate methods of numerical integration.

3D BEM IN FREQUENCY DOMAIN

This section briefly describes the general structure and the main components of the general 3-D frequency domain BEM code developed in this work.
The current code is designed for solving elastodynamic problems involving: i) Elastic and viscoelastic material behavior, ii) external harmonic loads and plane harmonic waves and iii) finite and infinite bodies (alone or being parts of a multibody system). The quantities that can be evaluated are the displacements, the tractions and the stresses on the boundary and the displacements and the stresses in the interior, in a pointwise fashion.

More specifically the program consists of the following main components:

- Input of model characteristics (geometry, boundary conditions, etc).
- Automatic generation of modelling details (selection of continuous or discontinuous elements, reconstruction of the input mesh, etc.).
- Construction of influence matrices by a collocation scheme and performance of numerical integrations (regular, nearly singular or singular integrals)
- Construction of total system equations by assembling subregion influence matrices and employing boundary conditions.
- Solution of the total system equations for boundary displacements and tractions.
- Calculation of stresses on the boundary.
- Calculation of interior domain quantities.

SPECIAL FEATURES OF PROGRAM

As it was mentioned in the introduction, the present computer program is characterised by main improvements in the areas of:
a) the treatment of corners, edges, interfaces and discontinuities in the boundary conditions.
b) the accurate and direct computation of nearly singular, singular and hypersingular integrals

When one has to treat corners, edges, interfaces and discontinuities in the boundary conditions, i.e. geometrical and physical discontinuities, numerical problems may arise when using continuous elements due to lack of uniqueness of the tangential plane at points of discontinuities. The usual techniques for overcoming these problems are the use of the ‘double nodes’ concept or the use of the local extra equations of elasticity in combination with the prescribed boundary conditions. Both techniques have been proved inefficient or very complicated in 3-D practical problems. One of the more straightforward ways of overcoming the previously described problems involves the use of discontinuous elements.
where the nodal and collocation points are assumed inside rather than on the element edges and corners. However, these elements present the following difficulties: i) the total number of degrees of freedom is greatly increased and ii) the numerical integration requires more refined procedures. To the authors' opinion, the best choice is the use of both continuous, discontinuous and partially discontinuous elements in order to combine the advantages of all types of elements. The family of continuous/discontinuous 3-D elements introduced by Rego Silva et al [6] is implemented in the present code with special attention to the nearly singular integrals appearing in discontinuous elements.

The treatment and evaluation of integrals is the most important problem in the BEM. Most of the difficulties are in connection with the evaluation of singular, nearly singular and hypersingular integrals. The problem of nearly singular integrals evaluation is resolved in a very efficient and accurate way by using Telle's self-adaptive transformation technique [9]. This method is used in the present code. For the evaluation of singular (Cauchy-Principal Value) and hypersingular integrals indirect methods, like the rigid body motion and the combination of dynamic and static integral equations to create regularized integral equations have been mostly employed. However, these methods have problems in cases involving 3-D infinite and semi-infinite bodies (e.g., use of enclosing elements complicates the solution) and cannot easily accommodate discontinuous elements or they are too complicated. The direct evaluation of these integrals seems to be the best approach. In flat elements this can be done easily. In higher order elements, the only way is the use of a limiting process and the analytical integration of these integrals. A full description of the methodology can be found in the works of Giuggini [7-8]. Here this methodology is successfully applied in large order 3-D problems.

NUMERICAL EXAMPLES

In order to demonstrate the performance of the present code, two characteristic 3-D elastodynamic problems with known solutions (analytical or numerical) are discussed in what follows.

Example 1: Elastic wave scattering by a spherical cavity in infinite space.
A steady-state, plane transverse wave (S-wave) propagating through an infinite elastic medium is scattered by a spherical cavity of radius \( a \). The Cartesian components of the displacement vector for
the incident wave propagating in the -x_3 -direction are given by \( u_1^I = u_0 \exp(-i K_s x_3) \), \( u_2^I = u_3^I = 0 \), where \( u_0 \) is the amplitude, \( i \) is the imaginary unit and \( K_s = \omega / C_s \) is the transverse wave number with \( \omega \) being the frequency and \( C_s \) the shear wave velocity.

The radial displacement amplitude around the periphery of the cavity (x_1 - x_3 plane) as obtained by the proposed method is shown in Figure 2 for two different dimensionless frequencies (\( K_p \alpha = 0.125, 0.913 \) where \( K_p = \omega / C_p \), with \( C_p \) being the compressional wave velocity). The results are in very good agreement with the analytical solution given by Einspruch et al [12] and the BEM results of Rezayat et al [13] who use an indirect method for the evaluation of CPV integrals. The observed slightly higher accuracy of the high frequency results of [12] is due to an adaptive Gauss integration scheme used there. Two meshes were employed in the present computations as shown in Figure 1 (discretization of the one octant). Mesh-1 was adequate for low frequencies, consisting of 24 fully continuous isoparametric 9-node quadratic quadrilateral Lagrangian elements corresponding to 98 nodes. However, the more refined mesh-2 consisting of 40 fully continuous isoparametric 9-node quadratic quadrilateral Lagrangian elements corresponding to 162 nodes was needed for obtaining accurate results for high frequencies.

![Figure 1: Discretizations of the Spherical Cavity using continuous 9-node quadratic quadrilateral elements: (a) mesh-1; (b) mesh-2.](image)

![Figure 2a: Amplitude of radial displacement versus polar angle (low frequency).](image)

The dynamic response of a 3-D surface, massless, flexible foundation to vertical external harmonic load is calculated. A square foundation of side $b = 152.4$ cm and thickness $h = 29.25$ cm is chosen to illustrate the present methodology. The elastic constants of the homogeneous, isotropic, and linear elastic foundation are: modulus of elasticity $E_f = 2.069 \times 10^5$ N/mm$^2$, Poisson ratio $\nu_f = 0.3$ and mass density $\rho_f = 7.84 \times 10^{-3}$ kg/cm$^3$. The corresponding elastic constants of the homogeneous, isotropic, and linear elastic half-space are: modulus of elasticity $E_s = 6.1 \times 10^4$ N/mm$^2$, Poisson ratio $\nu_s = 0.3$ and mass density $\rho_s = 3.01 \times 10^{-3}$ kg/cm$^3$. The above choice of elastic constants was made so that the results can be compared with those of Whittaker and Christiano [9] and Karabalis and Beskos [10]. For a harmonic vertical distributed load of amplitude $p_0 = 3.83 \times 10^5$ N/m$^2$ applied on the flexible foundation, the amplitude of the vertical displacement at the centre of the foundation - half-space interface plotted in Figure 3 versus the dimensionless frequency $\alpha_0 = \omega b / C_s$ with $\omega$ being the frequency and $C_s$ the shear wave velocity. The agreement of the present results with those of references [9] and [10] is very good, especially with those of reference [9]. A 2x2 foundation discretization with quadratic elements has been used in the present work, while 4x4 and 5x5 meshes with constant elements have been employed in reference [9]. The modelling of the problem consists of continuous/discontinuous/partially discontinuous elements in order to represent the edges of the foundation and the interface foundation - half-space properly.
Figure 3 : Vertical response of a square flexible foundation to a harmonic uniformly distributed vertical load versus frequency.

CONCLUSIONS

A general and advanced computer code for analysing 3-D harmonic elastodynamic problems has been developed. It is based on the frequency domain BEM and is characterised by two special features: accurate treatment of the problems of corners, edges, interfaces and discontinuities in boundary conditions by employing a combination of continuous and partially/full discontinuous quadratic elements and the accurate computing of singular, nearly singular and hypersingular integrals by employing direct and highly accurate methods of numerical integration.

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REFERENCES


