A 3D BEM modelling of human exposure to extremely low frequency (ELF) electric fields

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Abstract

The boundary element analysis of the human body exposed to extremely low frequency (ELF) electric fields is presented in this work. The human being is represented by a multidomain inhomogeneous body of revolution. The formulation of the problem is based on the quasi-static approximation and the related Laplace equation form of the continuity equation. This Laplace equation has been solved by the boundary element method (BEM). Solving the resulting Laplace equation for the electric scalar potential, the induced current density inside the human body is obtained. This quantity is the fundamental parameter in further analysis of possible biological effects of ELF exposures. An illustrated computational example of the human body exposed to the electric field generated by an overhead power line is also presented.

1 Introduction

The tremendous growth in the use of electrical energy for industrial applications is associated with the continuous presence of extremely low frequency (ELF) electric fields in the environment.

At low frequency exposures, i.e. when displacement currents can be neglected, the electric and magnetic fields are assumed to be decoupled. Generally, human being can be exposed to two kinds of electromagnetic fields generated by low frequency (LF) power systems: 1) low voltage/high intensity systems (The principal radiated field is the magnetic one, while the induced currents form close loops in the body); 2) high voltage/low intensity systems (The principal radiated field is the electric one while the induced currents have the axial character).
As the magnetic field penetrating into the human body remains mostly unchanged, the evaluation of human exposure to electric fields is much more difficult than to magnetic fields due to the electric field perturbation by the human body, other objects in the environment and the measuring device [5].

This paper deals with human exposure assessment to high voltage ELF fields. Generally, high voltage ELF fields are used for power utilities (transmission, distribution and applications) and for strategic global communications with submarines submerged in sea water. The impact of these fields on humans has caused an increasing public concern associated with the possible adverse health effects due to the exposure to these fields [1].

This subject generated much controversy on the possible link between the fields and health risk, particularly leukaemia and certain forms of cancer in human.

Basically, human exposure to high voltage ELF electric fields results in induced fields and currents in all organs. These induced currents and fields may give rise to thermal and nonthermal effects. While the thermal effects seem to be negligible, certain nonthermal effects often related to the cell level are still possible [1]-[4].

A number of epidemiological studies have been carried out to establish possible links between high voltage ELF exposure and cancer risk (leukaemia and nervous tissue tumour. Some of these studies are quoted in references [3]-[4]).

However, the biological effects of ELF fields still remain unclear as far as the long term effects are of interest.

In particular, the current density is, according to the ICNIRP basic restrictions [2], main parameter for quantifying the bio-effects for ELF exposures.

The calculation of the current density induced in the human body when exposed to ELF fields has been previously reported by several researchers, using either analytical [3]-[4], or numerical techniques [5]-[11].

King and Sandler [3]-[4] proposed the parasitic antenna model of the human body exposed to the ELF and VLF sources deriving some closed-form expressions for the induced current density inside the body. Chiba et al. [6] developed a Finite Element Method (FEM) based inhomogeneous body model for the calculation of the induced current density inside human beings exposed to 60Hz electric fields. Some advances in this model were reported in [7] and [8]. Gandhi and Chen [9]-[10] developed the realistic model of the human body and used the finite difference time domain (FDTD) method for the numerical solution of the problem. A simplified and numerically efficient thick-wire antenna model of the human body exposed to the electromagnetic fields has been proposed by Poljak and Rashed in [11].

On the contrary, not only to the simplified wire antenna approach [11], but also to the computationally very expensive domain methods such as FEM [6]-[8] and FDTD [9]-[10] this paper deals with an efficient multi-domain human body representation based on a multidomain implementation of the Boundary Element Method (BEM) [12]. This efficient BEM scheme is more sophisticated approximation than FDTD and at the same time computationally less expensive.
than FEM, as only the domain boundary has to be discretised.

It is worth pointing out, to the best of our knowledge, that this is one of the first application of the BEM to the modelling of the human body exposure to high voltage ELF electric fields. One of the biggest advantages of the domain decomposition techniques, such as BEM, further to its capabilities in dealing with piecewise homogeneous material properties, is that the final system of equations is sparse and highly bounded.

The formulation of the problem is based on the quasi-static approximation of the ELF electric field and on the related continuity equation for the induced current density. This continuity equation can be reduced to the Laplace equation for the electric scalar potential. The resulting equation is then numerically handled via the BEM based on the domain decomposition concept [11]-[12].

Once obtaining the electric scalar potential distribution along the body, as a solution of the corresponding Laplace equation, one can readily calculate the induced current density inside the body. An illustrative numerical example of human exposure to power line electric field is presented in this paper.

2 Multi-domain model of the human body

The human body consists of the various tissues and organs with varying electrical properties in terms of conductivity $\sigma$ and relative permittivity $\varepsilon_r$. At ELF frequencies near the power frequency $f = 50$ Hz (or 60 Hz in the US) all organs in the body behave as good conductors with related values of conductivity.

The human body model used in this work is similar to the one presented in [6] and consists of nine portions in total, as shown in Fig 1. (Dimensions are given in centimetres). The corresponding electrical conductivities of each part of the body are given in Table 1.

<table>
<thead>
<tr>
<th>Body portion</th>
<th>Region</th>
<th>Conductivity $\sigma$ [S/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head</td>
<td>I, II</td>
<td>0.12</td>
</tr>
<tr>
<td>Neck</td>
<td>III</td>
<td>0.6</td>
</tr>
<tr>
<td>Shoulders</td>
<td>IV</td>
<td>0.04</td>
</tr>
<tr>
<td>Thorax</td>
<td>V</td>
<td>0.11</td>
</tr>
<tr>
<td>Pelvis and crotch</td>
<td>VI</td>
<td>0.11</td>
</tr>
<tr>
<td>Knee</td>
<td>VII</td>
<td>0.52</td>
</tr>
<tr>
<td>Ankle</td>
<td>VIII</td>
<td>0.04</td>
</tr>
<tr>
<td>Foot</td>
<td>IX</td>
<td>0.11</td>
</tr>
</tbody>
</table>

It is worth emphasizing that dielectric properties are assumed to be negligible, i.e.: $\sigma \gg \omega \varepsilon$. Also, the conductivities of the body tissues are constant at any part of the body, as shown in Fig 1. The quasi-static approximation can be used since
the dimension of the body model is electrically short, i.e. rather small compared to the wavelength of the impressed field. Upon this assumption the static approximation for the ELF exposure model can be formulated via the Laplace type equation.

Figure 1: Multidomain model of the body and conducting properties of different parts at ELF exposures.

Nevertheless, the body is exposed to all six components of the electric field. It has already been shown in [3], [4] and [11] (and documented in more details in [13]) that the field component parallel to the upright cylindrical body axis, is the largest within the practical ranges of body height. Consequently, the other components can be neglected.

The impressed electric field generated by the power line is assumed to be spatially uniform along the body and varying with angular frequency $\omega$, i.e. the $\exp(j\omega t)$ time dependence is considered.

Therefore, the electric field over flat ground plane is assumed to be vertical and uniform near the ground level and, the human body is located between the parallel plate electrodes, in the middle of the lower one. A calculation domain with the corresponding boundary conditions is shown in Fig 2.

The lower plate electrode is assumed to be at zero potential while the upper plate electrode is assumed to be at the potential of a high voltage power line.

2.1 The equation of continuity

The continuity equation is usually given in the form:

$$\nabla J + \frac{\partial \rho}{\partial t} = 0$$

where $J$ is the current density and $\rho$ represents the volume charge density.
The induced current density can be expressed in terms of the scalar electric potential using the constitutive equation (Ohm’s Law) \( J = -\sigma \nabla \varphi \), where \( \sigma \) is the conductivity of the medium.

The volume charge density \( \rho \) and electric scalar potential \( \varphi \) are related through the Poisson equation:

\[
\rho \varphi = \nabla \cdot (\varepsilon \nabla \varphi) = 0
\]  

where \( \varepsilon \) is the corresponding permitivity of the medium.

Combining the equations (1)-(2), the equation of continuity becomes:

\[
\nabla \left( \sigma \nabla \varphi \right) + \frac{\partial}{\partial t} \nabla (\varepsilon \nabla \varphi) = 0
\]  

Finally, for the time-harmonic ELF exposures it follows:

\[
\nabla \left[ \left( \sigma + j\omega \varepsilon \right) \nabla \varphi \right] = 0
\]  

where \( \omega = 2\pi f \) is the operating frequency.

In the ELF range all organs behave as good conductors and the continuity equation (4), in accordance to the quasi-static approximation, simplifies into Laplace equation of the form:

\[
\nabla (\sigma \nabla \varphi) = 0
\]  

On the contrary, the surrounding air is a lossless dielectric medium and the corresponding governing equation is:

\[
\nabla (\varepsilon \nabla \varphi) = \nabla^2 \varphi = 0
\]
Solving the Laplace equation (6) in the body region, the induced current density can be obtained from Ohm’s Law.

### 2.2 The air-body interface conditions

The problem of human being coupling to high voltage ELF fields described by the continuity equation (5) can be analysed by specifying the boundary rules at discontinuities in material properties of conducting and dielectric medium.

The boundary condition for the tangential component of the electric field near the two-media interface is given by:

\[
\mathbf{n} \times (\mathbf{E}_b - \mathbf{E}_a) = 0
\]

where \( \mathbf{n} \) is the unit normal to the interface and \( \mathbf{E}_a \) and \( \mathbf{E}_b \) represent the fields in the air and in the body, respectively.

Expressing the electric field in terms of scalar potential, it follows:

\[
\mathbf{n} \times (\nabla \varphi_b - \nabla \varphi_a) = 0
\]

The boundary condition for the normal component of the induced current density near the body-air surface is given by:

\[
\mathbf{n} \cdot \mathbf{J} = -j \omega \rho_s
\]

where \( \rho_s \) denotes the surface charge density.

Expressing the current density in terms of scalar potential:

\[
\sigma_b \mathbf{n} \nabla \varphi_b = -j \omega \rho_s
\]

where \( \sigma_b \) is the corresponding tissue conductivity, and \( \varphi_b \) is the scalar potential at the body surface.

The boundary condition for the normal component of the electric flux density at the air-body surface is:

\[
\mathbf{n} \cdot \mathbf{D} = \rho_s
\]

or, expressing the electric flux density in terms of scalar potential it follows:

\[
\varepsilon_0 \mathbf{n} \nabla \varphi_a = \rho_s
\]

where \( \varphi_a \) and \( \rho_s \) denote the potential in the air in the close proximity of the body.
3 The boundary element method

The multidomain body model is handled numerically by means of a 3D implementation of the boundary element method (BEM) with domain decomposition [12]. This decomposition technique has been applied in order to deal with the inhomogeneous human body model, taking into account different conductivities. In case of complicated geometry, an extreme decomposition technique has been proposed in order to yield sparse system of equations, thus avoiding large fully populated matrices which are common in the standard BEM.

The problem consists of finding the solution of the Laplace equation in a non-homogenous media with prescribed boundary conditions

$$\nabla \cdot (\sigma \nabla \phi) = 0 \text{ on } \Omega$$

$$\phi = \phi^* \text{ on } \Gamma_1 \text{ and } \frac{\partial \phi}{\partial x_j} n_j = \frac{\partial \phi^*}{\partial n_j} \text{ on } \Gamma_2$$

(13)

The integration domain is considered piecewise homogeneous, so it can be decomposed into an assembly of $N$ homogeneous subdomains $\Omega_k (k = 1, m)$.

Using Green’s theorem for scalar functions, the following integral representation for a subdomain can be written:

$$c(\xi)\phi(\xi) + \int_{\Gamma_k} \phi \frac{\partial \phi^*}{\partial n} d\Gamma = \int_{\Gamma_k} \frac{\partial \phi}{\partial n} \phi^* d\Gamma$$

(14)

where $\phi^*$ is the 3D fundamental solution of Laplace equation, $\frac{\partial \phi^*}{\partial n}$ is the derivative in normal direction to the boundary, and $c(\xi)$ is the geometrically dependent free term accounting for the Cauchy type singularity of the integral on the left hand side of eqn (14).

Discretization of eqn (14) with $N_k$ elements leads to an integral relation:

$$c_i \phi_i + \sum_{j=1}^{N_k} \int_{\Gamma_{k,j}} \phi \frac{\partial \phi^*}{\partial n} d\Gamma = \sum_{j=1}^{N_k} \int_{\Gamma_{k,j}} \frac{\partial \phi}{\partial n} \phi^* d\Gamma$$

(15)

where $i$ stands for the source point and $\Gamma_{k,j}$ represents the $j$-th boundary element of $\Omega_k$. The present implementation is based on the isoparametric approach with quadratic interpolation functions defined for triangular elements.

With quadratic triangular elements, the potential, or its normal derivative, at any point of the $j$-th boundary element can be written in terms of their corresponding values at the 6 collocation nodes ($a=1,..,6$) by means of the interpolation functions $\psi_a$. 
The dimensionless coordinate $\xi$ spans from the computational square domain to the physical quadratic triangular element.

Combining equations (15) and (16) one obtains the following system of equations for each subdomain:

$$H\phi - G \frac{\partial \phi}{\partial n} = 0$$

(17)

where $H$ and $G$ are matrices defined by:

$$H = h_{ij}^a = \int_{\Gamma_{k,j}} \psi_a \left( \frac{\partial \phi^*}{\partial n} \right)_j d\Gamma$$

(18)

$$G = g_{ij}^a = \int_{\Gamma_{k,j}} \phi^* d\Gamma$$

(19)

where $a = 1, \ldots, 6$ stands for the collocation nodes inside the $j$-th field element ($j = 1, \ldots, 4$), and $i = 1, \ldots, 6 N_k$ stands for the source point, while $\phi$ is the vector of potentials at the collocation nodes, and $\frac{\partial \phi}{\partial n}$ is the vector of normal derivatives of potentials at the collocation nodes.

The next step is to assemble the individual systems of equations arising from each subdomain.

The needed matching conditions are provided by the continuity of potential and normal current density, imposed at the interface between two adjacent subdomains. The matching between two subdomains can be established through their shared nodes $\alpha = 1$ to 6, belonging to the common element $j$ according to

$$\phi^\alpha_j = \phi^\alpha_{j_B}, \quad \left(-\tau_A \frac{\partial \phi^\alpha_j}{\partial n} \right)_A = \left(\tau_A \frac{\partial \phi^\alpha_{j_B}}{\partial n} \right)_B$$

(20)

Each subdomain contributes with $6 N_k$ equations, and each collocation node carries two magnitudes, potential and flux. Finally, the total number of unknowns is equal to the number of equations $N = \sum_{i=1}^n 6 N_k(i)$.

One of the great advantages of the domain decomposition technique, in addition to its capabilities in dealing with piecewise homogeneous material properties, is that the final system of equations is sparse and highly banded.
4 Computational example

The numerical example deals with the well-grounded human body model of 175cm height exposed to the 60Hz overhead power line electric field. The height of the power line is 10m above ground.

The related external electric field, to which human being is exposed, equals 10kV/m. Fig 3 shows the corresponding boundary element mesh. The distribution of the induced current densities inside the body is shown in Fig 4. It can be clearly observed that the induced current density values increase at narrow sections such as ankle and neck.

The values of current densities induced in ankle obtained using BEM are compared with the results obtained by FEM [6] and with experimental results [6], as well. This is shown in Table 1.
Table 2: Comparison between the BEM, FEM and experimental results for the induced current density at various body portions, expressed in [nA/cm²].

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>Neck</td>
<td>452</td>
<td>462</td>
<td>466</td>
</tr>
<tr>
<td>Pelvis</td>
<td>232</td>
<td>227</td>
<td>225</td>
</tr>
<tr>
<td>Ankle</td>
<td>1891</td>
<td>1916</td>
<td>1866</td>
</tr>
</tbody>
</table>

The calculated results via BEM agree well with FEM and experimental results, as it is visible from Table 1.

5 Conclusion

Human exposure to high voltage ELF electric fields is analysed via the Boundary element Method (BEM). The 3D multidomain model of the human body has been used. The current density induced inside the human body, as a fundamental parameter in the analysis of human exposure to ELF fields, is obtained by solving the resulting Laplace equation via the Boundary Element Method with Domain Decomposition. This efficient BEM procedure is considered more accurate than FDTD and computationally less expensive than FEM, as only the domain boundary has to be discretised.

The BEM has been demonstrated to be a highly accurate technique according to the comparison with the experimental results. Numerical results obtained by the BEM are also in a good agreement with FEM results.

References


