2.5D Green’s functions for transient heat transfer by conduction and convection

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Abstract

This paper presents fundamental solutions for computing the heat field generated by spatially sinusoidal harmonic heat line sources placed in layered solid formations. These expressions model the transient heat transfer by conduction and convection in three-dimensional media. The temperatures and the heat fluxes at some point in the layered medium can both be computed when a heat line source produces energy.

The simulation is first performed in the frequency domain, while time responses are computed using inverse Fourier transforms.

The authors believe that these solutions can be useful as benchmark models for numerical applications, such as the Boundary Element Method. They are also of great value if used together with numerical models, since the full discretization of the solid layer interfaces is then unnecessary.
Keywords: transient heat transfer, conduction, convection, 2.5D Green’s Functions, layered formation.

1 Introduction

Most of the known techniques to solve transient convection-diffusion heat problems have been formulated in the time domain or using Laplace transforms. In the “time marching” approach the solution is assessed step by step at consecutive time intervals after an initially specified state has been assumed. Using the Laplace transform, a numerical transform inversion is required to calculate the physical variables in the real space, after the solution has been obtained for a sequence of values of the transform parameter. The Laplace transform technique has been broadly used for solving diffusion problems, but small truncation errors can be magnified in the inversion process and so the
accuracy depends on an efficient and precise inverse transform. Different inversion methods have been proposed over the years, such as the Stehfest algorithm [1].

This work presents Green’s functions for calculating the transient heat transfer wave field in layered solid formations, in the presence of conduction and convection. The problem is formulated in the frequency domain using time Fourier transforms. The proposed technique allows the use of any type of heat source, and deals with the unavoidable static response.

This work extends the work performed by the authors for the definition of the steady-state response of layered solid media subjected to a spatially sinusoidal harmonic heat conduction line source (e.g. Tadeu et al. [2]). The problem is now solved incorporating convection phenomena. The proposed fundamental solutions, relate the heat field variables (fluxes or temperatures) at some position in the solid domain caused by a heat source placed elsewhere in the media, in the presence of both conduction and convection phenomena.

As in the previous work, the technique requires the knowledge of the Green’s function for the unbounded media, which is written first as a superposition of cylindrical heat waves along one horizontal direction (z) and then as a superposition of plane waves, following a technique similar to the one used first by Lamb [3] for the propagation of elastodynamic waves in two-dimensional media. Other authors, such as Bouchon [4] and Tadeu and António [5], have used an equivalent approach to calculate three-dimensional elastodynamic fields using a discrete wave number representation.

The Green’s functions for a solid layered formation are formulated as the sum of the heat source terms equal to those in the full-space and the surface terms required to satisfy the boundary conditions at the interfaces i.e. continuity of temperatures and normal fluxes between solid layers, and null normal fluxes or null temperatures at the outer surface. The total heat field is achieved by adding the heat source terms, equal to those in the unbounded space, to that set of surface terms, arising within each solid layer and at each interface.

The Green’s functions for the case of a spatially sinusoidal, harmonic heat line source placed in an unbounded medium are developed by first applying a time Fourier transform to the time convection-diffusion equation for a heat point source and then a spatial Fourier transform to the resulting Helmholtz equation, along the z direction, in the frequency domain.

This methodology is verified comparing its results with the exact time solutions for one, two and three-dimensional point heat sources placed in an unbounded medium. Besides this verification, the responses of the Green’s functions for a solid layered formation are compared with solutions provided by the Boundary Element Method. The expressions presented in this work allow the heat field inside a layered solid medium to be computed, without either the discretization of the interior domain, which is necessary when using numerical techniques, such as the finite differences method, or even the discretization of the solid interfaces using boundary elements techniques.
2 3D problem formulation and Green’s functions in an unbounded medium

The transient convection-conduction heat transfer in solids with constant velocities along the $x$, $y$ and $z$ directions is expressed by the equation

$$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} T - \frac{1}{K} \left( V_x \frac{\partial T}{\partial x} + V_y \frac{\partial T}{\partial y} + V_z \frac{\partial T}{\partial z} \right) = \frac{1}{K} \frac{\partial T}{\partial t},$$

in which $V_x$, $V_y$ and $V_z$ are the velocity components in the direction $x$, $y$ and $z$ respectively, $T$ is temperature, $K$ is the thermal diffusivity, $k$ is the thermal conductivity, $\rho$ is the density and $c$ is the specific heat. Applying a Fourier transform in the time domain, one obtains

$$\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) - \frac{1}{K} \left( V_x \frac{\partial}{\partial x} + V_y \frac{\partial}{\partial y} + V_z \frac{\partial}{\partial z} \right) + \left( \frac{-i\omega}{K} \right)^2 \hat{T}(\omega, x, y, z) = 0,$$

where $i = \sqrt{-1}$ and $\omega$ is the frequency. Eqn (2) differs from the Helmholtz equation by the inclusion of a convective term. The fundamental solution of eqn (2) for a heat point source in an unbounded medium, located at $(0, 0, 0)$, can be expressed as

$$\hat{T}_f(\omega, x, y, z) = \frac{V_x V_y V_z}{2k} \frac{e^{-\frac{k}{2k}}}{\sqrt{x^2 + y^2 + z^2}}.$$

When the geometry of the problem remains constant along one direction ($z$) the full 3D problem can be expressed as a summation of simpler 2D solutions. This requires the application of a Fourier transformation along that direction, writing this as a summation of 2D solutions with different spatial wavenumbers $k_z$ (Tadeu & Kausel [6]). The application of a spatial Fourier transformation to

$$e^{-\frac{i\omega}{4K^2} V_x^2 + V_y^2 + V_z^2} \sqrt{x^2 + y^2 + z^2},$$

along the $z$ direction, leads to

$$\hat{T}_f(\omega, x, y, k_z) = -i e^{-\frac{2k}{4k}} H_0 \frac{V_x V_y V_z}{4k} \sqrt{V_x^2 + V_y^2 + V_z^2} - \frac{i\omega}{K} \left( k_z^2 r_0 \right),$$

where $H_0(\ )$ are Hankel functions of the second kind and order 0, and $r_0 = \sqrt{(x - x_0)^2 + (y - y_0)^2}$. The full three-dimensional solution is then achieved by applying an inverse Fourier transform along the $k_z$ domain. This inverse Fourier transformation can be expressed as a discrete summation if we assume the existence of virtual sources, equally spaced at $L_z$, along $z$, which enables the solution to be obtained by solving a limited number of two-dimensional problems.
with \( k_{2m} \) being the axial wavenumber given by \( k_{2m} = \frac{2\pi}{L_z} m \). The distance \( L_z \) is chosen so as to prevent spatial contamination from the virtual sources (Bouchon & Aki [7]). This technique reflects the adaptation of other mathematical and numerical formulations applied to solve problems such as wave propagation (Tadeu et al [8]).

The application of a spatial Fourier transformation along the \( z \) direction in eqn (2) leads to the following equation

\[
\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) - \frac{1}{K} \left( V_x \frac{\partial}{\partial x} + V_y \frac{\partial}{\partial y} \right) + \left( \sqrt{\frac{-i\omega}{K} - (k_z)^2} \right)^2 \tilde{T}(\omega, x, y, k_z) = 0 ,
\]

when \( V_z = 0 \). The fundamental solution of this equation is given by eqn (5) ascribing \( V_z = 0 \).

These same equations can be written as a continuous superposition of heat plane phenomena. Eqn 5, which results when a spatially sinusoidal harmonic heat line source is applied at the point \((x_0, y_0)\) along the \( z \) direction, subject to convection velocities \( V_x \), \( V_y \) and \( V_z \), is then given by the expression,

\[
\tilde{T}_f(\omega, x, y, k_z) = -i e^{-\frac{V_x x + V_y y + V_z z}{2k}} \int_{-\infty}^{\infty} e^{\frac{-V_x x - V_y y - i(\nu - \nu_0)}{\nu}} e^{-i\nu(x-x_0)} d\nu_0 ,
\]

where \( \nu = \sqrt{-\frac{V_x^2 + V_y^2 + V_z^2}{2K} - \frac{i\omega}{K} (k_z)^2 - k_z^2} \) with \( (\text{Im}(\nu) \leq 0) \), and the integration is performed with respect to the horizontal wave number \((k_x)\) along the \( x \) direction.

The integral in the above equation can be transformed into a summation if an infinite number of such sources are distributed along the \( x \) direction, at equal intervals \( L_x \). The above equation can then be written as

\[
\tilde{T}_f(\omega, x, y, k_z) = -i e^{-\frac{V_x x + V_y y + V_z z}{2k}} E_0 \sum_{n=-\infty}^{n+\infty} \left( \frac{E}{V_n} \right) E_d ,
\]

where \( E_0 = \frac{-i}{2kL_x} \), \( E = e^{-i(\nu - \nu_0)} \), \( E_d = e^{-ik_x(x-x_0)} \),

\[
\nu_n = \sqrt{-\frac{V_x^2 + V_y^2 + V_z^2}{4K^2} - \frac{i\omega}{K} (k_z)^2 - k_{2m}^2} \) with \( (\text{Im}(\nu_n) \leq 0) \), \( k_{2m} = \frac{2\pi}{L_z} n \), which can in turn be approximated by a finite sum of equations \((N)\). Notice that \( k_z = 0 \) corresponds to the two-dimensional case.

The heat in the spatial-temporal domain is calculated by applying a numerical inverse fast Fourier transform in \( k_z \), in the frequency domain. The computations are performed using complex frequencies with a small imaginary part of the form.
\[ \omega_c = \omega - i \eta \] (with \( \eta = 0.7 \Delta \omega \), and \( \Delta \omega \) being the frequency step) to prevent interference from aliasing phenomena. In the time domain, this effect is removed by rescaling the response with an exponential window of the form \( e^{\eta t} \). The time variation of the source can be arbitrary. The time Fourier transformation of the incident heat field defines the frequency domain where the BEM solution needs to be computed. The response may need to be computed from 0.0Hz to very high frequencies. However, as the heat responses decay very fast with increasing frequency, we may limit the upper frequency where the solution is required. The frequency 0.0Hz corresponds to the static response that can be computed when the frequency is zero. The use of complex frequencies allows the solution to be obtained because, when \( \omega_c = -i \xi \) (for 0.0Hz), the arguments of the equations are other than zero.

### 2.1 Verification of the solution

In order to verify the formulation presented above, the results are compared with the analytical response in the time domain. The exact solution of the convective diffusion, expressed by eqn (1), in an unbounded medium subjected to a unit heat source is well known and it allows the computation of the heat field given by both conduction and convection phenomena in the presence of three, two or one-dimensional problems. When the heat source is applied at point \((0,0,0)\) at time \(t=t_0\), the temperature at \((x,y,z)\) is given by the expression

\[
T(t,x,y,z) = e^{\frac{-((x-v_0 t)^2+(y-v_0 t)^2+(z-\rho t)^2)}{4k\tau}} \frac{4Kt}{\rho c(4\pi K\tau)^{d/2}}, \text{ if } t > t_0, \tag{10}
\]

where \(\tau = t - t_0\), \(r_{00}\) is the distance between the source point and the field point \((x,y,z)\), and \(d = 3\), \(d = 2\) and \(d = 1\) when in the presence of a three, two and one-dimensional problem, respectively (e.g. Carslaw & Jaeger [10]).

In the verification procedure, a homogeneous unbounded medium with the concrete thermal properties (\(k = 1.4 \text{ W.m}^{-1}\cdot\text{°C}^{-1}\), \(c = 880.0 \text{ J.Kg}^{-1}\cdot\text{°C}^{-1}\) and \(\rho = 2300 \text{ Kg.m}^{-3}\)) was excited at \(t = 277.8\)h, by a unit heat source placed at \((x = 0.0 \text{ m}, y = 0.0 \text{ m}, z = 0.0 \text{ m})\). The convection velocities applied in the \(x, y\) and \(z\) direction were equal to \(1 \times 10^{-6} \text{ m/s}\).

Figure 1 shows the temperature calculated by eqn (10) along a line of 40 receivers placed from \(y = -1.5 \text{ m}\) to \(y = 1.5 \text{ m}\), for cylindrical \((d = 2)\) and spherical \((d = 3)\) unit heat source, at different times.

The temperature responses along these receivers were computed using the Green’s functions formulation. The calculations were first performed in the frequency domain in the frequency range \([0, 1024 \times 10^{-7} \text{ Hz}]\) with a frequency increment of \(\Delta \omega = 10^{-7} \text{ Hz}\), which defines a time window of \(T = 2777.8\)h. The response for a spherical unit heat source has been computed dividing the results given by eqn (3) by \(2\pi\). The solution for the two-dimensional case (cylindrical
unit heat source) has been found with eqn (5), while the results for a plane unit heat source propagating along the \( y \) axis has been modelled ascribing \( k_z = 0 \) and \( k_\infty = 0 \) to eqn (9), multiplied by \( L_x \). Complex frequencies of the form \( \omega = \omega - i0.7\Delta\omega \) have been used to avoid the aliasing phenomenon. The spatial period has been set as \( L_x = L_z = 2\sqrt{k/(\rho c \Delta f)} \). In Figure 1, the solid line represents the exact time solution given by eqn (10) while the marks show the response obtained using the proposed Green’s functions. There is good agreement between these two solutions.

![Figure 1: Temperature along a line of 40 receivers, at different times (350 h, 450 h, 550 h and 650 h): a) for a cylindrical (\( d = 2 \)) unit heat source; b) for a spherical (\( d = 3 \)) unit heat source.](image)

### 3 Green’s functions in a layered formation

The Green’s functions for a multi-solid layer are established using the required boundary conditions at the solid-solid interfaces.

Consider a system built from a set of \( m \) solid plane layers of infinite extent bounded by two flat, semi-infinite, solid media. The top semi-infinite medium is denominated medium 0, while the bottom semi-infinite medium is assumed to be the medium \( m+1 \). The thermal material properties and thickness of the different layers may differ. Different vertical convection velocities can be ascribed to each solid layer. The convection is computed assuming that the origin of convection coincides with the conduction source. The system of equations is achieved considering that the multi-solid layer is excited by a spatially sinusoidal heat source located in the first solid layer (medium 1). The heat field at some position in the solid domain is computed taking into account both the surface heat terms generated at each interface and the contribution of the heat source term.

For the solid layer \( j \), the heat surface terms on the upper and lower interfaces can be expressed as
\[ \tilde{T}_j (\omega, x, y, z) = E_{0j} e^{-2k_j \frac{y - y_0}{V_{nj}}} \sum_{n=-\infty}^{\infty} \left( \frac{E_{ij}}{V_{nj}} \right) A_n^j e^{i \frac{k_n h_j}{V_{nj}}} E_d, \]

\[ \tilde{T}_{j2} (\omega, x, y, z) = E_{0j} e^{-2k_j \frac{y - y_0}{V_{nj}}} \sum_{n=-\infty}^{\infty} \left( \frac{E_{ij}}{V_{nj}} \right) A_n^j e^{i \frac{k_n h_j}{V_{nj}}} E_d, \]

where \( E_{0j} = \frac{-i}{2k_j L_s} \),

\[ E_j = e^{-iv_j \frac{y - y_0}{V_{nj}}} \],

\[ E_{j2} = e^{-iv_j \frac{y - y_0}{V_{nj}}} \]

\[ V_{nj} = \sqrt{\left( \frac{V_j}{2K_j} \right)^2 + \frac{\rho_j}{K_j} - k_z^2 - k_{xm}^2}, \]

with \( \text{Im} (V_{nj}) \leq 0 \) and \( h_j \) is the thickness of the layer \( l \). Meanwhile, \( K_j = k_j/\left( \rho_j c_j \right) \) is the thermal diffusivity in the solid medium \( j \) (\( k_j, \rho_j \) and \( c_j \) represent the thermal conductivity, the density and the specific heat of the material in the solid medium, \( j \), respectively). The heat surface terms produced at interfaces 1 and \( m+1 \), governing the heat that propagates through the top and bottom semi-infinite media, are respectively expressed by

\[ \tilde{T}_{02} (\omega, x, y, z) = E_{00} e^{-2k_2 \frac{y - y_0}{V_{n0}}} \sum_{n=-\infty}^{\infty} \left( \frac{E_{0i}}{V_{n0}} \right) A_n^0 e^{i \frac{k_n h_i}{V_{n0}}} E_d, \]

\[ \tilde{T}_{(m+1)2} (\omega, x, y, z) = E_{0(m+1)} e^{-2k_2 \frac{y - y_0}{V_{n(m+1)}}} \sum_{n=-\infty}^{\infty} \left( \frac{E_{(m+1)i}}{V_{n(m+1)}} \right) A_n^{(m+1)} e^{i \frac{k_n h_i}{V_{n(m+1)}}} E_d. \]

A system of \( 2(m+1) \) equations is assembled, ensuring the continuity of temperatures and heat fluxes along the \( m+1 \) solid interfaces between layers. Each equation takes into account the contribution of the surface terms and the involvement of the incident field. All the terms are organized according to the form \( Fa = b \)

\[
\begin{bmatrix}
  c_{10}c_{30} & -c_{21}c_{41} & -c_{11}c_{41}c_{51} & \cdots & 0 & 0 & 0 \\
  0 & -c_{21}c_{51} & -c_{11}c_{51}c_{61} & \cdots & 0 & 0 & 0 \\
  0 & 0 & -c_{11}c_{61} & \cdots & 0 & 0 & 0 \\
  \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
  0 & 0 & 0 & \cdots & -c_{2n}c_{4n} & -c_{1n}c_{4n}c_{5n} & 0 \\
  0 & 0 & 0 & \cdots & -c_{2n}c_{5n} & -c_{1n}c_{5n}c_{6n} & 0 \\
  0 & 0 & 0 & \cdots & c_{2n}c_{5n}c_{6n} & c_{1n}c_{5n} & -c_{2(n+1)}c_{4(n+1)} \\
  0 & 0 & 0 & \cdots & c_{1n}c_{5n} & 0 & -c_{2(n+1)}c_{6(n+1)} \\
  \end{bmatrix}
\begin{bmatrix}
  A_{n0} \\
  A_{n1} \\
  A_{n2} \\
  \vdots \\
  A_{nm} \\
  A_{nm} \\
  A_{n(m+1)} \\
  \end{bmatrix} =
\begin{bmatrix}
  c_{11}c_{41}e^{-iv_{10}} \\
  -c_{21}c_{51}e^{-iv_{10}} \\
  \vdots \\
  -c_{31}c_{61}e^{-iv_{10}} \\
  -c_{1n}c_{4n}e^{-iv_{10}} \\
  -c_{2n}c_{5n}e^{-iv_{10}} \\
  \vdots \\
  -c_{3n}c_{6n}e^{-iv_{10}} \\
  -c_{2(n+1)}c_{4(n+1)}e^{-iv_{10}} \\
  \vdots \\
  -c_{2(n+1)}c_{6(n+1)}e^{-iv_{10}} \\
  \end{bmatrix}
\]

(13)
where \( c_{i,j} = \left[ \frac{V_{ij}^y + iV_{nj}}{2K_j} \right] \), \( c_{2,j} = \left[ \frac{V_{ij}^y - iV_{nj}}{2K_j} \right] \), \( c_{3,j} = \frac{e^{2K_j}}{V_{nj}} \), \( c_{4,j} = \frac{e^{-2K_j}}{V_{nj}} \), and \( c_{5,j} = e^{-i\nu_h} \).

The resolution of the system gives the amplitude of the surface terms in each solid interface. The temperature field for each solid layer formation is obtained by adding these surface terms to the contribution of the incident field, leading to the following equations:

**Top semi-infinite medium (medium 0)**

\[
\tilde{T}(\omega, x, y, k_z) = E_{00}e^{-\omega \mu_0 A_{n_0}} E_d, \quad \text{if } y < 0;
\]

**Solid layer 1 (source position)**

\[
\tilde{T}(\omega, x, y, k_z) = \frac{-i}{4k_1} e^{-\omega \mu_0 A_{n_1}} \left( E_{01}e^{-\omega \mu_0 A_{n_0}} + E_{11}e^{\omega \mu_0 A_{n_1}} \right) E_d, \quad \text{if } 0 < y < h_1;
\]

**Solid layer j (j ≠ 1)**

\[
\tilde{T}(\omega, x, y, k_z) = E_{0j}e^{-\omega \mu_0 A_{n_j}} \left( E_{1j}e^{-\omega \mu_0 A_{n_1}} + E_{2j}e^{\omega \mu_0 A_{n_j}} \right) E_d, \quad \text{if } \sum_{l=1}^{j-1} h_l < y < \sum_{l=1}^{j} h_l;
\]

**Bottom semi-infinite medium (medium m + 1)**

\[
\tilde{T}_{(m+1)2}(\omega, x, y, k_z) = E_{0(m+1)}e^{-\omega \mu_0 A_{n_{(m+1)}}} \left( E_{1(m+1)}e^{\omega \mu_0 A_{n_{(m+1)}}} \right) E_d.
\]

Notice that when the position of the heat source is changed, the matrix \( F \) remains the same, while the independent terms of \( b \) are different. However, as the equations can be easily manipulated to consider another position for the source, they are not included here.

### 3.1 Verification of the solution

The accuracy of the formulation presented in this paper is verified by comparing its results with the solution of the BEM model for a certain problem. The BEM code, which involves the discretization of all solid interfaces, makes use of the Green’s Functions for an unbounded medium. The BEM code has been validated by applying it to a cylindrical circular ring core, since the analytical solutions have been derived for this particular case. In order to avoid the unlimited discretization of the solid interfaces in the BEM model a damping factor is considered. This factor uses complex frequencies with a small imaginary part of the form \( \omega = \omega - i\eta \) (with \( \eta = 0.7\Delta \omega \)) [Bouchon and Aki [9], Phinney [11]]. In the present case the elements are distributed along the surface up to
\[ L_{\text{dist}} = 2 \frac{\sqrt{k_i}}{\left( \rho_c \omega \right)} \], using the thermal material properties from the solid medium that lead to the largest spatial distance.

A flat concrete layer, 3m thick, bounded by two semi-infinite steel media, as displayed in Figure 2, is used to evaluate the accuracy of the proposed formulation. The convection velocities applied to the three media were \(5 \times 10^{-7} \text{ m/s}, 8 \times 10^{-7} \text{ m/s} \) and \(1 \times 10^{-6} \text{ m/s} \) for the top medium, concrete layer and bottom medium, respectively. The thermal material properties used are presented in Table 1.

The calculations have been performed in the frequency domain from 0Hz to \(32 \times 10^{-7} \text{ Hz} \), with a frequency increment of \( \Delta \omega = 1 \times 10^{-7} \text{ Hz} \) and considering a single value of \( k_z \) equal to \(0.4 \text{ rad/m} \). The amplitude of the response for two receivers placed in two different media was computed for a heat point source applied at \((x = 0.0 \text{ m}, y = 1.0 \text{ m})\). The real and imaginary parts of the response at receiver \(1 \ (x = 0.2 \text{ m}, y = 0.5 \text{ m}) \) and receiver \(2 \ (x = 0.2 \text{ m}, y = 3.5 \text{ m}) \) are displayed in Figure 2, when the imaginary part of the frequency has been set to \( \eta = 0.7 \Delta \omega \). The solid lines represent the analytical responses, while the marked points correspond to the BEM solution. The square and the round marks designate the real and imaginary parts of the responses, respectively. The two solutions seem to be in very close agreement.

### Table 1: Thermal material properties.

<table>
<thead>
<tr>
<th>Solid layer (concrete)</th>
<th>Lower solid medium (steel)</th>
<th>Top solid medium (steel)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Thermal conductivity</strong></td>
<td>(k_i = 1.4 \text{ W.m}^{-1} \cdot \text{C}^{-1})</td>
<td>(k_2 = 63.9 \text{ W.m}^{-1} \cdot \text{C}^{-1})</td>
</tr>
<tr>
<td><strong>Density</strong></td>
<td>(\rho_i = 2300 \text{ Kg.m}^{-3})</td>
<td>(\rho_2 = 7832 \text{ Kg.m}^{-3})</td>
</tr>
<tr>
<td><strong>Specific heat</strong></td>
<td>(c_i = 880 \text{ J.Kg}^{-1} \cdot \text{C}^{-1})</td>
<td>(c_2 = 434 \text{ J.Kg}^{-1} \cdot \text{C}^{-1})</td>
</tr>
</tbody>
</table>

Figure 2: One solid layer bounded by two semi-infinite solid media: a) Receiver 1. b) Receiver 2.
4 Conclusions

2.5 Green’s functions, for computing the transient heat transfer by conduction and convection in an unbounded medium and layered media, have been presented. In this approach the calculations are first performed in the frequency domain. The results for a layered formation are obtained adding the heat source term and the surface terms, required to satisfy the interface boundary conditions (temperature and heat fluxes continuity).

The verification of the unbounded medium formulation was obtained comparing its time responses and the exact time solutions. In turn, the analytical solutions used in the solid layered media formulation were verified using a BEM algorithm. Using these two approaches together can be useful in the resolution of engineering problems, such as inclusions placed in layered formations.

References

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