Calculation of base station antenna radiation pattern using the weak Galerkin-Bubnov FEM formulation for integro-differential operators

D. Vucicic, D. Poljak & V. Doric
Department of Electronics, University of Split, Croatia

Abstract

The aim of this paper is to develop an accurate and efficient model of base station antenna systems for analysis purposes, due to a growing concern regarding their potentially hazardous impact on human health. The formulation of the problem is based on the set of Pocklington integro-differential equations. This equation has been numerically treated by the weak Galerkin-Bubnov formulation of the Finite Element Method for integro-differential operators (sometimes referred to as the indirect Boundary Element Method). The electric near-field and electric far-field results have been calculated throughout this work and have been compared to the results obtained using the commercial software NEC Win-Pro, and to other results available in references. Results obtained using the model presented in this paper show good compliance with respect to referent values. The model developed in this work could be further extended for handling even more complex geometries, but it is already applicable to many real world problems.

1 Introduction

An increasing number of GSM and UMTS base station antenna systems and potential hazardous influence on human health has recently caused a growing public concern. Therefore, the analysis of the intensity and form of radiated electromagnetic energy is of great interest to biological studies [1].

Some simplified analytical approaches to base station antenna analysis have been reviewed in [1], while interesting numerical treatment has been presented in [2] and [3].
In this paper an efficient numerical model for the treatment of an antenna array in front of reflector problem has been developed. This model is based on the Pocklington integro-differential equation that is solved by the weak Galerkin-Bubnov scheme of the Finite Element Method for integro-differential operators [4]-[6]. Problem is solved for two different cases: antenna array in front of a wire grid reflector and antenna array in front of a half-space reflector.

The analysis starts with a calculation of the equivalent current distribution along the wires. Once the current distribution is known, it is easy to compute electric near-field and far-field values. These field values are calculated using far-field approximation, thus avoiding the Sommerfeld integrals that are quite difficult and time-consuming to solve. By doing this, some error is introduced, so one has to take that into account when using this procedure. Various illustrative numerical results are presented in the paper.

2 Mathematical model

Two different models are used in this paper. In the first case antenna array is set up in front of wire grid reflector and this model represents an accurate problem that can be found in praxis. The second problem represents simplified solution of antenna in front of reflector case. In this case antenna is set up in front of conducting half-space, which represents the reflector. This approach is less time-consuming and less accurate, respectively. The analysis starts with a case of a free space. The both methods can be obtained as an extension of the free space case. Figure 1 shows a vertical antenna array of $M$ dipoles of length $L_m$, distance between centres of adjacent dipoles $d_c$, and horizontal distance between antennas $d_h$, insulated in free space. The currents induced along the antenna elements is governed by the set of coupled Pocklington integro-differential equations [1]:

\[
E_{zm}^i = -\frac{1}{j4\pi\omega\varepsilon_0} \left( \frac{\partial^2}{\partial z^2} + k_i^2 \right) \sum_{n=1}^{M} \int_{-L_n/2}^{L_n/2} I_n(z')g_{0mn}(z,z')dz', \quad m=1,2,...,M \quad (1)
\]

where $I_n(z')$ is the current distribution along the n-th wire, Green’s function $g_{0mn}(z,z')$ is given by:

\[
g_{0mn}(z,z') = \frac{e^{jkR_{mn}}}{R_{mn}} \quad (2)
\]

and

\[
R_{mn} = \sqrt{(z-z')^2 + a_m^2} \quad m = n \quad R_{mn} = \sqrt{(z-z')^2 + D_{mn}^2} \quad m \neq n \quad (3)
\]

Equation (1) is solved following the procedure given in [5], [6] and equivalent current distribution along antenna is obtained. Once the equivalent current distribution is known, the radiated electric field can be computed [6].
3 Numerical results: far-field pattern analysis

The far-field pattern results of an antenna located in front of perfectly conducting half-space and wire grid reflector are presented. Imperfectly conducting half-space reflector is dropped out because of the restrictions introduced by Fresnel's coefficient (if a distance between reflector and antenna is less than $0.5\lambda$, results are not accurate) [2]. In practice the distance is up to $0.25\lambda$, and if the distance is greater a primary aim of this configuration is changed, i.e. radiation pattern changes completely and the reflector purpose is lost. A comparison of results will be made for the following geometries.

First example is related to a driven antenna array consisting of eight half-wave dipole antennas spaced by $0.75\lambda$ (between centres), with radius of $0.004\lambda$. This array is distanced from reflector by $0.25\lambda$. Two reflectors are used: perfectly conductive half-space – in this case there is an image of driven antenna array; and wire grid reflector – it consists of 46 wires in both wings plus mast. Wires radius is $0.004\lambda$, and mast radius is $0.0762\lambda$. Wire reflector is flat - angle between wings is $180^\circ$. Height of this reflector is $6.25\lambda$. Calculations are performed using the unity voltage generators located in a feeding gap of each dipole and the applied frequency is $900$ MHz.

Figure 2 shows the vertical radiation patterns for the models used. There is a difference in gain in main lobe direction, as well as, in side lobes directions. This difference is around $1$ dB, in favour of perfectly conducting half-space reflector.
There is also the rear lobe for the case when wire grid reflector is used. This lobe is created by a radiation from the wire grid structure, because a current induced on reflector wires is due to the interaction with the driven antenna array. In the Figure 3 a horizontal radiation pattern is depicted.

Figure 2: Vertical radiation pattern.

Figure 3: Horizontal radiation pattern.
The horizontal radiation pattern shows more obvious discrepancy between results obtained via different models. In the case of half-space reflector half-power beam width (HPBW) is wider and equal to 120° and in the case of the wire grid reflector it is a few degrees less. There is also some radiation behind the antenna that is taken into account with the wire grid reflector. If a gain in some narrow angle in the main lobe direction is of interest, then half-space approximation can be used. One must bear in mind the gain overestimation in that case. That overestimation is around 1 dB.

Another example is based on configuration reported in [3] where some simulation results for base station antenna far and near fields are available. Far field results for the model presented in [3] will be calculated using the BEM research code and compared to results available. In this case there are eight dipoles set up along z-axis in front of wire grid reflector with an angle between reflector wings equal to 180°. Complete geometry data is given in the Table 1.

<table>
<thead>
<tr>
<th>Mast diameter(D)</th>
<th>0.0762 λ₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflector angle (α)</td>
<td>180°</td>
</tr>
<tr>
<td>Wire radius</td>
<td>0.004 λ₀</td>
</tr>
<tr>
<td>Driven dipole length</td>
<td>0.5 λ₀</td>
</tr>
<tr>
<td>Reflector height (h_{ref})</td>
<td>6.25 λ₀</td>
</tr>
<tr>
<td>Reflector width (L)</td>
<td>0.185 λ₀</td>
</tr>
<tr>
<td>Distance between driven dipole and mast surface (y_{dia})</td>
<td>0.25 λ₀</td>
</tr>
</tbody>
</table>

Figure 4: Vertical gain pattern.
Reflector is consisted of vertical wires. In each wing there are 23 wires, excluding mast. Calculations were performed with 100W of antenna input power.

Results obtained using BEM software are shown in Figure 4 and Figure 5.

![Figure 4: Horizontal gain pattern.](image)

Figure 5: Horizontal gain pattern.

Results, obtained using BEM research code slightly differ from the results published in [3].

When considering vertical pattern, gains in the main lobe direction and in the first side lobe direction are lower for BEM results by order of 0.65 - 1 dB. The difference increases even more in other side lobe directions. There is also a certain difference in the rear lobe direction.

In the case of BEM results the rear lobe gain is -7dB, and in [3] the rear lobe gain is -4 dB. In horizontal pattern also shows a difference in the rear lobe direction. These differences in results arise due to the difference in the wire grid reflector modelling. A discrepancy due to the different solution methods used for calculations must be considered, as well.

4 Numerical results: near-field pattern analysis

The near field of the antenna geometry given in [3] is calculated and analyzed. Calculations were carried out along 0°, 90° and 180° azimuth axes of the antenna up to distance of 10m from the antenna, and heights from 0 to -10m from the antenna centre, with resolution of 0.05m.

Figure 7 shows the electric field along the x-axis at z=0m. Electric field results are given for points distanced up to 10 m from antenna. Results around x=0 must be discarded due to the error caused by quasi-singularity problems.
arising in the Green’s kernel. In the Figure 8 and Figure 9 electric field along x-axis is shown for the heights $z=-5m$ and $z=-10m$ below antenna centre. From the electric field it is obvious that the field decreases as the distance from the antenna increases. Figures 10 to 12 show the electric field along positive y-axis for three heights: $z=0m$, $z=-5m$ and $z=-10m$. When the electric field strength is calculated for $z=0m$ a quasi-singularity occurs due to the small radius. To avoid the problem it is necessary to shift the calculation path along z-axis for some value. A test has been carried out in order to obtain the most appropriate value and the results obtained are given in Figure 13.

![Figure 6: Areas of near field calculation.](image)

![Figure 7: Results for $\phi = 0^\circ, 180^\circ$, $z = 0m$.](image)
From the Figure 13 it is obvious that for $z=-0.1m$ the electric field is the highest and thus the most accurate one.

Results attained by these calculations slightly differ from those given in paper [3]. This is due to the model differences and due to a different approach used for these calculations. But the important fact is that shapes of these near field patterns look very similar to those given in paper [3].

5 Conclusion

The paper deals with an analysis of base station antenna radiation pattern. The mathematical model is based on the corresponding set of Pocklington integro-differential equations. This set of equations is numerically handled via the
Galerkin-Bubnov variant of the indirect boundary element method. The near and far field pattern analysis has been carried out and the various illustrative computational results have been presented throughout this work. Due to the unavailability of real antenna geometry data, method used in this paper is checked against the already proven method. The resemblance between results obtained using two different methods is judged to be satisfying.

![Figure 10](image1.png)

**Figure 10:** Results for $\phi = 90^\circ$, $z = 0$ m.

![Figure 11](image2.png)

**Figure 11:** Results for $\phi = 90^\circ$, $z = -5$ m.

The model developed in this work could be further extended for handling even more complex geometries, but it is already applicable to many real-world problems. In future this work could be extended to even more realistic models, depending on the level of accuracy needed.
Figure 12: Results for $\phi = 90^\circ$, $z = -10$ m.

Figure 13: Comparison of shift values (along z-axis).

References


[3] Šarolić, Antonio; Modlic, Borivoj; Poljak, Dragan. Installation Uncertainty of a Base Station Antenna Radiation Pattern Proc 12th


