On consolidation of layered soils
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ABSTRACT

Biot's consolidation analysis of plane strain layered soils is investigated. The soil is modelled as an isotropic fluid saturated layered porous medium. The time marching method, uncoupled boundary element method, and successive stiffness method are applied for the numerical study. The results show that the land subsidence due to surface loading is mainly influenced by the stiffness of the soil layer. The soil layer with smaller stiffness and thicker soft layer will induce larger land subsidence.

INTRODUCTION

The settlement of structures and foundations is the consequence of dissipation of pore water pressure. The consolidation analysis of this problem has been an active engineering research area. In geotechnical engineering, it is frequently necessary to investigate the effect of layered soils on the rate of consolidation.

Biot first presented the general theory of three-dimensional linear consolidation. However, because of the complexity of the coupled set of partial differential equations, most of the analytical solutions of Biot's model are limited to single uniform soils subjected to specialized loads and boundary conditions. Recently, Tarn and Lu presented an analytical solution of land subsidence due to a point sink in an anisotropic poroelastic half space. For the
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more general situations, researchers resort to numerical techniques. Sandhu and Wilson\(^6\) first applied the finite element method to poroelasticity. Over the years, numerous refinements and extensions have been made\(^7-8\). However, for many geotechnical problems, the domain extends indefinitely. The implementation of finite element method induces a very large set of simultaneous algebraic equations. Thus the boundary element method becomes a more attractive alternative applied by researchers\(^9-15\). Banerjee and Butterfield\(^9\) investigated the staggered boundary element method of poroelasticity. Similar study was presented by Brebbia\(^10\). Kuroki et al.\(^11\), on the basis of staggered procedures, proposed the uncoupled boundary element method for Biot's linear consolidation theory. Aramaki\(^12\) applied the same uncoupled scheme to the consolidation problems having thin layers with high permeability. Cheng and Detournay\(^13\) presented a direct boundary element method for plain strain poroelasticity. Recently, Nishmura and Kobayashi\(^14\) presented a two-dimensional time domain direct boundary integral equation method for Biot's linear consolidation theory and discussed the singularities of integration. Dargush and Banerjee\(^15\) proposed an alternative time domain boundary element formulation for two- and three-dimensional quasi-static poroelasticity. In addition to the finite element method and the boundary element method, Booker and Small\(^16-17\), on the basis of integral transforms scheme, proposed the method of finite layer analysis to study the consolidation of layered soils.

This paper investigates Biot's consolidation analysis of plane strain layered soils subjected to surface loading. The time marching method\(^10\), uncoupled boundary element method\(^11\), and successive stiffness method\(^18\) are applied for the numerical study. The numerical analysis model is presented first. Then the effect of layered soil on the rate of consolidation is fully examined.

PROBLEM FORMULATION

The governing equations of a quasi-static isotropic fluid saturated porous medium\(^19\) are

\[
GV^2 u_i + \frac{G}{1-2\nu} e_{,i} - p_{,i} + F_i = 0
\]  

(1)
where

\[ \lambda = \frac{k K}{\gamma_w(1 + n \beta K)} \]

\( u_i \) is the component of displacement, \( G \) is the shear modulus, \( \nu \) is Poisson's ratio, \( e \) is the volumetric strain, \( p \) is the excess pore pressure (compression is positive), \( F_i \) is the body force, \( k \) is the coefficient of permeability, \( \gamma_w \) is the unit weight of pore fluid, \( K \) is the bulk modulus, \( n \) is the porosity, \( \beta \) is the compressibility of pore fluid, \( \sigma_{vol} = K \epsilon \) is the total volumetric stress, and \( t \) represents time, respectively.

Let \( n_i \) be the external normal to the boundary \( \Gamma \) of the domain \( \Omega \). The boundary conditions are given by

\[ u_i = \bar{u}_i \quad \text{on } \Gamma_u \]  
\[ \sigma_{ij} n_j = \bar{t}_i \quad \text{on } \Gamma_\sigma \]  
\[ p = \bar{p} \quad \text{on } \Gamma_p \]  
\[ q_i n_i = \bar{q} \quad \text{on } \Gamma_q \]  

where \( \Gamma_u \cup \Gamma_\sigma = \Gamma, \Gamma_p \cup \Gamma_q = \Gamma, \bar{u}_i, \bar{t}_i, \bar{p}, \) and \( \bar{q} \) are the prescribed displacement, traction, excess pore pressure, and flux, respectively.

Eqs. (1) and (2) are coupled partial differential equations of Biot's model\(^1\). These equations are solved numerically, and the time-marching method\(^10\) is adopted. The excess pore pressure \( p \) in Eq. (1) and the total volumetric stress rate \( (\partial \sigma_{vol} / \partial t) \) in Eq. (2) are assumed to be constant in each time step\(^11\). Thus, Biot's model becomes uncoupled, and the uncoupled boundary element method\(^11\) and the successive stiffness method\(^18\) are applied for the numerical study.

Referring to Fig. 1, the uncoupled boundary integral
equations for the solid skeleton displacement and excess pore pressure of the represented \(i\)-th soil layer\(^9\) are

\[
c_{ij}(\xi)u_j(\xi) + \int_\Gamma T^*_ij(x;\xi)u_j(x)\,d\Gamma = \int_\Gamma T^*j_i(x;\xi)u^*_j(x;\xi)\,d\Gamma + \int_\Omega p\delta^{kili}_kl^*(x;\xi)\,d\Omega \tag{7}
\]

\[
c(\xi)p^*(\xi,t) = \int_\Gamma \int_0^t \left[ p(x,\tau)q^*(x,t-\tau;\xi) - p^*(x,t-\tau;\xi)q(x,\tau) \right] \,d\tau\,d\Gamma
\]

\[+ \int_\Omega \int_0^t \frac{\partial \sigma_{\text{vol}}}{\partial t} \,d\tau\,d\Omega
\]

\[+ \frac{k}{\lambda \gamma_w} \int_0^t \int_\Omega p^*(x,t;\xi) p_0(x,0)\,d\Omega \tag{8}
\]

where \(u^*\), \(T^*\), \(p^*\), and \(q^*\) are the fundamental solutions\(^10\) of displacement, traction, excess pore pressure, and flux, respectively; and \(p_0(0,0)\) is the initial excess pore pressure. The internal effective stress\(^19\) is given as

\[
\sigma'_{ij}(\xi) = \int_\Gamma u^*_{ijk}(x;\xi)[T^*_k(x) + p n_k]\,d\Gamma - \int_\Gamma T^*_ijk(x;\xi)u_k(x)\,d\Gamma
\]

\[- \int_\Omega u^*_{ijk}(x;\xi) p_{.k}\,d\Omega \tag{9}
\]

where \(u^*_{ijk}(x;\xi)\), and \(T^*_ijk(x;\xi)\) are kernels\(^10\).

The discretized boundary integral equations are expressed in the following forms.

\[
H^iP^i = G^iQ^i + A^i
\tag{10}
\]

\[
H^iU^i = G^iT^i + M^i
\tag{11}
\]

where \(P^i\), \(Q^i\), \(U^i\), and \(T^i\) are vectors of nodal excess pore pressures, fluxes, displacements and tractions,

\[
h_{ij} = \frac{1}{2} \delta_{ij} - \int_\Gamma \int_{t_{f-1}}^{t_f} q^*(x,t_f-\tau;\xi^i)\,d\tau\,d\Gamma
\tag{12}
\]

\[
g_{ij} = - \int_\Gamma \int_{t_{f-1}}^{t_f} p^*(x,t_f-\tau;\xi^i)\,d\tau\,d\Gamma
\tag{13}
\]

\[
a_i = - \frac{1}{K} \int_\Omega \int_{t_{f-1}}^{t_f} p^*(x,\tau;\xi^i)\,d\tau\,d\Omega \left( \frac{\partial \sigma_{\text{vol}}^{-1}}{\partial t} \right)_j
\]
where $\bar{h}_{ij}$ and $\bar{g}_{ij}$ are 2 x 2 matrices, $\mathbf{m}_i$ is 2 x 1 vector. The total volumetric stress rate $\partial \sigma_{\text{VOL}} / \partial t$ is represented by the backward difference. In Eqs. (12)-(14), the time variable integral is integrated analytically, while the integral of space variable is integrated numerically. To ensure the numerical convergence, the time increment $\Delta t$ and element size $\Delta \Gamma$ are chosen to satisfy the following relation\(^1\)

$$\Delta \Gamma = \sqrt{2\lambda/\Delta t}$$

The nodal fluxes $Q^i$ and tractions $T^i$ are expressed in terms of excess pore pressures $P^i$ and displacements $U^i$ by inverting the matrices $\overline{G}^i$ and $\overline{G}^i$, respectively\(^1\)

$$Q^i = (\overline{G}^i)^{-1} (H^i P^i - A^i) = F^i P^i - D^i$$

$$T^i = (\overline{G}^i)^{-1} (H^i U^i - M^i) = K^i U^i - N^i$$

Eqs. (19) and (20) are then partitioned,

$$\begin{bmatrix}
q^i_t \\
q^i_b \\
q^i_s
\end{bmatrix} =
\begin{bmatrix}
F^i_{tt} & F^i_{tb} & F^i_{ts} \\
F^i_{bt} & F^i_{bb} & F^i_{bs} \\
F^i_{st} & F^i_{sb} & F^i_{ss}
\end{bmatrix}
\begin{bmatrix}
p^i_t \\
p^i_b \\
p^i_s
\end{bmatrix} -
\begin{bmatrix}
D^i_t \\
D^i_b \\
D^i_s
\end{bmatrix}$$

$$\begin{bmatrix}
t^i_t \\
t^i_b \\
t^i_s
\end{bmatrix} =
\begin{bmatrix}
K^i_{tt} & K^i_{tb} & K^i_{ts} \\
K^i_{bt} & K^i_{bb} & K^i_{bs} \\
K^i_{st} & K^i_{sb} & K^i_{ss}
\end{bmatrix}
\begin{bmatrix}
u^i_t \\
u^i_b \\
u^i_s
\end{bmatrix} -
\begin{bmatrix}
N^i_t \\
N^i_b \\
N^i_s
\end{bmatrix}$$
where the subscripts \(\text{t}, \text{b},\) and \(s\) represent the top, bottom, and side boundaries of each separate layer. The interface continuity conditions are

\[
\begin{align*}
\mathbf{u}_t^i &= \mathbf{u}_b^{i+1}, \quad t_t^i + t_b^{i+1} = 0, \\
\mathbf{p}_t^i &= \mathbf{p}_b^{i+1}, \quad q_t^i + q_b^{i+1} = 0,
\end{align*}
\]  
\(i = 1,\ldots,L-1\)  
(23)

If the side boundaries \(\Gamma_i\) are located in the undisturbed regions, then

\[
\mathbf{u}_s^i = 0, \quad p_s^i = 0, \quad (i = 1,\ldots,L)  
\]  
(24)

According to the permeable/or impermeable top \((i = L)\) and bottom \((i = 1)\) boundaries, the boundary conditions and governing equations\(^{19}\) are written as follows.

(1) The top surface is permeable, and the rigid bottom boundary is impermeable. Thus the boundary conditions are

\[
\begin{align*}
\mathbf{u}_b^1 &= 0, \quad t_t^1 = f(x), \quad p_t^1 = 0, \quad q_b^1 = 0
\end{align*}
\]  
(25)

where \(f(x)\) is surface loading. Substituting Eqs. (23)-(25) into Eqs. (21) and (22) yields

\[
\begin{align*}
q_b^i &= \mathbf{F}_i \mathbf{p}_b^i - \mathbf{D}_i \\
\mathbf{t}_t^i &= \mathbf{K}_i \mathbf{u}_t^i - \mathbf{N}_i
\end{align*}
\]  
(26)  
(27)

where

\[
\begin{align*}
\mathbf{F}_i &= \mathbf{F}_{bb}^i - \mathbf{F}_{bt}^i \left( \mathbf{F}_{tt}^i + \mathbf{F}_{tt}^i \right)^{-1} \mathbf{F}_{tb}^i \quad \text{(28)} \\
\mathbf{D}_i &= \mathbf{D}_b^i - \mathbf{F}_{bt}^i \left( \mathbf{F}_{tt}^i + \mathbf{F}_{tt}^i \right)^{-1} \left( \mathbf{D}_{tt}^i + \mathbf{D}_{tt}^i \right) \quad \text{(29)} \\
\mathbf{K}_i &= \mathbf{K}_{tt}^i - \mathbf{K}_{tb}^i \left( \mathbf{K}_{tt}^i + \mathbf{K}_{tt}^i \right)^{-1} \mathbf{K}_{bt}^i \quad \text{(30)} \\
\mathbf{N}_i &= \mathbf{N}_t^i - \mathbf{K}_{tb}^i \left( \mathbf{K}_{tt}^i + \mathbf{K}_{tt}^i \right)^{-1} \left( \mathbf{N}_{tt}^i + \mathbf{N}_{tt}^i \right) \quad \text{(31)}
\end{align*}
\]

The excess pore pressure at the rigid boundary \((i = 1)\) can be obtained as
and the displacement at the top surface \((i = L)\) is

\[
\mathbf{u}_L = \left( \mathbf{K}^L \right)^{-1} \left( \mathbf{t}^L_t + \hat{\mathbf{N}}^L \right)
\]  

(2) Both the top surface and bottom boundary are permeable. The boundary conditions are

\[
\mathbf{u}_b = 0, \quad t^L_t = f(x), \quad \mathbf{p}_t^L = 0, \quad p_b^L = 0
\]

Using similar procedures as in case (1), one obtains the same displacement field equations (Eqs. (27) and (33)), while the excess pore pressure equation is

\[
q_t^i = \mathbf{F}^i \mathbf{p}_t^i - \mathbf{D}^i
\]

where

\[
\mathbf{F}^i = \mathbf{F}_{tt}^i - \mathbf{F}_{tb}^i \left( \mathbf{K}^{-1} + \mathbf{F}_{bb}^i \right)^{-1} \mathbf{F}_{bt}^i
\]

\[
\mathbf{D}^i = \mathbf{D}_{tt}^i - \mathbf{F}_{tb}^i \left( \mathbf{K}^{-1} + \mathbf{F}_{bb}^i \right)^{-1} \left( \mathbf{D}^{-1} + \hat{\mathbf{D}}_b^i \right)
\]

The flux at the top permeable surface \((i = L)\) is

\[
q_t^L = -\hat{\mathbf{D}}^L
\]

(3) The top surface is impermeable, and the rigid bottom boundary is permeable. The boundary conditions are

\[
\mathbf{u}_b = 0, \quad t^L_t = f(x), \quad \mathbf{q}_t^L = 0, \quad p_b^L = 0
\]

With similar procedures as in previous cases, the displacement field equations are found to be the same as Eqs. (27) and (33), and the excess pore pressure equation the same as Eq. (35). The excess pore pressure at the top surface \((i = L)\) is

\[
\mathbf{p}_t^L = \left( \mathbf{F}^L \right)^{-1} \hat{\mathbf{D}}^L
\]

RESULTS AND DISCUSSION
The Young's modulus of soil is between 2 MPa (soft clay) and 1400 MPa (gravel), Poisson's ratio is between 0.15 (sand) and 0.5 (medium clay), and the coefficient of permeability is between $10^{-9}$ m/sec (soft clay) and $10^{-3}$ m/sec (gravel). To study the effect of soil layer on the consolidation, the representative Young's modulus and Poisson's ratio of soil are chosen as $E = 20$ MPa, $\nu = 0.3$. The product of porosity $n$ and compressibility of pore fluid $\beta = n\beta = 6.5 \times 10^{-11}$ m$^2$/N, and the ratio of permeability and unit weight of pore fluid is $k/\gamma_w = 2.6 \times 10^{-13}$ m$^4$/Ns, respectively.

Fig. 2 shows the comparison of the degree of settlement-time due to strip load in the single layer soil between the present model and Booker and Small (1982). The Young's modulus and coefficient of permeability are $E$ and $k$, respectively; and the ratio of layer thickness and half width of loading is 2. It can be seen that the result of present model agrees well with that of Booker and Small (1982). Fig. 3 is the comparison of the excess pore pressure on centreline due to strip load in the single layer soil between the present model and Booker and Small (1982). The soil layer is the same as Fig. 2, but the ratio of layer thickness and half width of loading is 1. Since the time increment of present model (Eq. (18)) is not equal to that of Booker and Small (1982), the excess pore pressures are different, but they possess the same trend.

The degree of settlement-time due to strip load in the single layer soil for three kinds of boundary conditions is shown in Fig. 4. The soil layer and the ratio of layer thickness and half width of loading are the same as Fig. 2. One can observe that the rate of surface settlement for the permeable top surface and bottom boundary (case 2) is faster than those of the other two cases. But the surface settlement approaches a constant for all cases when the consolidation reaches the steady state.

Fig. 5 shows the degree of settlement-time due to strip load in the four-layer soil for the permeable top surface and impermeable bottom boundary (case 1). The ratio of total depth $H$ and half width of loading $b$ is 3; the layer thicknesses are $h_1 = h_2 = b$, and $h_3 = h_4 = 0.5b$; the Young's
moduli are $E_1 = E_2 = E_4 = E$, and $E_3 = 5E$. It is found that the rate of surface settlement is apparently affected by the coefficient of permeability of the third layer. The rate of settlement is faster for the third layer with larger permeability coefficient, while the surface settlement approaches a constant when the consolidation reaches the steady state. The excess pore pressure on centreline of the same soil layer of Fig. 5 is shown in Figs. 6-8. One finds that the excess pore pressure decreases with time, and pore pressure of the third layer is obviously affected by the variation of permeability, but the change of pore pressure of the remaining layers is not significant.

The degree of settlement-time due to strip load in the four-layer soil with different third layer soils is investigated in Fig. 9. The soil layer and the ratio of total depth $H$ and half width of loading $b$ are the same as those in Fig. 5, but the third layer has different content. The top surface is permeable, but the bottom boundary is impermeable (case 1). As expected, the settlement of layered foundation with a softer inner layer is larger than that of foundation with a stiffer inner layer. Furthermore, we observe that the change is very significant. The effect of inner soft layer thickness on the settlement is studied in Fig. 10. As expected, the settlement of layered foundation with a thicker soft inner layer is larger than that of foundation with a thinner soft inner layer. But the effect of soft layer thickness is not as significant as that of soil type (Fig. 9).

CONCLUSION

The Biot's consolidation analysis of plane strain layered soils subjected to surface loading is investigated. The soil is modelled as an isotropic fluid saturated layered porous medium. The time-marching method is applied for the numerical analysis, and the excess pore pressure and the total volumetric stress rate are assumed to be constant in each time step. Thus, Biot's model becomes uncoupled. The uncoupled boundary element method and the successive stiffness method are adopted. The results of numerical examples show good agreement with previously published ones. And the results show that the land subsidence due to surface loading is mainly influenced by the stiffness of the soil layer. The boundary conditions of top and bottom
boundaries only affect the rate of consolidation, but the final land subsidence is not apparently affected by these conditions. The soil layer with smaller stiffness and thicker soft layer will induce larger land subsidence.

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REFERENCES

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Fig. 1 Schematic configuration of layered soil

Fig. 2 Comparison of the degree of settlement-time due to strip load in the single layer soil between the present model and Booker and Small (1982).
Fig. 3 Comparison of the excess pore pressure on centreline due to strip load in the single layer soil between the present model and Booker and Small (1982).

Fig. 4 The degree of settlement-time due to strip load in the single layer soil for three kinds of boundary conditions.
Fig. 6. The excess pore pressure on confined due to strip load in the four-layer soil (Case 1: permeable top and impermeable bottom boundary).

Fig. 5. The degree of settlement-time due to strip load in the four-layer soil (Case 1: permeable top and impermeable bottom boundary).
Fig. 7 The excess pore pressure on centreline due to strip load in the four-layer soil (Case 1: permeable top surface and impermeable bottom boundary, $k_3 = 10k$).

Fig. 8 The excess pore pressure on centreline due to strip load in the four-layer soil (Case 1: permeable top surface and impermeable bottom boundary, $k_3 = 100k$).
Fig. 9 The degree of settlement-time due to strip load in the four-layer soil with different third layer soil.

Fig. 10 The degree of settlement-time due to strip load in the four-layer soil with different thickness of soft third layer.