Boundary element analysis of FRP-concrete delamination

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Abstract

The FRP-concrete delamination problem is studied making use of BEM. Cohesive crack model has been adopted for the interface, whereas linear elasticity is used for the two materials outside the process zone. Numerical analyses are carried out by means of symmetric Galerkin boundary element method for cohesive interfaces, adopting the arc-length technique to follow the equilibrium path beyond its critical point. Two different test setups have been numerically simulated and results are compared with experimental tests. For bond lengths longer than minimum anchorage length, tests may exhibit a snap-back branch after the attainment of the maximum force.

Keywords: SG-BEM, cohesive interface, FRP-concrete delamination, arc-length.

1 Introduction

The effectiveness of strengthening concrete structures with composite materials can be compromised if debonding of Fiber-Reinforced Polymer (FRP) plate occurs. It is then very important to accurately predict the debonding failure load as well as the post-failure behavior.

Recently, boundary element method (BEM) has received great attention, especially in fracture mechanics. This method is particularly attractive in the present case, since all non linearities are localized on a boundary, the interface, whereas the two materials, FRP plates and concrete, can be considered as elastic domains.

The role of the interface is essential to the stress transfer between FRP and concrete. A comprehensive way of modelling FRP-concrete debonding phenomena is not yet available. At present, experimental data can be used to define mode II
shear stress - tangential slip law [1], [2]. However these laws must be modified to be used for 2D or 3D problems, since compliance of concrete cover contributing to interface slip is implicitly included in these laws.

The incremental symmetric boundary integral formulation for cohesive interface problems is briefly described in section 3 (see [3] for details). Riks arc-length technique with local control is used for the solution of non linear problems [4].

Several numerical simulations have been performed, concerning two different setups of delamination tests of FRP-plates bonded on concrete substrate. Due to the presence of a softening branch in shear stress-slip law, the behaviour of a specimen undergoing delamination may be strongly non linear, causing very brittle failure mechanisms, and possibility of snap-back behaviour after the attainment of maximum load. Results are found in good agreement with experimental results reported in the literature. Moreover, crack nucleation and growth can be numerically followed, until complete delamination.

2 Interface law

In the so-called cohesive crack model, the fracture process zone is modelled as a fictitious crack, representing a region where stresses can be transmitted, whose values depend on crack amplitude. In this study, a mode II cohesive crack law is adopted for the FRP-concrete interface.

This law must correctly reproduce elastic stiffness for low slip levels, maximum shear stress and softening branch for high slips. An important starting point is the correct definition of kinematical variables involved, that is interface slips and dual static variables (interface stresses). Hence, different models, where slips represent different kinematical variables, must adopt different interface laws. For instance, for 1D bond - slip models based on the assumption of plane strain profile for both concrete and plate (see for instance [1]), the non linear interface law must involve compliances of both adhesive layer and external concrete cover. A mode II power fractional interface law has been proposed in [2] by post-processing experimental data of delamination tests, giving the local bond stress \( \tau \) as a function of slip \( s \) (see Fig. 1):

\[
\frac{\tau}{\bar{\tau}} = \frac{s}{\bar{s}} \left( n - 1 \right) + \left( \frac{s}{\bar{s}} \right)^n
\]

where \((\bar{\tau}, \bar{s})\) denote peak shear stress and corresponding slip, and \(n\) is a free parameter mainly governing the softening branch. According to (1), linear compliances of both adhesive and concrete cover for low slip values as well as failure and subsequent post-failure softening behaviors are modelled at the interface level.

A non linear constitutive interface law for FRP-concrete suitable for multi-dimensional domain decomposition method, such as FEM of BEM, is not yet available in literature. In the present study, the interface law has been defined starting from that proposed in [2]. That law has been modified for 2D analysis by subtracting the elastic contribution of concrete cover compliance, since this contribution is...
Figure 1: Mode II non-linear FRP - concrete interface laws: the proposed (2D) interface law, a 1D bond-slip law (from [2]) and a bilinear law.

explicitly taken into account in the BEM. Hence, the initial stiffness of 2D interface law, obtained from (1) as:

\[
\frac{\partial \tau}{\partial s} (s = 0) = K_0^{2D} = \frac{\bar{\tau}}{\bar{s}} \frac{n}{n - 1}
\]  (2)

is given by: \(1/K_0^{2D} = 1/K_0^{1D} - 1/K_c\), where \(K_0^{1D}\) is the initial stiffness of 1D bond-slip law, and \(K_c = G_c/h_c\) is the stiffness of external concrete cover, with \(G_c\) being shear modulus and \(h_c = 26\) mm the equivalent thickness of concrete contributing to interface compliance [1].

Moreover, the value of fracture energy of 1D law has been preserved, since that value was calibrated from experimental tests in order to give the value of maximum transmissible force by an anchorage of infinite length [5]. Fracture energy of interface law (1) can be written as:

\[
G_f = \bar{\tau} \bar{s} \pi [1/(n - 1)]^{(1 - \frac{2}{n})} \csc (2\pi/n)
\]  (3)

Hence, for prescribed values of \(K_0^{2D}\) and \(G_f\), the parameters \(\bar{s}\) and \(n\) of 2D interface law can be easily obtained from eqns (2), (3). The interface law for 2D analyses is reported in Fig. 1. In the same figure, a bilinear law is also depicted, obtained by prescribing the same peak values \((\bar{\tau}, \bar{s})\) and fracture energy \(G_f\) of 2D power fractional law.

It is worth noting that, at present, no experimental information are available to calibrate a non linear law for normal stress-crack opening displacement. Therefore, in the present analysis, a linearly elastic law is adopted, \(\sigma = k_p w\), with compliance due to adhesive deformation, i.e., \(k_p = E_g/h_g\), \(E_g\) and \(h_g\) being the elastic modulus and the thickness of the adhesive layer respectively.

Denoting with \(\mathbf{p}^{def} = \{\sigma, \tau\}\) the cohesive tractions vector and with \(\mathbf{w}^{def} = \{w, s\}\) the relative opening displacements vector, the interface constitutive equation can
be written in an incremental form making use of the tangent stiffness matrix of cohesive law, denoted with \( D_T: \dot{p} = D_T(w) \dot{w} \).

### 3 Boundary integral formulation

#### Single-zone boundary integral equations - Consider a homogeneous solid with domain \( \Omega \subset \mathbb{R}^2 \) and boundary \( \Gamma = \Gamma_u \cup \Gamma_p \). Assuming small strains and displacements, consider the following quasi-static external actions: tractions \( \bar{p}(x) \) on \( \Gamma_p \), displacements \( \bar{u}(x) \) on \( \Gamma_u \); the constitutive law is assumed to be isotropic and linear elastic. Boundary integral equations (BIEs) for the linear elastic problem can be derived from Somigliana’s and “hypersingular” identities by performing the boundary limit \( \Omega \ni x \to x^o \in \Gamma \). Somigliana’s identity, which is the boundary integral representation of displacements in the interior of the domain \( x \in \Omega \), is based on Green’s functions which represent components \( u_i \) of the displacement vector \( u \) in a point \( x \) due to: i) a unit force concentrated in space (point \( y \)) and acting on the unbounded elastic space \( \Omega_\infty \) in direction \( j \) (such functions are gathered in matrix \( G_{uu} \)); ii) a unit relative displacement concentrated in space (at a point \( y \)), crossing a surface with normal \( l(y) \) and acting on the unbounded elastic space \( \Omega_\infty \) (in direction \( j \)) (gathered in matrix \( G_{up} \)). The hypersingular identity, which is the boundary integral representation of tractions on a surface of normal \( n(x) \) in the interior of the domain, involves Green’s functions (collected in matrices \( G_{pu} \) and \( G_{pp} \)) which describe components \( (p_i) \) of the traction vector \( p \) on a surface of normal \( n(x) \) due to: i) a unit force concentrated in space (point \( y \)) and acting on the unbounded elastic space \( \Omega_\infty \) in direction \( j \); ii) a unit relative displacement concentrated in space (at a point \( y \), crossing a surface with normal \( l(y) \) and acting on the unbounded elastic space \( \Omega_\infty \) (in direction \( j \). The two integral equations are usually referred to as displacement and traction equations and read as follows [6] for \( x \in \Gamma \), having set \( r = x - y \):

\[
C(x)u(x) + \int_{\Gamma} G_{up}(r; l(y))u(y) \, dy = \int_{\Gamma} G_{wu}(r)p(y) \, dy \quad (4)
\]

\[
D(x)p(x) + \int_{\Gamma} G_{pp}(r; n(x); l(y))u(y) \, dy = -\int_{\Gamma} G_{pu}(r; n(x))p(y) \, dy \quad (5)
\]

As seen from equations (4)-(5), singularities of Green’s functions are triggered off in the limit process, thus generating “free terms” \( C(x) \) and \( D(x) \) and giving to the integrals the distributional connotation of finite part of Hadamard and of Cauchy’s principal value.

**Incremental multi-zone formulation** - Consider \( N \) domains \( \Omega^n, n = 1, 2, \ldots, N \) connected to each other by \( M \) interfaces. Quasi-static external tractions \( \bar{t}^n \) are imposed on the Neumann boundary \( \Gamma^n_p \) of each domain \( \Omega^n \), whereas displacements \( \bar{u}^n \) are imposed on the Dirichlet boundary \( \Gamma^n_u \) of each domain \( \Omega^n \). The incremental BIE formulation of the problem has been proposed in [3] and for paucity of space will be here merely briefly summarized starting from its compact form:

\[
M\ddot{u} = \dot{p}
\]
Equation (6) represents a set of BIEs obtained from the incremental form of equations (4), (5) on the Dirichlet and Neumann boundaries of each domain, as well as suitable linear combinations, by means of the transposed tangent stiffness matrix \( D_T^T \), of equations (4), (5) written on the two faces of each interface (see [3] for further details). It has been proved [3] that if \( D_T = D_T^T \) then the integral operator \( M \) is symmetric with respect to a suitable bilinear form \( A \), that is: \( A(M \dot{u}, \dot{v}) = A(\dot{u}, M \dot{v}) \). It is then possible to give a variational formulation to problem (6), whose solution is a critical point of the functional

\[
\Psi[\dot{u}] = \frac{1}{2} A(\dot{u}, M \dot{u}) - A(\dot{u}, \dot{p})
\]

(7)

From the algebraic point of view, let \( \psi_u(y) \) be a matrix of shape functions and \( \dot{u}_h(y) = \psi_u(y) \dot{u} \) be discrete approximations of \( \dot{u}(y) \). From the stationarity condition of (7), the linear system

\[
M \dot{u} = \dot{p}
\]

(8)

can be obtained, where \( M = A(\psi_u, M \psi_u) \) and \( \dot{p} = A(\psi_u, \dot{p}) \).

**Integration in time** - Problem (8) is a set of first order differential equations. Supplied with initial conditions, a unique solution \( \dot{u} \) exists for any given term \( \dot{p} \) provided that discrete operator \( M(u) \) is invertible. Such a condition, in the analysis of non-linear responses of structures, is not a priori fulfilled if the interface law is not stable in a second order sense (i.e. \( D_T \) is not negative definite). Solution \( u \) of (8) in the range of \( D_T \) negative definite may be approximated [4] by standard integration schemes: explicit Runge-Kutta methods of order two and four have been adopted in the present paper.

**Arc-length method** - Consider the non-linear incremental problem (6) where the structural response \( u \) is obtained for \( p \) varying from a null initial value and evolving quasi statically in time by means of a load factor \( \lambda(t): \dot{p} = \dot{\lambda} \bar{p} \), supposing that \( \lambda \) at least initially increases. The problem under consideration presents a limit points and conventional incremental algorithms, based on a fixed value of load, may fail to overcome such points. Hence, path-following techniques must be used, whereby the load factor \( \lambda \) is added to the set of unknowns. Equation (8) becomes therefore:

\[
M \dot{u} = \dot{\lambda} \bar{p}
\]

(9)

The number of unknowns increases by one and the set of governing equations is given by (9) plus an additional constraint equation which accounts for the increment of the load factor. The arc-length type method is based on the introduction
of a control function giving a measure of the evolution of the loading process. The pioneering form of the constraint equation, proposed originally in [8], reads as follows:

\[ ||\dot{u}||^2 + \dot{\lambda}^2 = c(\lambda, u)^2 \quad (10) \]

Huge efforts have been made in the last two decades - see [9] for an exhaustive review - in order to improve computational efficiency of constraint equation (10). In the present paper, a local control function is used, analogous that proposed in [10], with arc-length suitably adapted by the curvature along the equilibrium path. Differently from all previously cited papers, the present formulation is differential in time whereby the solution is reached by (explicit) time integration strategies. Details of the adopted approach, coupling boundary integral equations and arc-length technique, are reported in [4].

4 Simulation of delamination problems via BEM

The proposed BE model for cohesive interfaces has been used to simulate some FRP - concrete delamination tests. Two different test setups are considered, depicted in Figs. 2a and 7a, together with mechanical and geometrical properties of specimens. For FRP-concrete interface, the nonlinear interface law reported in Fig. 1 has been adopted.

4.1 Simulation of Setup N. 1 delamination test

Fig. 2a shows a typical configuration for pull-pull delamination tests. Left and bottom sides have been constrained in order to have no displacements in the direction normal to the surface and free displacement tangent to it. Two different numerical tests have been performed.

4.1.1 Comparison with experimental tests

Experimental test performed by Chajes et al. [11] have been considered. Experimental results were post-processed in Savoia et al. [1]: FRP - concrete shear stresses and slips along the bond length have been derived from experimental strains in FRP plate. In these tests, FRP plate width (25.4 mm) was much smaller than concrete width. Nevertheless, in the present simulation a plane stress problem has been considered.

Comparison between experimental data and numerical results obtained through the present BE model are reported in Figs. 3a-b, for different bonding lengths. In each figure, strains in FRP plate along the bonding length are reported, for different values of applied force. The highest load corresponds to experimental failure load. All figures show good agreement between numerical and experimental results both at low and high loading levels. Agreement at low loadings (the interface behavior being almost linear) means that the initial interface stiffness is correctly defined. The agreement is very good also for high loadings, where plate delamination is occurring.
Moreover, delamination failure load as a function of bonding length is reported in Fig. 6a. Experimental results are compared with numerical predictions obtained from the proposed BE solution and those given by a recently proposed 1D bond-slip model [2]. The two different models give practically the same results, for both small and long bonding lengths. This circumstance confirms that the corresponding interface laws are actually equivalent. In the same figure, dashed line indicates failure loads obtained by adopting the bilinear interface law (also reported in Fig. 1) together with 2D BE solution. It is clearly shown that failure loads obtained using the bilinear law are significantly overestimated. The asymptotic value (for bonding length approaching infinity) is practically the same, since the same value of fracture energy of interface law has been adopted.

### 4.1.2 Other numerical tests

With reference to the same test setup and mechanical properties of specimens depicted in Fig. 2a, different bond lengths have been considered \((L = 50 \text{ and } 200 \text{ mm})\), whereas the plate width is 50 mm. In this case load-displacement curves obtained from numerical simulations are reported in Figs. 4a-b, compared with results given by 1D model described in [1]. Abscissa refers to relative axial displacement \(u\) between the initial and the final section of FRP plate (axial elongation of the plate).

It is worth noting that, for long bonding lengths (see Fig. 4b), snap-back branch occurs after the attainment of maximum load, due to elastic shortening of FRP.
plate when axial loading is decreasing during delamination. Of course, this simulation cannot be performed with a standard force or displacement control procedure, and more refined techniques are required, as the arc-length technique described in Section 3. Snap-back branch is not present in the case of short bonding lengths (see Fig. 4a).

For the 200 mm bonding length case, slip and shear stress profiles along the anchorage are reported in Figs. 5a-b. Curves refer to three different equilibrium points (A, B, C) underlined in Fig. 4b. Straight line in Fig. 5a shows the slip value corresponding to peak shear stress of interface law \( s = 0.0421 \) mm. Hence, when slip is higher, softening branch in shear stress profile occurs (see Fig. 5b).

Finally, for the 200 mm bonding length, load-plate elongation curve is reported in Fig. 6a, adopting the proposed power fractional law and the bilinear law for the interface. The figure confirms that, adopting the bilinear law, higher values of delamination load are predicted; moreover, the snap-back branch is given by a straight line until complete delamination.

4.2 Simulation of Setup N. 2 test

In Setup N. 2, depicted in Fig. 7a, the specimen is symmetric and the load is applied at two external FRP plates. Moreover, both concrete and FRP plates are restrained at the left end section. Bond length is \( L = 600 \) mm.
Figure 6: a) Maximum transmissible force vs. bond length: experimental and numerical results obtained with different interface laws ($b_p = 25.4$ mm). b) Pull-pull delamination test for $L = 200$ mm: solution obtained with different interface laws ($b_p = 50$ mm).

The corresponding load-displacement curve is reported in Fig. 7b. The figure clearly shows the transition between State 1 condition for low level loads (both stiffness contributions of concrete and FRP plates are present) and State 2 condition after complete FRP delamination (FRP plates only contribute to specimen stiffness). Hence, in this case the post-delamination branch is stable and mono-

Figure 7: Geometrical properties of the specimen and structural response.

Figure 8: Setup N.2: slip $s$ and shear stress $\tau$ distributions along the interface.
tonically increasing. Figs. 8a-b show the profiles along the bonded length of FRP-concrete slip and shear stresses, with reference to three different equilibrium points A, B, C underlined in Fig. 7b. Point A indicates the beginning of delamination process, point C a condition where almost the whole plate is delaminated, and point B an intermediate condition. Fig. 8b clearly shows that, during delamination, the portion of plate contributing to the anchorage translates along the specimen from the (right) loaded end to the (left) restrained section.

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References