Numerical calculation of magnetic dissipation at power transformers

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Abstract

The analysis of the power transformer electromagnetic dissipation has been based on the analysis of electromagnetic process distribution in the stationary regime. The finite element method was used in the area of the numerical calculation of magnetic field. The 2D calculation of magnetic field distribution in transformers was realized through the application of the FEM2D software. From the numerical calculation of the magnetic field, the dissipating inductivity is calculated through the application of the energy method and the linked fluxes method. Based on the laboratory measures conducted on the model of a low-voltage transformer, the values of the numerical calculation have been provided. Results of the numerical calculation of the dissipating inductivities were compared to the measured values obtained in the short circuit regime with the application of very precise instrumentation. Some proportional deviations were noted. Conducted analysis on the numerical calculation of magnetic dissipation from the point of data accuracy, and complexity of the calculation brings the energy method into first place. The acceptable results have also been exercised with the linked fluxes method.

Keywords: magnetic dissipation, dissipating inductance, dissipating fluxes, method of finite elements, method of flux linkage, energy method.

1 Introduction

The numerical methods of determining the magnetic dissipation for different positions and shapes of windings and magnetic core are very often used by the constructors in the transformers manufacture. Special attention during the
calculation and analysis of the magnetic dissipation has to be dedicated to the non-linearity of the transformer magnetic circle \((\mu=f(B))\). Formerly, these weaknesses, exercised at resolving the Poisson’s differential equation by analytical methods, were avoided by utilization of the graphical methods based on the orthogonal characteristics of lines of field and lines of the constant potentials. In case one wants to perform the numerical calculation of magnetic field in transformer, which is the basis for calculation of dissipating inductivities, this non-linearity has to be considered. Result of this is further complication of the entire calculation. For achieving the satisfactory accuracy of the numerical calculation, it is necessary to determine the magnetization characteristic of magnetic material of the transformer core. Numerical calculation of the transformer magnetic field was done by using the method of finite elements. This method enables determination of allocation of static or time changing field in linear or non-linear, isotropic or anti-isotropic type of material with electric current or permanent magnetic stimulus. The finite elements analysis is divided on pre-processing, resolution and post-processing phases. A result accuracy of magnetic field depends on modeling and discretization of the problem, determined edge conditions and parameters of the materials used. It is hard to find general methodology for discretization and determining the edge conditions. With the application of the finite elements method, the field function, described by differential equation, cannot be directly determined from the differential equation, but it provides data on minimization of an appropriate function. From the numerical calculation of magnetic field, the dissipation inductivity is determined by the application of the energy method and linked fluxes method.

### 2 Defining the dissipation field at power transformers

Dissipation field at the even distribution of magnetic-stimulated forces per height of transformer windings is shown on the Figure 1. Dissipation flux goes through the air cut between the windings, comes to fastening rings and shackles, than, it closes through the magnetic core and wall of kettle. Dissipation field is conditionally divided in two components: axial component of dissipating field directed along the axis of winding, and radial component of dissipating field directed along radial axis of winding. Diagrams of radial and axial components of dissipation field and magnetic-stimulated forces of winding of double-winded power transformer are shown on the Figure 2.

As it can be seen from the Figure 2.a, at an even distribution of the magnetic-stimulated forces, the radial component of dissipation field has the largest value at the edges of winding, while the axial component of dissipation field has the largest value in the middle of winding. It is noted that at the un-even distribution of magnetic-stimulated forces Figure 2.b, caused by different height of windings, there is a huge increment of maximum of radial component of dissipation field. The dissipation field induces whirlpool currents in all conducting parts through which it extends. At windings these currents are closing within particular conductors, and, unlike the load currents, they do not leave the winding. At transformer, windings are made of couple of parallel conductors in coil. At the
beginning and at the end of a section conductors are combined, creating closed electric circle through which a dissipation field extends. In each circle axial component of a dissipation field induces voltages under which influence currents that flow in the circles are created. They do not leave the circle area. Beside parallel conductors, at big transformers it is often case to combine parallel sections of windings. That kind of combining also creates current circles in which whirlpool currents are being induced. Figure 3 shows parallel combining of two sections of windings that are made of one conductor. Axial component of dissipation field induces currents only in circle that is made by parallel conductors, and a radial component of dissipation field induces currents only in circle of parallel sections of windings. Whirlpool currents are added to or deducted from load current that leads to unequal heating of sections of windings, thermal exertion of isolation and its faster ageing.

![Figure 1: Dissipation field of a transformer: 1-column; 2-winding LV; 3&7-shackle; 4-fastening rings; 5-winding HV; 6-wall of kettle.](image)

The dissipation field, as previously said, increases additional losses in transformer, windings and elements of construction, reduces useful power, and coefficient of useful effect of the transformer. Also, it reduces voltage on the transformer secondary windings, protects transformer at short circuits, reduces electromagnetic forces, and limits currents and heating of windings. Having in mind the role of dissipation field, it is of great importance to perform an accurate measurement and analysis according the influence it has at voltage and currents in windings at short circuits of a transformer. Nominal frequency voltage that should be applied at one winding, while having a short circuit at another winding, needed for creating nominal currents $I_1$ and $I_2$ at windings, is called relative voltage of short circuit of transformers. It is designated with $u_k$ and expresses relation between voltage of short circuit $U_k$, which responds to current of short circuit that is equal to nominal, and nominal voltage. Considering an important role of dissipation field in transformer, voltage of short circuit cannot be arbitrary. Sometimes it can be high (consumers with frequent short circuits) or relatively small (at transformers with quiet work regime). Voltages of short circuits established for each group of transformers, are result of fundamental
research of constructions, and they need to be sufficient for limitation of currents of short circuits and for self-protection of transformer, but not that high to significantly increase losses and reduce transformation power.

Figure 2: Diagrams of axial and radial components of dissipation field: a) At an even distribution of magnetic forces, b) At an un-even distribution of magnetic forces, c) Section of winding under the influence of axial and radial component of dissipation field.

Figure 3: Whirlpool currents dispositioning in sections of winding.
Calculation of dissipation inductivities is complex because it is hard to separate dissipating lines of force from the main magnetic flux lines of force. Because of that, calculation of dissipation inductivities is performed in the driving stage where only dissipating magnetic fluxes exist, while the main magnetic flux does not exist at all. This is the case of an “ideal short circle” that is achieved by the flow of the same electric currents $I_k$ through the transformer with the equal number of coils at primary and secondary winding arranged in serial order. The coils are connected in such a way that the magnetic-stimulated forces cancel each other, there is no main magnetic field, only two dissipating magnetic fields. Described condition can be achieved also in transformers with any ratio in regard to primary and secondary coils, when the electric current $I_k$ is flown through the primary coil.

For all driving conditions that are characterized with neglecting the magnetization current we can use values of dissipating inductivities at “ideal short circuit”. From the energy of dissipating magnetic field the equivalent dissipating inductivity can be calculated from the linked flux or from the energy of dissipating magnetic field, and after the numerical calculation of magnetic field, it is determined by application of energy and linked fluxes method.

### 3 Energy method

In most of practical cases, calculation of field cannot be conducted with the satisfactory accuracy by using the analytical methods, thus it is necessary to use some of the numerical methods. If we anticipate that the calculation of magnetic field is done, it remains to calculate the energy accumulated in static magnetic field that, depending on the nature of problem, can be resolved in different ways:

- By field vector, integration along the entire volume in which the field exists,
- By known density of currents and vector magnetic potentials, integration along the volume of the area where the currents flow,
- By currents and magnetic fluxes for the system of thin power currents loops of constant section (cut) in a linear material.

Energy of magnetic field is determined by integration of magnetic field strength and magnetic induction, i.e. through the density of electric current and magnetic potential.

\[
W_m = \iiint_{(V)} \left( \vec{B} \cdot \vec{H} d\vec{B} \right) dV \Rightarrow W_m = \frac{1}{2} \iiint_{(V)} \vec{H} \cdot \vec{B} dV
\]  
\[
W_m = \iiint_{(S)} \left( \frac{dA \times \vec{H}}{\vec{S}} \right) \cdot d\vec{S} + \iiint_{(V)} \left( \frac{d\vec{J} \cdot \vec{A}}{\vec{A}} \right) \cdot dV = \frac{1}{2} \iiint_{(V)} \vec{J} \cdot \vec{A} dV
\]  

Since this is about two-dimensional (2D) magnetic fields, than the vectors, density of currents, magnetic potentials and strength of magnetic field can be presented by product of the calculation length area unit vectors and scalars of given values:
\[ \vec{J} = \vec{J}_z J(x, y), \quad \vec{A} = \vec{J}_z A(x, y), \quad \vec{H} = \vec{J}_x H_x(x, y) + \vec{J}_y H_y(x, y) \]  

(3)

d\mathbf{V} = l \cdot dS \tag{4}

Final result is energy per unit of length of the calculation observed area:

\[ W_m' = \iint_{(S)} \left\{ \int \vec{J} \cdot dA \right\} \cdot dS \tag{5} \]

Equation (5) for the linear magnetic materials would be as follows:

\[ W_m' = \frac{l}{2} \iint_{(S)} \vec{J} \cdot \vec{A} \cdot dS \]  

(6)

Given equations significantly reduce domain of calculation compared to the equations we have started with. With discretization of the calculation area to the large number of finite elements, the integration by which the energy of the magnetic field is calculated approximates with the final sum:

\[ W_m' = \frac{l}{2} \iint_{(S)} \vec{J} \cdot \vec{A} \cdot dS \Rightarrow W_m' = \frac{l}{2} \sum_{i=p}^{i=z} J_i \Delta S \cdot A_i = \frac{l}{2} \sum_{i=p}^{i=z} A_i \Delta S \tag{7} \]

where: \( \overrightarrow{A_i} = (A_u + A_b + A_e) / 3 \) - middle potential of \( i \)-th element. Indexes \( i_p \) and \( i_z \) mark starting and ending element of a net where exists current density “\( J \)”.

### 4 Linked fluxes method

Determination of dissipating inductivities from the linked fluxes represents defining the magnetic fluxes that exist in a power transformer. Figure 3 shows in a simplified manner the magnetic fluxes in a power transformer. Determination of dissipating fluxes is not an easy job, having in mind that different fluxes penetrate through different sectors of the same winding. It is necessary to define an area across which an integration of magnetic induction vector is to be performed, with the goal to calculate the magnetic flux. Windings are divided on “\( n \)” sectors that are small enough to obtain the magnetic flux, penetrating through the subject area, to be approximately constant. Inducted electromotive force, due to dissipating flux, through the one of the windings in this case can be calculated as follows:

\[ e_R = -\frac{d}{dt} \left( \sum_{i=1}^{n} n_i \cdot \left( \Phi_i - \Phi_g \right) \right) = -n \cdot \frac{d}{dt} \left( \phi_i \right) \]  

(8)

where \( \Phi_i \) is equivalent dissipating magnetic flux, and \( \Phi_g \) is partial flux through an \( i \)-th sector of winding. Since:
\[ \sum_{i=1}^{n} n_i \cdot (\Phi_i - \Phi_g) = n \cdot \phi_f \]  

(9)

where \( n \) is total number of coils in a winding.

Relation (9) can be written in a shape:

\[ \sum_{i=1}^{n} n_i \cdot \Phi_i - n \cdot \Phi_g = n \cdot \phi_f \]  

(10)

there fore a magnetic flux is:

\[ \phi_f = \frac{\sum_{i=1}^{n} n_i \cdot \Phi_i}{n} - \Phi_g \]  

(11)

If we consider that every sector has a same number of coils \( \omega' \), i.e. equal division of windings per sectors, than:

\[ n = \sum_{i=1}^{n} n_i = \sum_{i=1}^{n} n' = n \cdot n' \]  

(12)

Finally magnetic flux is:

\[ \phi_f = \frac{\sum_{i=1}^{n} \Phi_i}{n} - \Phi_g \]  

(13)

According to relation (13) it is possible to determine so called equivalent dissipating magnetic flux that will serve to determine the dissipating inductivities. Let us anticipate that the field is resolved, i.e. that the components of magnetic induction in every loop of finite elements are known. To confirm the accuracy of the numerical procedures, the best would be to undertake the parallel tests on one low-voltage air-core power transformer. Power transformer whose magnetic field needs to be calculated has a core with non-linear characteristic of magnetization (Figure 5).

![Illustration of magnetic fluxes in single-phase transformer.](image-url)
If we want to do the calculation of magnetic field in transformer, this non-linearity has to be taken into consideration. The result of it is that the entire calculation becomes more complex. The resolution of the edge problem gives data on magnetic field in transformer: the task is to determine unknown function \( A = A(x,y) \) of the calculation area that satisfies the Poisson’s equation:

\[
\frac{\partial}{\partial x} \left( \frac{1}{\mu} \frac{\partial A}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{1}{\mu} \frac{\partial A}{\partial y} \right) = -J
\]  

(14)

with an edge condition: \( A(x,y) = 0 \), \( (x,y) \in \Gamma \), Dirichlet’s type.

Function of the currents density is:

\[
J(x,y) = \begin{cases} 
+J_p, & (x,y) \in S_{PL}, \\
-J_p, & (x,y) \in S_{PD} \\
-J_S, & (x,y) \in S_{SL}, \\
+J_S, & (x,y) \in S_{SD}
\end{cases}
\]

(15)

Magnetic permeability in the calculation area is:

\[
\mu(x,y) = \begin{cases} 
\mu(H), & (x,y) \in S_{FE} \\
\mu_0, & (x,y) \notin S_{FE}
\end{cases}
\]

(16)

Described edge problem is resolved by the application of the finite elements method. Discretization of the problem per this method, as a result, gives non-linear system of equations such as \( [K(U)]U = F \). Non-linear system of equations is resolved in most of cases by the application of Newton-Raphson’s iterative procedure. The results of this procedure are data on magnetic vector potential in every loop of the net of finite elements that covers the calculation area. Calculation of non-linear magnetic field with the method of finite elements is done with the application of FEM2D software. Given geometry of power transformer is divided on the finite elements, by using the triangular elements. The entire cross-section of a transformer is divided on areas with the same type of material. This procedure is used in order to assign values of magnetic permeability (for linear materials), magnetization curve (for non-linear materials).
and electric current density (for areas that represent windings) to the elements on which certain areas are divided.

Since the equipotent lines of magnetic vector potential in case of 2D field, at the same time are the lines of force of magnetic induction, between the primary and secondary winding exists the line of zero potential. Also, the same line exists through the middle of the transformer core. Line of zero potential through the middle of magnetic core of transformer, shows that the edge problem is symmetrical to this direction. That means that it is possible to analyze only one half of a transformer.

5 Results of calculation of magnetic field and dissipating inductivities (short circuit regime)

Regions in which the named currents of primary and secondary winding are assigned are: \( I_1 = 1.36 \text{A}, \quad J_1 = \frac{I_1 \omega_1}{S_1} = 3.845 \text{e}6 \quad \text{A/m}^2, \quad I_2 = 12.46 \text{A}, \quad J_2 = \frac{I_2 \omega_2}{S_2} = -3.845 \text{e}6 \quad \text{A/m}^2. \)

It can be seen that between the primary and secondary winding exist the line of zero potential, equipotent lines of magnetic vector potential are symmetrically closed around the winding and part of the magnetic core (Figure 6).

From the Figure 7, it can be seen that the small part of magnetic core is used for closing the dissipating magnetic fluxes, and the bigger part (blue color) has the magnetic induction of very small value. Magnetic field exists in narrow areas of the air space between the primary and secondary windings. It also exists around the air space where the induction is the strongest. That gives the picture of energy flow from the primary to secondary winding. Results of dissipating inductivities calculation (Table 1) show that the difference between the two essentially different methods is very small (Table 2), i.e. accuracy of calculation is very high; this points on high-quality and dense net of finite elements.

Since it is about numerical calculations, whose accuracy is not easy to estimate, another experiment of a transformer short circuit with the application of a very precise instrument was done. This instrument gave the precise data that were used to experimentally determine the value of dissipating inductivity. Numerically calculated values will be compared with the result obtained through
the application of the power analyzer, to determine the proportional deviation. That will be used as an accuracy standard of every numerical procedure. Table 3 gives comparison of calculation by using the energy method (EM) and linked fluxes method (LFM) with measures on the low-voltage air-core power transformer. Proportional errors are determined in regard to the measured results. Based on the presented results, it can be seen that the energy method is more accurate. However, the accuracy achieved through the method of linked fluxes is also satisfactory.

Table 1: Results of dissipating inductivities calculation.

<table>
<thead>
<tr>
<th>Winding</th>
<th>Energy Method</th>
<th>Linked Fluxes Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary</td>
<td>$W_p = 0.0016\ (J/m)$ $L_p = 2W_p l_0/l^2 = 1.761\ (mH)$</td>
<td>$A_{SR} = 3.9799\times10^{-5}\ (Tm)$ $L_p = l_0 A_{SR}/l = 1.719\ (mH)$</td>
</tr>
<tr>
<td>Secondary</td>
<td>$W_s = 0.0014\ (J/m)$ $L_s = 2W_s l_1/l^2 = 2.061\ (mH)$</td>
<td>$A_{SR} = 3.4654\times10^{-5}\ (Tm)$ $L_s = l_0 A_{SR}/l = 2.011\ (mH)$</td>
</tr>
<tr>
<td>Total</td>
<td>$W_m = 0.003\ (J/m)$ $L = L_p + L_s = 3.821\ (mH)$</td>
<td>$A_{SR} = 7.4453\times10^{-5}\ (Tm)$ $L = L_p + L_s = 3.73\ (mH)$</td>
</tr>
</tbody>
</table>

Table 2: Difference in results of the two numerical methods.

\[
\delta_p = (L_{p_{en}} - L_{p_{ut}})/L_{p_{en}} \cdot 100 = 2.385\ (%) \\
\delta_s = (L_{s_{en}} - L_{s_{ut}})/L_{s_{en}} \cdot 100 = 2.426\ (%) \\
\delta = (L_{e_{en}} - L_{l_{ut}})/L_{e_{en}} \cdot 100 = 2.381\ (%)
\]

Table 3: Comparison of calculated and measured values.

<table>
<thead>
<tr>
<th>Type of transformer</th>
<th>Dissipating inductivity L = L_p + L_s (mH)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EM</td>
</tr>
<tr>
<td>Low-voltage transformer</td>
<td>3.821</td>
</tr>
</tbody>
</table>
6 Conclusion

Here we have shown the utilization and direct application of the energy method and method of linked fluxes for calculation of dissipating inductivities on a concrete example of low-voltage air-core power transformer. Accuracy of the conducted numerical methods are confirmed through the experimental surveying of dissipating inductivities at the above mentioned power transformer. Comparison of the calculation results with the surveying results confirms applicability of the given mathematical model and developed numerical procedures. Results of research obtained through this work have both theoretical and practical utilization value. They also represent realistic assumption for realization of further research in the field of magnetic dissipation in power transformers. Numerical calculation of power transformer, versus the analytical methods, enables insight to the inside of device, all of its parts, which ease their optimization and adjustment. It is possible to almost for sure, anticipate “behavior” of the device, which enables shortening the time needed to implement the idea, and also avoiding the experiments. Shown is numerical and graphical presentation of results of complex distribution of magnetic field in clear and various forms. Achieved results can be calculated for any regime of power transformer work. This is important because with the change in the regime of work of a transformer, i.e. with the change of currents through the windings, due to the non-linearity of magnetic materials, the dissipating inductivity of the primary and secondary windings changes. Everything mentioned above, justifies introduction and further development of these and similar numerical calculations for resolving the practical problems.

References