Bridging forces analysis in composite laminate tensile fracture by boundary element method

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ABSTRACT

Cohesive Crack Model (CCM) is presented to verify the best $\sigma$-$\nu$ curve for center-cracked composite laminate in tension. The behaviour of cohesive forces for the numerical analysis is assumed to follow four different stress-crack tip opening displacement (CTOD) paths: constant, linear and two piece-wise linear ones. Stiffness matrix calculation for BEM input differs from standard FEM procedure according to the absence of point loads, so that crack surface displacements has been related to corresponding boundary element tractions. In two cases numerical results and experimental data fit approximately well; the crack opening displacement (COD) has been the comparing parameter.

INTRODUCTION

Fracture control of fiber composites requires accurate methods for predicting the strength of laminates with cut-outs, surface scratches and parththrough cracks. One important step towards such a technique is to develop a methodology for predicting the fracture load, the stress redistribution due to damage growth, and the changes in stiffness with increased damage for laminates with various notch types. A more general method, requiring only basic properties of the laminate such as unnotched strength, apparent fracture energy and stiffness is here used to predict the strength of laminates with cracks of various shapes and sizes. In the model, called the cohesive crack model (CCM) \cite{1}, the damage zone is approximated by a crack with cohesive stresses acting on its surfaces. Damage in the material is taken into account by reducing the cohesive stresses with increased crack opening, which in turn corresponds to increased separation of the material. The predicted fracture loads correlate very well, in some cases, with experimental values. Has been demonstrated that CCM method is accurate and versatile on laminates with circular, oval and rectangular holes of various sizes and also on laminate with
cracks [3,4]. In this paper the CCM is applied to glass-fiber epoxy-resin composite laminates. Moreover for the present work boundary element procedure has been used, in spite of the usual finite element one, assembling different stiffness matrix.

**THEORETICAL MODEL**

The CCM method is based on two fundamental parameters the unnotched tensile strength \( \sigma_0 \) and the apparent fracture energy \( G^* \). During increasing external load on a laminate with a notch, a damage zone will form and grow in the stress-intense region at the notch. Damage is here used as a collective expression for delamination, fiber and matrix fracture, matrix yielding, fiber debonding, etc. To include the effect of large scale crack-tip nonlinearity, a crack of a given (traction free) length \( a \) is replaced by an (elastic) effective crack of length \( a^{\text{eff}} (=a+l_p) \) where \( l_p \) is the length of process zone. The stresses at the tip of the effective crack are assumed to be distributed on the length \( l_p \) of the effective crack.

![Figure 1: Physical mechanics of craze and crack propagation for the present material](image)

If the process zone is assumed to be localized in a band of narrow width, then the closing stress distribution can be obtained from the uniaxial tensile stress-displacement relationship. The crack initiation is assumed to occur when the crack-front displacement at the tip of the traction free crack equals the critical crack-displacement value \( v_c \).

It is important to note that the area below the \( \sigma-v \) curve is equal to the apparent fracture energy \( G^* \) and \( \sigma_0 \) is the unnotched tensile strength of the laminate. \( G^* \) represents the sum of all energies dissipated in the various failure mechanisms. In the virgin, unloaded material there is no damage and hence no equivalent crack in the model. When the load is increased such that the stress at point A in Fig.1 reaches \( \sigma_0 \) the equivalent crack is assumed to form. Upon further loading, this crack opens and growth into the laminate. The stress at point A is then assumed to follow one of the relationships shown in Fig. 1. The stress \( \sigma(x) \) at point P along
an a priori determined crack path is treated in the same way. Through this procedure, nucleation, stable growth and unstable growth of the crack are modelled in a series of calculations using a condensed stiffness matrix for the structure.

PROBLEM FORMULATION

We now try to reduce the physical problem to a mathematically manageable one through approximations. Observing several experimental tests by the optical method of Caustic we deduced that crack propagation, which is parallel to load application direction in our case, is preceded by formation of crazes that tend, during increasing external load, to nucleate into microdefects or inclusion and grow, by the formation of fibrils from the bulk polymer, parallel to preformed crack: i.e. we initially have an horizontal fictitious crack and, as CTOD reaches critical value, a vertical propagation. We examine how the various characteristics of a given $\sigma$-$\nu$ relation affect mechanics of craze respect to experimental results. To study craze and/or crack growth, one needs to know whether $\sigma$-$\nu$ relation changes as propagation proceeds and, if so, how. The question can only be answered by experiments. Since no experimental data on this particular topic is available, we feel free to assume further that $\sigma$-$\nu$ relation is "invariant" with respect to both the craze and the crack lengths for quasi-static propagation under a "controlled" environment.

EXPERIMENTAL DETAILS AND RESULTS

The material used in the experimental program consisted of unidirectional laminae glass fiber in epoxy resin. The prepreg material were hand laid-up for the desired $[0/0]_{\text{sym}}$ stacking sequence and cured according to the manufacturer’s recommended procedure. Preformed crack was carried on by a circular diamond saw of 2.0 cm diameter to obtain 0.125 $a/w$ ratio as shown in figure 2 together with geometries and dimensions.

![Figure 2: Geometry of the specimen and one quarter boundary element scheme](image)
The static tensile test for unnotched material was performed in laboratory environment under load control by the rate of 1kN every 1.25 minute: we obtained 1300 MPa for the unnotched tensile strength and, with the same procedure, 160 MPa (4.5% of average dispersion among 5 tests) for the notched tensile strength. Load vs COD curve was acquired using MTS 632.02 clip-on displacement gage. From plotted diagrams (fig.5) we noted that linearity exist until 0.233 mm. and 10795 N. and we captured vertical crack propagation, i.e. catastrophic failure, at 0.360 mm. (1.3% of average dispersion) and 12000 N.

BEM SIMULATIONS OF CRAZE AND CRACK PROPAGATION

Due to symmetry only one quarter of the laminate is analysed and the crack path is assumed to be horizontal, starting from the sharp corner of the crack. In the CCM calculations, the specimen is represented by a boundary element orthotropic structure in plane stress condition of thickness 0.9 mm. The length of all the elements belonging to craze is constant and equal to 0.1 mm. No quarter point technique has been used.

According to standard procedure for CCM applied to FEM, it's supposed to calculate a stiffness matrix relating external deformation $d$ and crack opening $v_i$ to external load $P$ and cohesive forces $F_i$. On the other hand, for boundary element method utilization, we build a stiffness matrix relating the above displacements to element boundary tractions $T$ and node boundary tractions $t_i$: furthermore only displacements boundary conditions has been implemented for the solution scheme developed in problem formulation part of this work to study how the fibril (cohesive) $\sigma$-$v$ characteristics affect the mechanics of craze and crack growth. The geometry of the problem is depicted in fig. 2. The craze length is denoted by $l_p$, the crack length by $a$ and the vertical boundary displacement of craze by $v$. The remotely applied stress is $\sigma_{00}$ and the fibril cohesive reaction is represented by $\sigma$. Given the remotely applied stress and a closing distribution for the fibrils, the problem is to find $\sigma(x)$, $v(x)$ and crack length such that both equilibrium and the smooth closure (Barenblatt’s) condition are satisfied simultaneously. Quasi-static craze and crack growth are simulated by varying the craze length $l_p$ continuously.

$$\begin{bmatrix} T \\ t_1 \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & \ldots \\ k_{21} & k_{22} & \ldots & \ldots \\ k_{31} & \ldots & \ldots & \ldots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} d \\ v_1 \\ v_2 \\ v_3 \\ \vdots \end{bmatrix}$$

Figure 3: CCM stiffness matrix for boundary element, relating displacements to boundary tractions.
With regard to apparent fracture energy \( G^* \), Rice integral applied to the notched specimen subjected to his ultimate strength, experimentally evaluated, gave us the value of 27.17 kJ/m². The effect of the generic stress-displacement relation of the form \( \sigma(x) = \sigma(\nu(x)) \) will be examined. We start by first with the Barenblatt-Dugdale model.

Figure 4: Four different cohesion-laws for boundary element input: constant, linear and two piecewise linear models

The Barenblatt-Dugdale model. The simplest model assumes constant cohesive stress throughout the cohesive zone as in fig. 4. In this case, corresponding to the critical value \( \nu_c \) we have crack opening displacement value (0.287 mm.) 23.1% more than the experimental one; on the other hand external load \( P \) (13193 N) exceeds maximum experimental value of 22.2% while craze length \( l_p \) is 0.2 mm. It should be emphasised that we suppose to have no crack propagation, i.e. \( \nu < \nu_c \), until we remain in the linear part of P-COD experimental curve and as the linearity ends, corresponding to 0.233 mm., i.e. \( \nu = \nu_c \), crack propagation starts without instability which later on takes place (corresponding to 0.360 mm.) and catastrophic failure ensues.

Linear model. All the \( \sigma-\nu \) models are designed so that the area under each \( \sigma-\nu \) curve \( G^*_c \) is constant and equal to 27.17 kJ/m² the same as that for Dugdale model employed. When \( \nu \) reaches his critical value \( \nu_c \), COD and \( P \) values (0.229-10477) remain under experimental ones respectively of -1.7% and -3%; \( l_p \) results of 0.3 mm.

Piece-wise linear models. We now try to increase critical CTOD moving to quadratic \( \sigma-\nu \) relation approximated by piece-wise linear curves as in fig. 4. In these cases we reach 0.237 mm. and 0.240 mm. for critical COD values, respectively 1.7% and 3% higher than experimental ones, and we acquired a decreasing of notched strength (10592 N) equal to -1.9% for model 1 and (10420 N) to -3.6% for model 2. This confirm other researcher’s results, i.e. shape of \( \sigma-\nu \) curve has more influence on the notched strength for smoothly notched laminates than for laminates with sharp notches. Process zone reach respectively 0.5 mm. and 0.8 mm. for model 1 and model 2.
Figure 5: Experimental and boundary element resulting plot for external load $P$ vs. crack opening displacement (COD) curve.

Figure 6: Vertical stress vs. crack tip distance for boundary element solution at $v = v_c$. 
CONCLUDING REMARKS

The sensitivity of the Cohesive Crack Model (CCM) has been studied by boundary element analysis for several variation of $\sigma-v$ curve shape in a fiber-glass epoxy-resin laminate. Non standard stiffness matrix has been used for boundary element calculations in order to relate crack surface displacement to node boundary tractions. Except constant model that shows great variations from experimental values, moving from linear to piecewise linear models there is no remarkable difference on critical COD values, while we record a small decreasing on notched strength. It could be interesting to experimentally verify the cohesive zone length $l_p$ by the most suitable optical method in order to detect numerical respondence. Anyway the exact model seems to be placed between linear and piecewise linear ones, but linear relationship still seems to play the lead role, that is seems to be the more realistic model in tensile fracture mechanics even for glass fiber and epoxy resin composite materials. For further confirmation, experiments needs to be carried out for a wider range of crack length to determine if the fibril $\sigma-v$ relation is invariant with respect to $l_p$ and $a$.

REFERENCES