A simplified calculation of transient plane waves in a presence of an imperfectly conducting half-space

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Abstract

Transient behaviour of plane waves in the presence of a two media configuration is analysed in this work. The time domain formulation of the problem is based on simplified reflection and transmission coefficient expressions arising from the modified image theory. Knowing the impulse response of the finitely conducting half-space the reflected and transmitted fields respectively, can be obtained by using a convolution operator. Some illustrative numerical results for transient earth-reflected waves are presented.
Keywords: Transient analysis, plane waves, reflection and transmission coefficients, modified image theory, imperfectly conducting half-space.

1 Introduction

In many electromagnetic compatibility (EMC) applications it is important to determine the transient behaviour of plane waves reflected from imperfectly conducting earth [1] or transient waveform fields penetrating into a lossy medium [2]. The analysis of electromagnetic waves behaviour in the presence of lossy media can be modelled via the rigorous Sommerfeld integral approach [3], [4] or via the approximate reflection and transmission coefficient approach [5], [6] and can be carried out in the frequency or in the time domain, respectively.
One of the most cited papers dealing with a frequency domain analysis of transient waves reflected from a lossy half-space is the work performed by Dudley et al. [7].

Yet direct time domain modelling is a more demanding task in both analytical and numerical sense, it certainly provides a deeper insight into the particular

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electromagnetic phenomenon itself and some computational advantages, as well [8]-[10].

A key point in a direct time domain calculation of reflected and transmitted waves is the knowledge of a lossy medium impulse response. The impulse response of a dissipative half-space is given in terms of the time dependent reflection and transmission coefficients, respectively. Barnes and Tesche [1] have derived the approximate analytical expressions for the transient earth-reflected impulsive plane waves for both vertical and horizontal polarization.

This paper deals with a simplified calculation of time domain reflection and transmission coefficients waveforms arising from the modified image theory [11]. First, the frequency domain expressions for the reflection and transmission coefficients are inverted into the time domain performing the analytical inverse Laplace transform thus providing the impulse response of a finitely conducting half-space. Transient plane waves are then obtained taking the convolution of the incident field. Some illustrative numerical results are given in the paper.

2 Transient fields in a presence of a lossy medium

The earth-reflected electric field or field transmitted into the earth $E^{ref,tr}(s)$ can be obtained from an incident electric field E^{inc} as follows [1]:

$$E^{ref,tr}(s) = \Gamma(s)E^{inc}(s) \tag{1}$$

where $\Gamma(s)$ is the reflection or transmission coefficient, respectively, while s denotes complex frequency Laplace transform variable.

Figure 1 shows the incident field decomposed into components having vertical and horizontal polarization of the electric field vectors.

The transient counterpart of equation (1) for the reflected and transmitted electric field component is given as the convolution of the incident field and the inverse Laplace transform of the reflection or transmission coefficient, respectively, as follows:

$$E^{ref,tr}(t) = \int_0^t \Gamma(\tau)E^{inc}(t-\tau)d\tau \tag{2}$$

where $\Gamma(t)$ is the corresponding temporal reflection or transmission coefficient, respectively.

From the linear system point of view, the time domain function $\Gamma(t)$ can be referred to as the impulse response of a finitely conducting half-space.

The total field at an arbitrary point above the interface is composed of both incident and reflected field, respectively. Thus, the total electric field vector in the air can be written as:

$$\vec{E}^{tot}(t) = \vec{E}^{inc}(t) + \vec{E}^{ref}(t^*) \tag{3}$$

where t^* stands for a retarded time, required for the reflected field to travel from the interface to a nearby observation point.

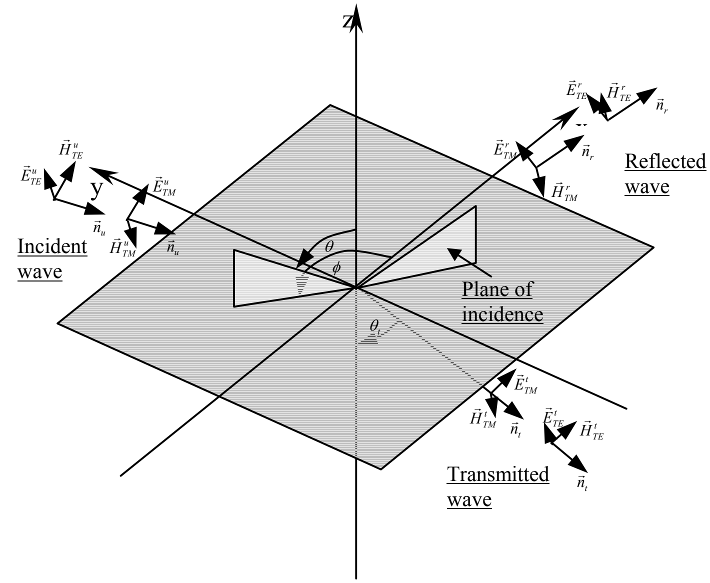

Figure 1: Incident, reflected and transmitted wave.

3 Impulse response of a lossy half-space

To obtain the related impulse responses of a finitely conducting earth one starts from the frequency dependent reflection and transmission coefficients.

Simplified expressions for frequency dependent reflection and transmission coefficients can be obtained by using the modified image theory [11]. This approximation is stated to be valid for the frequencies up to order of MHz [12]. The reflection coefficient due to an air-earth interface is given by:

$$R = \frac{\varepsilon_{eff} - \varepsilon_0}{\varepsilon_{eff} + \varepsilon_0} \tag{4}$$

where ε_{eff} is the complex permittivity of the earth given by:

$$\varepsilon_{eff} = \varepsilon_r \varepsilon_0 + \frac{\sigma}{j\omega} = \varepsilon_r \varepsilon_0 + \frac{\sigma}{s} \tag{5}$$

Similarly, the reflection coefficient by which an earth-air interface is taken into account is given by:

$$R = -\frac{\varepsilon_{eff} - \varepsilon_0}{\varepsilon_{eff} + \varepsilon_0} \tag{6}$$

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If the fields penetrating into lossy media are of interest a transmission coefficient is needed. The transmission coefficient air-earth is given by:

$$T = \frac{2\varepsilon_{eff}}{\varepsilon_{eff} + \varepsilon_0} \tag{7}$$

Finally, the transmission coefficient by which the earth-air interface is taken into account can be written as:

$$T = \frac{2\varepsilon_0}{\varepsilon_{eff} + \varepsilon_0} \tag{8}$$

Contrary to the most of the methods cited in the available publications the calculation of reflection coefficients (4) and (6) and transmission coefficients (7) and (8) can be performed analytically.

The time domain counterparts of these coefficients, i.e. the related impulse responses, are given by applying the inverse Laplace transform to the relations (4), (6) and (8).

The air-earth or earth-air reflection coefficients can be re-arranged as follows:

$$R(s) = \pm\frac{\tau_1 s + 1}{\tau_2 s + 1} \tag{9}$$

where τ_1 and τ_2 are the time constants characterized by a lossy medium and are defined by relations:

$$\tau_1 = \frac{\varepsilon_r - 1}{\sigma}\varepsilon_0 \tag{10}$$

$$\tau_1 = \frac{\varepsilon_r + 1}{\sigma}\varepsilon_0 \tag{11}$$

To invert the expression (9) into the time domain one must use the following inverse Laplace transform theorem:

$$L^{-1}\left[F(s+a)\right] = e^{-at} f(t) \tag{12}$$

The inverse Laplace transform applied to the equation (9) yields:

$$R(t) = \pm\left[\frac{\tau_1}{\tau_2}\delta(t) + \frac{1}{\tau_2}\left(1 - \frac{\tau_1}{\tau_2}\right)e^{-t/\tau_2}\right] \tag{13}$$

where $\delta(t)$ denotes the Dirac delta impulse.

The mathematical details for the analytical evaluation of the inverse Laplace transform for equation (9) are given in Appendix, for completeness.

In addition, the air-earth transmission coefficient (7) can be, for convenience, rewritten as;

$$T(s)=\frac{\tau_3 s}{\tau_2 s+1} \tag{14}$$

where τ_3 is the time constant given by:

$$\tau_3=\frac{2\varepsilon_r}{\sigma}\varepsilon_0 \tag{15}$$

The corresponding temporal counterpart of equation (14) obtained by means of the inverse Laplace transform is then given by:

$$T(t)=\frac{\tau_3}{\tau_2}\delta(t)+\frac{1}{\tau_2}\left(1-\frac{\tau_3}{\tau_2}\right)e^{-t/\tau_2} \tag{16}$$

Finally, the earth-air transmission coefficient (8) can be written as:

$$T(s)=\frac{\tau_4 s}{\tau_2 s+1} \tag{17}$$

where τ_4 is the time constant given by:

$$\tau_4=\frac{2\varepsilon_0}{\sigma} \tag{18}$$

Applying the inverse Laplace transform to equation (17) yields:

$$T(t)=\frac{\tau_3}{\tau_2}\left[\delta(t)-\frac{1}{\tau_2}e^{-t/\tau_2}\right] \tag{19}$$

It is worth noting that expressions (13), (16) and (17) contain two terms; the Dirac impulse term which is independent of earth conductivity, and the term exponentially decaying in time.

4 Calculation example: Transient plane wave reflected from the lossy earth

The reflection of double exponential transient electric field is considered as it is shown in Figure 2.

The calculation example is related to the case of the normal incidence (ψ=0, θ=0). The incident wave is given in the form of the double exponential transient electric field:

$$E^{inc}(t) = E_0\left(e^{-at} - e^{-bt}\right) \tag{20}$$

where the EMP parameters are: E_0=52.5kV/m, a=4*10^6s^{-1}, b=4.78*10^8s^{-1}.

Inserting the expressions (13) and (20) into the convolution integral (2) for a general incident field results in the following expression for the earth reflected field:

$$E^{ref}(t) = \int_0^t \left[\frac{\tau_1}{\tau_2}\delta(t) + \frac{1}{\tau_2}\left(1 - \frac{\tau_1}{\tau_2}\right)e^{-\tau/\tau_2}\right]E_0\left(e^{-a(t-\tau)} - e^{-b(t-\tau)}\right)d\tau \tag{21}$$

The solution of the convolution integral (18) can be carried out in a close form and it is given in the form:

$$\begin{aligned} E^{ref}(t) &= \frac{\tau_1}{\tau_2}E_0\left(e^{-at} - e^{-bt}\right) + \\ &E_0\left(1 - \frac{\tau_1}{\tau_2}\right)\left[\left(\frac{1}{\tau_2 a - 1} - \frac{1}{\tau_2 b - 1}\right)e^{-t/\tau_2} - \left(\frac{e^{-at_2}}{\tau_2 a - 1} - \frac{e^{-bt_2}}{\tau_2 b - 1}\right)\right] \end{aligned} \tag{22}$$

As only the reflected field is considered, the incident field term, field vectors, and the propagation time delay are ignored in this work.

Figure 2 shows the transient earth-reflected field for the various values of the earth conductivity. The relative permittivity of the earth is assumed to be constant and equals ε_r=10.

Typical values of a lossy earth conductivity σ vary from 1 to 100mS/m while the corresponding permittivity values range from 10 to 30 [1].

These results for transient reflected fields seem to be in a satisfactory agreement with the results published in [1].

5 Conclusion

The time domain analysis of a transient plane waves in the presence of a lossy half-space is presented throughout this work. The formulation of the problem is based on the simplified reflection and transmission coefficients, respectively by which the dissipative half-space have been taken into account.

This approach involves first the evaluation of the lossy half-space impulse response and then the calculation of transient electric fields obtained by the use of convolution of this impulse response with the incident field waveform.

Some illustrative numerical results are also presented in the paper.

A future work will be devoted to the application of transient analysis to an arbitrary wire configuration radiating above or buried in the medium with finite conductivity that is a problem of great practical importance in electromagnetic compatibility.

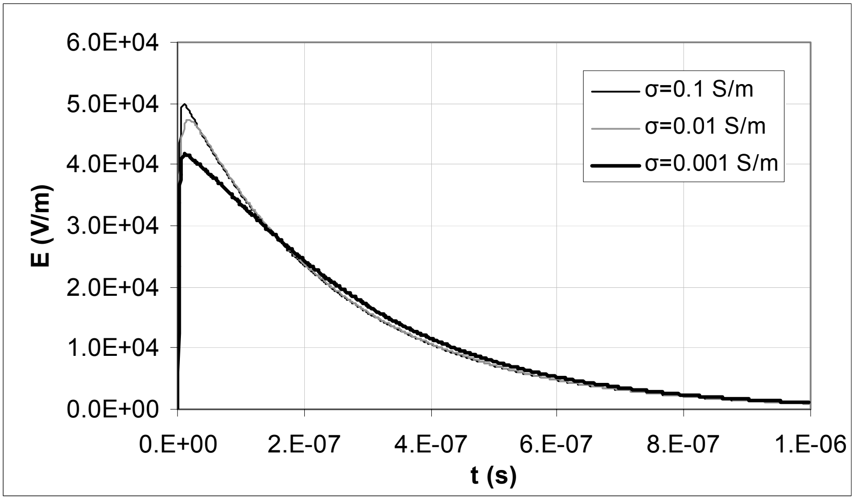

Figure 2: Transient plane wave reflected from the lossy ground with ε_r=10 for the various values of conductivity σ.

Appendix

The expression for air-earth and earth-air reflection coefficient in the Laplace domain is written in the form:

$$R(s) = \pm \frac{\tau_1 s + 1}{\tau_2 s + 1} \tag{A1}$$

For convenience, it can be re-written as follows:

$$R(s) = \pm\left(\tau_1 \frac{s}{\tau_2 s + 1} + \frac{1}{\tau_2 s + 1}\right) = \pm\left(\frac{\tau_1}{\tau_2}\frac{s}{s + \dfrac{1}{\tau_2}} + \frac{1}{\tau_2}\frac{s}{s + \dfrac{1}{\tau_2}}\right) \tag{A2}$$

Now, using the addition-subtraction technique one obtains the following expression:

$$R(s) = \pm \frac{\tau_1}{\tau_2}\left(1 - \frac{1}{\tau_2}\frac{1}{s + \frac{1}{\tau_2}}\right) + \frac{1}{\tau_2}\frac{s}{s + \frac{1}{\tau_2}} \tag{A3}$$

Using the well-known shifting theorem of the inverse Laplace transform:

$$L^{-1}[F(s+a)] = e^{-at} f(t) \tag{A4}$$

the time domain counterpart of the equation (A3) becomes:

$$R(t) = \pm\left[\frac{\tau_1}{\tau_2}\delta(t) + \frac{1}{\tau_2}\left(1 - \frac{\tau_1}{\tau_2}\right)e^{-t/\tau_2}\right] \tag{A5}$$

Similarly, the air-earth and earth-air transmission coefficient in the Laplace domain is given by:

$$T(s) = \frac{\tau_n s}{\tau_2 s + 1} \tag{A6}$$

where τ_n stands for time constants τ_3 or $\tau 4$, respectively.

Furthermore equation (A6) can be re-written as:

$$T(s) = \tau_n \frac{s}{\tau_2 s + 1} = \frac{\tau_n}{\tau_2}\frac{s}{s + \frac{1}{\tau_2}} \tag{A7}$$

and again, using the addition-subtraction technique results in:

$$T(s) = \frac{\tau_n}{\tau_2}\left(1 - \frac{1}{\tau_2}\frac{1}{s + \frac{1}{\tau_2}}\right) \tag{A8}$$

Finally, use of the shifting theorem (A4) leads to the time domain counterpart of the equation (A8):

$$T(t) = \frac{\tau_n}{\tau_2}\left[\delta(t) - \frac{1}{\tau_2}e^{-t/\tau_2}\right] \tag{A9}$$

References

[1] P.R. Barnes, F.M. Tesche: "On the Direct calculation of a Transient Plane Wave Reflected from a Finitely Conducting Half-Space", *IEEE Trans. EMC,* Vol. 33, No 2., pp. 90-96, May 1991.

[2] G.E. Bridges: "Transient Plane Wave Coupling to Bare and Insulated Cables Buried in a Lossy Half-Space", *IEEE Trans. EMC,* Vol. 37, No 1., pp. 62-70, Feb. 1995.

[3] G.J. Burke, E.K. Miller, "Modelling Antennas near to and Penetrating a Lossy Interface", *IEEE Trans. AP,* Vol. 32, No 10, pp. 1040-1049, Oct. 1984.

[4] D. Poljak, V. Roje, "The integral equation method for ground wire input impedance", in *Integral Methods in Science and Engineering, Vol I Analytic Methods,* C. Constanda, J. Saranen, S. Seikkala (Ed), Longman 1997, pp. 139-143.

[5] E.K. Miller, A.J. Poggio, G.J. Burke, E.S. Selden, "Analysis of Wire Antennas in the Presence of a Conducting Half-Space, Part II. The Horizontal Antenna in Free Space", *Canadian Journal of Physics*, Vol. 50, pp 2614-2627, 1972.

[6] D. Poljak, "Electromagnetic Modeling of Finite length Wires Buried in a Lossy Half-Space", *Engineering Analysis with Boundary Elements*, Vol 26, pp. 81-86, Jan 2002.

[7] Dudley et al: "On the Interaction of a Transient Electromagnetic Plane Wave and a Lossy Half-Space", *Jape. Phys.* Vol. 45, No 3., March 1974.

[8] D. Poljak, E.K. Miller, C.Y. Tham: "Time Domain Energy Measures for Thin-Wire Antennas and Scatterers", *IEEE Antennas & Propagation Magazine,* Vol 44, No1, pp 87-95, Feb 2002.

[9] D. Poljak, C.Y. Tham, *Integral Equation techniques in Transient Electromagnetics, WIT Press*, Southampton-Boston, 2003.

[10] D. Poljak, C.Y. Tham, A. McCowen: "Transient Response of Nonlinearly Loaded Wires in a Two Media Configuration", *IEEE Trans, EMC* vol. 46, 1, pp. 2004.

[11] T. Takashima, T. Nakae, R. Ishibasi: "Calculation of Complex Fields in Conducting media", *IEEE Trans. El. Insul,* Vol. 15, No 1., pp. 1-7, Feb. 1980.

[12] L.D. Grcev, F.E. Menter: "Transient electromagnetic Fields Near Large Earthing Systems", *IEEE Trans. Magnetics,* Vol. 32, pp. 1525-1528, May 1996.