Dual porosity DRM formulation for flow and transport through fractured porous media

T. Samardzioska & V. Popov
Wessex Institute of Technology, UK

Abstract

The main objective of this work is to develop a model for flow and solute transport in fractured porous media based on the dual porosity approach and the Dual Reciprocity Method. The developed model is compared to the results obtained using the equivalent continuum model and the discrete fracture/non-homogeneous model. The comparison is performed for a square porous domain with regular mesh of three parallel fractures combined with a further three fractures perpendicular to the first ones. The results helped to draw some conclusions in respect to the similarity of potentials as well as fluxes for the different methods. In this research the Boundary Element Dual Reciprocity Method scheme (BE DRM) has been used, in combination with the dual-porosity model.

Keywords: fractured porous media, dual porosity model, DRM, BEM.

1 Introduction

Fractured porous media could be modelled with a non-homogeneous model where porosity and permeability are allowed to vary discontinuously and rapidly, as both quantities are significantly larger in the fractures than in the porous rock. This approach is considered to be more accurate than the dual porosity model, since it introduces a smaller number of simplifications or approximations. However, the computational and data requirements for treating such a model are often too large, which makes this approach not suitable for every practical case.

As an alternative, the discontinuous nature of the permeability and porosity can be avoided by replacing them by averaged values. Such simplification is used in the equivalent continuum model [1], which does not contain fractures,
but the results may be quite different from the ones obtained using the non-
homogeneous model for a given geometry and properties of the fracture network.

A progress in the continuum model approach has been achieved with dual
porosity models [2]. The model assumes that the fractured porous media is a
continuum consisting of two overlapping regions [3]. The coupling term between
the two pore systems was derived, among the others, by Gerke and van
Genuchten [4, 5], and this approach has been followed in this work. The
numerical approach used is based on the Boundary Element Dual Reciprocity
Method – Multi Domain scheme (BE DRM-MD) [6, 7].

2 The dual porosity flow and transport model for fractured
porous media

2.1 Dual porosity model

Flow and transport in structured porous media are frequently described using
dual-porosity (or double-porosity) models. In such a model the void space of the
fractures is considered as a continuum while the void space within the blocks is
regarded as another continuum. The dual-porosity medium is considered to be a
superposition of these two systems over the same volume. The two pore systems
interact by exchanging water and solutes in response to pressure head and
concentration gradients. Macroscopically, two flow velocities, two pressure
heads, and two solute concentrations characterize the porous medium at any
point in time and space. Dual-porosity (DP) models assume that both water flow
and solute transport can be described by two equations, which are coupled
through a term that describes the exchange of fluid or solute between the two
pore regions.

Since flow in the fractures is much more rapid, the early dual porosity models
neglected the flow through the matrix block system [2]. The model used in this
work is based on the one of Gerke and van Genuchten [4, 5] that solves the full
models in both systems, porous matrix and fracture network.

2.2 Equations for flow

Assuming applicability of Darcy’s law, saturated water flow in fractured porous
media is described by a coupled pair of equations [4, 5, 8, 9]:

\[
\begin{align*}
C_f \cdot \frac{\partial h_f}{\partial t} &= K_f \cdot \nabla^2 h_f - \frac{\Gamma_w}{w_f} \\
C_m \cdot \frac{\partial h_m}{\partial t} &= K_m \cdot \nabla^2 h_m + \frac{\Gamma_w}{1 - w_f}
\end{align*}
\]

where \( h \) is pressure head [L], \( C \) is specific storativity [L\(^{-1}\)], \( K \) is hydraulic
conductivity [LT\(^{-1}\)], \( t \) is time [T] and \( w_f \) is relative volumetric proportion of the
fracture pore system. The parameter \( \Gamma_w \) is the water transfer term (T\(^{-1}\)), and is
given as
\[
\Gamma_w = \alpha_w \left( h_f - h_m \right) \tag{2}
\]

where \( \alpha_w \) is first-order mass transfer coefficient for flow \([L^{-1}T^{-1}]\) and is given as

\[
\alpha_w = \alpha_w^* \cdot K_a = \frac{\beta}{a^*} \gamma_w K_a \tag{3}
\]

where \( a \) is half width of the matrix block, or distance from the centre of the fictitious matrix block to the fracture boundary \([L]\), \( \beta \) is dimensionless factor depending on the geometry of the aggregates and usually takes values between 3, for rectangular slabs, and 15 for spheres, \( \gamma_w \) is 0.4 and is relatively independent of the aggregate geometry and the applied initial pressure and conditions, and \( K_a \) is effective hydraulic conductivity of the matrix at the fracture/matrix interface and is defined in the present case as

\[
K_a = 0.5 \cdot \left| K_{af} + K_{am} \right| \tag{4}
\]

Substituting expression (2) for \( \Gamma_w \) into (7) and considering that \( w_m = 1 - w_f \) yields:

\[
\nabla^2 h_f = \frac{C_f}{K_f} \cdot \frac{\partial h_f}{\partial t} + \frac{\alpha_w \left( h_f - h_m \right)}{w_f \cdot K_f} \tag{5a}
\]

\[
\nabla^2 h_m = \frac{C_m}{K_m} \cdot \frac{\partial h_m}{\partial t} + \frac{\alpha_w \left( h_m - h_f \right)}{w_m \cdot K_m} \tag{5b}
\]

### 2.3 Equations for transport

In the similar manner as for the flow, the solute transport in a saturated fractured porous medium is described using two coupled dual-porosity advection-dispersion equations:

\[
\nabla^2 c_f = \frac{1}{D_f} \left( \frac{\partial c_f}{\partial t} + V_{f_x} \frac{\partial c_f}{\partial x} + V_{f_y} \frac{\partial c_f}{\partial y} + \Gamma_s \right) \tag{6a}
\]

\[
\nabla^2 c_m = \frac{1}{D_m} \left( \frac{\partial c_m}{\partial t} + V_{m_x} \frac{\partial c_m}{\partial x} + V_{m_y} \frac{\partial c_m}{\partial y} - \frac{\Gamma_s}{w_m} \right) \tag{6b}
\]

where \( c \) is solute concentration \([ML^{-3}]\), \( D \) is dispersion coefficient \([L^2T^{-1}]\), \( V_{f_x} \) and \( V_{f_y} \) are velocities in the fractures in \( x \) and \( y \) directions \([MT^{-1}]\), respectively, \( V_{m_x} \) and \( V_{m_y} \) are velocities in the matrix blocks in \( x \) and \( y \) directions \([MT^{-1}]\), respectively, and \( \Gamma_s \) is solute mass transfer term \([ML^{-3}T^{-1}]\) given as

\[
\Gamma_s = \alpha_s \left( 1 - w_f \right) \left( c_f - c_m \right) + \begin{cases} \Gamma_w c_f & \Gamma_w \geq 0 \\ \Gamma_w c_m & \Gamma_w \leq 0 \end{cases} \tag{7}
\]
where $\alpha_s = \frac{\beta}{a^2} D_a$ is first-order solute mass transfer coefficient [T$^{-1}$], $\Gamma_w$ is water transfer term, and $D_a$ is effective diffusion coefficient [L$^{2}$T$^{-1}$], calculated as average value for the two regions.

3 The numerical implementation

In the examples shown in this work, linear variation of the field variables is assumed over the boundary elements.

3.1 Flow model

The DRM representations of (1a) and (1b) are shown below

\[
\begin{align*}
%Hh_f - Gq_f = S_f \left[ C_f \cdot \frac{\partial h_f}{\partial t} + \frac{\alpha_w (h_f - h_m)}{w_f} \right] \quad (8a)

%HH m - Gq_m = S_m \left[ C_m \cdot \frac{\partial h_m}{\partial t} + \frac{\alpha_w (h_m - h_f)}{w_m} \right] \quad (8b)
\end{align*}
\]

where the following notation is used

\[
S_i = (H\bar{U} - G\bar{Q}) \cdot F^{-1} \frac{1}{K_i}
\] (9)

Next a finite difference approximation for the time derivative and a linear variation of $u$ and $q$ within each time-step are introduced yielding

\[
\begin{align*}
\left( H - \frac{S_f \alpha_w}{w_f} \right) \theta_h - \frac{S_f C_f}{\Delta t} h_f^{n+1} - G\theta q_f^{n+1} =
\]

\[
= - \left( H - \frac{S_f \alpha_w}{w_f} \right) \left( 1 - \theta_h \right) - \frac{S_f C_f}{\Delta t} h_f^n + G \left( 1 - \theta_q \right) q_f^n - \frac{S_f \alpha_w}{w_f} h_m^n \quad \text{(10)}
\]

\[
\left( H - \frac{S_m \alpha_w}{w_m} \right) \theta_h - \frac{S_m C_m}{\Delta t} h_m^{n+1} - G\theta q_m^{n+1} =
\]

\[
= - \left( H - \frac{S_m \alpha_w}{w_m} \right) \left( 1 - \theta_h \right) - \frac{S_m C_m}{\Delta t} h_m^n + G \left( 1 - \theta_q \right) q_m^n - \frac{S_m \alpha_w}{w_m} h_f^n \quad \text{(11)}
\]
3.2 Transport model

The DRM representations of (6a) and (6b) are shown below

\[
H \cdot c_f - G \cdot q_f = S_f \left\{ \frac{\partial c_f}{\partial t} + V_{fx} \frac{\partial c_f}{\partial x} + V_{fy} \frac{\partial c_f}{\partial y} + \frac{1}{w_f} \left[ \alpha_s \cdot w_m (c_f - c_m) + \Gamma_w \cdot c_f \right] \right\} 
\]

(12a)

\[
H \cdot c_m - G \cdot q_m = S_m \left\{ \frac{\partial c_m}{\partial t} + V_{mx} \frac{\partial c_m}{\partial x} + V_{my} \frac{\partial c_m}{\partial y} - \frac{1}{w_m} \left[ \alpha_s \cdot w_m (c_f - c_m) + \Gamma_w \cdot c_f \right] \right\} 
\]

(12b)

where:

\[
S_i = (H\bar{U} - G\bar{Q}) \cdot F^{-1} \frac{1}{D_i}
\]

(13)

Next, the partial derivatives are represented using the usual DRM approximation [10], the time derivative is represented using finite difference approximation and \( c \) and \( q \) are represented by using a linear variation within each time-step, yielding

\[
\left[ (H - P_f) \theta_c - \frac{S_f}{\Delta t} \right] c_f^{n+1} - G \theta_q q_f^n = \left[ - \left( H - P_f \right) \left( 1 - \theta_c \right) - \frac{S_f}{\Delta t} \right] c_f^n + G \left( 1 - \theta_q \right) q_f^n \frac{S_f \alpha_s w_m}{w_f} c_m^n
\]

(14)

\[
\left[ (H - P_m) \theta_c - \frac{S_m}{\Delta t} \right] c_m^{n+1} - G \theta_q q_m^n = \left[ - \left( H - P_m \right) \left( 1 - \theta_c \right) - \frac{S_m}{\Delta t} \right] c_m^n + G \left( 1 - \theta_q \right) q_m^n - S_m \left( \alpha_s + \frac{\Gamma_w}{w_m} \right) c_f^n
\]

(15)

where:

\[
P_f = S_f \left( V_{fx} \frac{\partial F}{\partial x} F^{-1} + V_{fy} \frac{\partial F}{\partial y} F^{-1} + \frac{\alpha_s w_m}{w_f} + \frac{\Gamma_w}{w_f} \right)
\]

(16)

\[
P_m = S_m \left( V_{mx} \frac{\partial F}{\partial x} F^{-1} + V_{my} \frac{\partial F}{\partial y} F^{-1} + \alpha_s \right)
\]

(17)
The complete flow and transport problem involves four variables for each of the two domains, fractures and porous block, consisting of one hydraulic head, one flow flux, one solute concentration and one solute flux.

### 3.3 Solution procedure for the DP model

The system of equations (10) and (11), which also applies to the system equations (14) and (15), can be solved simultaneously, as shown in (18), or by using an iterative scheme, as shown in Table 1. In (18) BC represents boundary conditions and \( u \) represents hydraulic head or solute concentration. In the present study the iterative scheme was selected as a better choice as in this way the possibility of obtaining an ill conditioned system of equations due to large difference in the hydraulic conductivities, or advective terms, in the two different porous media, fractures and porous matrix, is avoided.

\[
\begin{bmatrix}
[F] & 0 \\
0 & [MB]
\end{bmatrix}
\begin{bmatrix}
u_f \\
qu_f \\
u_m \\
q_m
\end{bmatrix}^{n+1} = 
\begin{bmatrix}
[F_1] & 0 \\
0 & [MB_1]
\end{bmatrix}
\begin{bmatrix}
u_f \\
qu_f \\
u_m \\
q_m
\end{bmatrix}^n + [BC]
\] (18)

### 4 Numerical examples

In this example a square area is analysed with dimensions 0.46 x 0.46m\(^2\), where 24.4% of the volume is occupied by fractures [11]. Two different meshes were analysed, one with horizontal and vertical fractures, in this example referred to as the “original” mesh, and the other one with “rotated” mesh of fractures under 45° angle in respect to the original mesh. In the both cases care has been taken for the analysed parts of the domains corresponding to porous media and fractures to be equivalent in terms of volume, and also the mutual distance between the fractures to be the same. The numerical scheme has been tested and has shown good agreement towards a 1D analytical solution for homogeneous domain, prior to solving this example. The dual porosity model parameters where defined as: half-distance between the fractures \( a = 0.06 \text{m} \), volumetric factor \( \omega_f = 24.4\% \), geometry of the matrix blocks \( \beta = 3.0 \), \( \gamma_w = 0.4 \), and effective hydraulic conductivity \( K_a \) taken as an average value of both conductivities in the matrix block \( K_m = 8.64 \times 10^{-5} \text{m/d} \), and in the fractures \( K_f = 8.64 \times 10^{-3} \text{m/d} \). The value of the specific storativity used for all the models is \( 10^{-4} \text{m}^{-1} \). Dispersion coefficients \( D_f = 0.05 \text{m}^2/\text{d} \) and \( D_m = 0.005 \text{m}^2/\text{d} \) were used for the transport simulation.

The boundary conditions for flow are: hydraulic head \( h_f(0,y,t) = h_m(0,y,t) = 1.05 \text{m} \) at the inlet surface and \( h_f(L,y,t) = h_m(L,y,t) = 1.0 \text{m} \) at the outlet surface, and normal derivatives \( q_f(x,0,t) = q_m(x,0,t) = 0 \) and \( q_f(x,b,t) = q_m(x,b,t) = 0 \) at the lower \((y = 0)\) and the upper \((y = b)\) boundaries of the domain, respectively. For the solute transport: concentration \( c_f(0,y,t) = c_m(0,y,t) = 1.2 \) and \( c_f(L,y,t) = ...
Here, \( L \) is the length of the domain in the direction of the flow and \( b \) is its width. The initial conditions in the fractures and the matrix pore system are \( h(x,y,0) = 1.0 \text{m} \) for the flow and \( c(x,y,0) = 1.0 \text{m} \) for the transport.

In Figure 2 a comparison is shown between the results obtained using the present dual porosity (DP) model and the previous results [11] obtained using the equivalent continuum (EC) and non-homogeneous (NH) models for hydraulic head profiles in the fractures. The results obtained with the three models are in good agreement.

The comparison of estimated total flow fluxes on the outlet of the domain is shown in Figure 3. The flux for the DP model was calculated by using the volumetric factor \( w_f \) of the fractures, which is representative for the whole domain, in the figure referred to as DP (wf), or by using the exact aperture of the fractures participating in the considered cross section, in the figure referred to as DP. It is apparent that both NH solutions obtained using two different meshes

<table>
<thead>
<tr>
<th>Iteration</th>
<th>( u_f, q_f ) – fracture network (first step in each iteration)</th>
<th>( u_m, q_m ) – matrix block (second step in each iteration)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( u_f(t), u_m(t) ) (BC + IC) ( q_f(t), q_m(t) ) (BC + IC)</td>
<td>( u_f(t + \Delta t), u_m(t) ) ( q_f(t + \Delta t), q_m(t) )</td>
</tr>
<tr>
<td></td>
<td>( \frac{1}{u_f(t + \Delta t)} = ? ) ( \frac{1}{q_f(t + \Delta t)} = ? )</td>
<td>( \frac{1}{u_m(t + \Delta t)} = ? ) ( \frac{1}{q_m(t + \Delta t)} = ? )</td>
</tr>
<tr>
<td>( k+1 )</td>
<td>( u_f^{k+1}(t + \Delta t), u_m^{k+1}(t + \Delta t) ) ( q_f^{k+1}(t + \Delta t), q_m^{k+1}(t + \Delta t) )</td>
<td>( u_f^{k+1}(t + \Delta t), u_m^{k+1}(t + \Delta t) ) ( q_f^{k+1}(t + \Delta t), q_m^{k+1}(t + \Delta t) )</td>
</tr>
<tr>
<td></td>
<td>( \frac{1}{u_f^{k+1}(t + \Delta t)} = ? ) ( \frac{1}{q_f^{k+1}(t + \Delta t)} = ? )</td>
<td>( \frac{1}{u_m^{k+1}(t + \Delta t)} = ? ) ( \frac{1}{q_m^{k+1}(t + \Delta t)} = ? )</td>
</tr>
</tbody>
</table>

IF: \( \Delta u_f = u_f^{n+1}(t + \Delta t) - u_f^n(t + \Delta t) \rightarrow 0 \) and \( \Delta u_m = u_m^{n+1}(t + \Delta t) - u_m^n(t + \Delta t) \rightarrow 0 \)

THEN: stop the iterations
and the DP model using the exact aperture of the fractures participating in the considered cross section show flux results that are in good agreement.

Figure 1: Square mesh with discrete fracture zones: a) original mesh; b) rotated mesh

Figure 2: Hydraulic head profiles inside the fractures estimated using the three models (t=0.001 days; $\alpha_w=1.454$, $w_f=0.244$, $K_f =10^{-7}$ m/s, $K_m =10^{-9}$ m/s, $C_f=C_m=10^{-4}$ m$^{-1}$)

Figure 4 shows the concentration profiles in the fractures and matrix blocks obtained with the three different models.

The comparison of estimated total solute fluxes on the outlet of the domain showed similar behaviour as the ones for the flow.
5 Conclusions

A model was developed for flow and solute transport in fractured porous media based on the dual porosity approach and the Dual Reciprocity Method. The ill-conditioning of the system matrix due to very large difference in the permeabilities of the fractures and porous matrix is avoided by solving the equations in the porous matrix and fractures through an iterative scheme. The developed model was validated using analytical solutions for simple 1D cases and then compared to the results previously obtained using the equivalent continuum model and the discrete fracture/non-homogeneous model [11], showing good agreement for both, hydraulic head and concentration profiles in the domain.

The results show that the fluxes should be evaluated with special care when the DP model is used. While the NH model can accurately estimate the fluxes...
since the total cross section of the fractures which participate in the considered
cross section of the porous media is accurately estimated, in the case of the DP
model in a general case the fluxes would be estimated by using the volumetric
factor of the fractures, which introduces errors. The problem is that in such case,
not only the “active” fractures, which contribute towards the flux, are taken into
account, but also the fractures with stagnant water, providing a significant
overestimate of the flux. The EC model shows the same problems when
calculating fluxes, but for the case of the EC model used in [11] it is more
difficult to improve the results, as the error is inherent in the solution of the EC
model itself and is due to the way that the equivalent media properties are
estimated.

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