Transient plane wave coupling to a finite length wire buried in a conductive ground

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Abstract

The calculation of current induced on a straight wire buried in a lossy ground due to a transient plane wave is presented. A frequency domain model is based on the Pocklington integral equation arising from scattering theory and the thin wire approximation. Influence of the nearby air-earth interface is taken into account via the reflection coefficient appearing within the corresponding integral equation kernel. The Pocklington integral equation is solved using the indirect Galerkin-Bubnov variant of the boundary element method. Time domain results are obtained by means of the inverse Fourier transform. Some illustrative numerical examples are presented.

1 Introduction

The calculation of the current induced along power and communication cables due to a plane wave is important issue in various EMC studies. Although there are a large number of papers dealing with plane wave coupling to the overhead wires [1]-[3], there are only few dealing with the buried wires [4, 5].

Most of the studies related to this problem are based on an approximate transmission line (TL) approach. Although quite efficient for the long (infinite) cables, TL approach fails for the finite length wires [6]. Alternative approach is based on the wire antenna theory by which the radiation effects are taken into account. Antenna theory approach provides accurate solutions for the finite length wires. On the other hand the solution becomes significantly time consuming when very long cables are considered.

Through the antenna theory approach the effects caused by the air-earth interface are taken into account through exact Sommerfeld integral formulation.
Since evaluation of the Sommerfeld integrals is analytically demanding and numerically time consuming [7, 8], a simplified approach based on reflection coefficient approximation is sometimes used [3, 5].

The wire antenna approach to the analysis of the plane wave coupling to the buried cable with the effect of the half space included through reflection coefficient (RC) approximation has been proposed in [5].

This paper extends the model presented in [5] to the more complex case of a plane wave excitation with an arbitrary angle of incidence. The formulation is based on the Pocklington integral equation. This integral equation is solved via the indirect Galerkin-Bubnov Boundary Element Method (GB-BEM) [8]. Furthermore, the transient response of the wire is computed using the inverse Fourier transform.

## 2 Frequency domain formulation

The geometry of interest is the horizontal straight wire of length $L$ and radius $a$, buried in a lossy medium at depth $d$, and it is shown in figure 1.

![Figure 1: Straight wire buried in a dissipative medium.](image)

In accordance to the wire antenna theory and reflection coefficient (RC) approximation the current distribution along the single straight wire antenna embedded in a lossy ground is governed by the Pocklington integro-differential equation [3], [8]:

$$
E_{z}^{ex} = -\frac{1}{j4\pi\omega\varepsilon_{ref}} \int_{-L/2}^{L/2} \left[ \frac{\partial^2}{\partial x' \partial z} + k_z^2 \right] \left[ g_0(x, x') - R_{TM} g_1(x, x') \right] I(x') dx'
$$

(1)

where $E_{z}^{ex}$ is tangential component of the imposed electric field at the wire surface, $I(x')$ is the unknown current distribution induced along the wire axis, and $g_0(x, x')$ denotes the lossy medium Green function:

$$
g_0(x, x') = \frac{e^{-jk_z R_i}}{R_i}
$$

(2)
while \( g_i(x,x') \) is, in accordance to the image theory, given by:

\[
g_i(x,x') = \frac{e^{-jk_2R_1}}{R_2} \tag{3}
\]

where \( k_2 \) is the propagation constant of the lower medium and \( R_1 \) and \( R_2 \) are distances from the source point and from the corresponding image to the observation point defined by:

\[
R_1 = \sqrt{(x-x')^2 + a^2}, \tag{4}
\]

\[
R_2 = \sqrt{(x-x')^2 + 4d^2}
\]

The influence of a nearby ground interface is taken into account by means of the Fresnel plane wave reflection coefficient:

\[
R_{TM} = \frac{1}{n} \frac{1 - \cos \theta}{1 - \sin^2 \theta} ; \quad \theta = \arctg \frac{|x-x'|}{2d} ; \quad n = \frac{\varepsilon_{ef}}{\varepsilon_0} \tag{5}
\]

Imposed electric field at the wire surface, due to transmitted plane wave Fig 2, is given by:

\[
E_x^{exc} = E_x^r = E_0 (\Gamma_{TE} \sin \phi - \Gamma_{TM} \cos \phi) e^{-jk_2r_r} \tag{6}
\]

where \( \alpha \) is the angle between \( E-field \) vector and the plane of incidence and \( \theta_i \) is determined by the Snell low:

\[
k_i \sin \theta = k_2 \sin \theta_i \tag{7}
\]

where \( k_i \) is free space propagation constant.

Furthermore, \( \Gamma_{TM} \) and \( \Gamma_{TE} \) are the vertical and horizontal Fresnel transmission coefficients at the air-earth interface given by [9]:

\[
\Gamma_{TM} = \frac{2 \sqrt{n} \cos \theta}{n \cos \theta + \sqrt{n - \sin^2 \theta}} \tag{8}
\]

\[
\Gamma_{TE} = \frac{2 \cos \theta}{\cos \theta + \sqrt{n - \sin^2 \theta}}
\]

and \( \vec{n}_r \cdot \vec{r} \) is distance from the origin point to the observation point at the wire surface.
\[ \vec{n}_i \cdot \vec{r} = -x \sin \theta_i \cos \phi - y \sin \theta_i \sin \phi - z \cos \theta_i \]  

(9)

The Pocklington integral equation (1) is solved by means of the indirect Galerkin-Bubnov boundary element procedure and equivalent current distribution at the given frequency is obtained.

Figure 2: Incident, reflected and transmitted wave from an air-earth interface.

The mathematical details regarding this numerical procedure can be found elsewhere, e.g. in [7], or more recently in [8].

3 Transient response calculation

The knowledge of the current distribution along a buried wire for a wide spectrum of frequencies provides the frequency domain counterpart of the system impulse response, also referred to as the transfer function of the system, \( H(f) \).

To examine the transient response of the buried wire, an incident field in a form of a single exponentially decaying function is used:

\[ e_x''(t) = E_0 e^{-at} \]  

(10)

The Fourier transform of the excitation function (10) is given by:

\[ E_x''(f) = \frac{E_0}{a + j2\pi f} \]  

(11)
Now, the current distribution $I(f)$ is obtained by multiplying the frequency spectrum of the excitation function with the previously calculated transfer function $H(f)$:

$$I(f) = H(f)E_x^r(f)$$

(12)

In order to obtain a time dependent current distribution the Inverse Fourier Transform (IFT) is to be applied to the function $I(f)$ [10, 11]:

$$i(t) = \int_{-\infty}^{\infty} I(f)e^{j2\pi ft}d\omega$$

(13)

However, due to discrete character of the calculated transform function $H(f)$, the actual frequency response $I(f)$ is represented by a discrete set of values. Therefore, the integral (13) cannot be solved analytically and one has to deal with the Discrete Fourier transform (or in this case Fast Fourier Transform) i.e.:

$$i(t) = \text{IFFT}(I(f))$$

(14)

Thus, the discrete set of the time domain current values is defined by [10, 11]:

$$i(n\Delta t) = F \cdot \sum_{k=0}^{N-1} I(k\Delta f)e^{jkn\Delta f\Delta t}$$

(15)

where $F$ is the highest frequency taken into account, $N$ is the total number of frequency samples, $\Delta f$ is sampling interval and $\Delta t$ is the time step.

## 4 Numerical results

For the comparison purposes, firstly the current distribution induced along a conducting cylinder, immersed in seawater, is computed assuming a unit incident field. The water is characterized with $\varepsilon_r=80$ and $\sigma=4\text{S/m}$, and the operating frequency is $f=1\text{MHz}$. The cylinder length is $L=120\text{m}$ and $L=160\text{m}$ with a radius of $a=0.6\text{m}$ and $a=0.8\text{m}$ respectively. In Fig 3, the calculated current distributions are compared to the analytical results available from [12]. Agreement is judged to be satisfactory.

The next example is related to the straight wire of length $L=5\text{m}$ and radius $a=1\text{cm}$ embedded in the dielectric half-space with $\varepsilon_r=10$ at the depth $d=1\text{m}$. The wire is illuminated by the transmitted part of the electromagnetic pulse (EMP) incident waveform: $E_{inc}(t) = E_0(e^{\alpha t} - e^{\beta t})$ where the EMP parameters are: $E_0=52.5\text{kV/m}$, $a=4*10^6\text{s}^{-1}$, $b=4.78*10^8\text{s}^{-1}$.

The transient current induced at the wire center is shown in Fig 4. The transient response obtained by using the proposed indirect frequency domain approach seems to be in a satisfactory agreement with the results computed via the direct time domain approach [13].

Further numerical examples are related to the transmission lines buried in a lossy ground with permittivity $\varepsilon_r=10$ and conductivity $\sigma=0.001\text{S/m}$. Conductor
radius is $a=1\text{cm}$ while length $L$ and burial depth $d$ are varied. The wire is excited by the transmitted plane wave with a single exponential decaying form defined with equation (10). The time constant of the exponential function is chosen to be $a=(10 \cdot 8.854^{-8}\text{s})^{-1}$.

![Graph of Abs(I) vs 2x/L for different L values](image)

**Figure 3:** Comparison of numerical results obtained with different solution methods.

![Current waveform](image)

**Figure 4:** Transient current induced at the center of the straight wire ($L=5\text{m}$, $a=1\text{cm}$, $\varepsilon_r=10$, $d=1\text{m}$).

The current induced at the center of wires with various lengths for the normal incidence is shown in Fig 5. The burial depth is $d=1\text{m}$. The influence of the
transient current reflections from the wire ends is clearly visible, particularly for the shorter wires.

Figure 5: Transient response for different wire lengths.

Figure 6: Transient response for different burial depth.

Figure 6 shows the influence of the burial depth to the induced current on the 200m long wire. Obviously, the time shift and the attenuation of the signal are increased with the depth. The only exception occurs for the case of \(d=0.1m\), where the induced current is lower then for the case of \(d=1m\). This is due to the influence of the nearby ground-air interface.

Figure 7 examines the transient response of the wire of length \(L=30m\) buried at depth \(d=1m\) for various angles of incidence \(\theta=0^\circ, 60^\circ, 80^\circ\). For the higher
angle $\theta$, amplitude of the induced current is smaller, since the tangential component of the transmitted electric field is also decreased.

![Graph showing transient response for different angle of incidence.](image)

Figure 7: Transient response for different angle of incidence.

5 Conclusion

The paper deals with the transient plane wave coupling to the transmission line buried in the lossy ground. The frequency domain model is based on the Pocklington integral equation arising from the wire antenna theory. The influence of a lossy half space has been taken into account via the reflection coefficient (RC) approximation instead of rigorous Sommerfeld integral approach. This integral equation is numerically solved via Galerkin-Bubnov boundary element scheme and the frequency spectrum of the induced current is obtained. The related transient response of the buried wire is readily computed applying inverse Fast Fourier Transform.

The future work on the subject will be related to the more complex multiple wire configurations.

References


