QNDE using complete frequency information from ultrasound

G. Rus\textsuperscript{1}, S.-C. Wooh\textsuperscript{1} & R. Gallego\textsuperscript{2}
\textsuperscript{1}NDE Lab, Department of Civil and Environmental Engineering, Massachusetts Institute of Technology, Cambridge, MA 02139, USA
\textsuperscript{2}Department of Structural Mechanics, University of Granada, Spain

Abstract

Practical NDE techniques by ultrasonics are based on A, B and C scan images processed and filtered in a variety of ways, from which the characterization of the defects are quantitatively extracted by heuristic methods or even by visual interpretation of the image. Apart of the arbitrariness of the choice of this threshold, a great amount of information contained in the image and therefore in the measured signal is being lost. This rich data may be crucial to combat the noise that hides difficult defects. This paper presents an effort to integrate all the information recorded in the measurements in a generalized processing or inversion scheme. QNDE was originated as an application of the fast developing numerical methods to so-called inverse problems. A number of works have been developed for idealized probes with emphasis on the numerical techniques, but using a vaguely developed link to the measurement and search procedure. In this paper, such a numerical procedure is extended and deployed to an experimental case. The principle to be used is the measurement and inversion of frequency-domain information instead of classical time-domain delays or vibration eigenmodes or eigenvalues, together with the use of a reduced set of output data understood as a regularization technique to drastically overcome noise problems. A deconvolution scheme from a sane specimen overrides uncertainties about the input signal and other coherent noise. The main strength of this approach is that it is not necessary to visually be able to identify the portion of the signal that contains the information about the flaw, which may be hidden under many complicated patterns or other waves. The approach is used for the experimental case of an aluminum specimen with a defect in the form of a side drilled hole. The ultrasonic measurements are using a simulated array of transmitters as well as receivers. A wedge transducer has been developed and manufactured for this technique to achieve point contact. Neither of them are placed right above the defect, and the magnitude of surface waves hides the echo from the defect. Despite these facts, this inversion technique successfully finds the depth and size of the defect.
1 Introduction

When seeking defects inside a body, any physical magnitude that propagates inside it and manifests on an accessible part of it may be studied in principle in order to obtain information about what is happening inside. This work is aimed at the study of **acoustic response**, i.e. the propagation of elastic waves, which governs the well known ultrasonic detection techniques.

To obtain a quantitative characterization, a more or less detailed model of the propagation of the ultrasound is usually made (see Wooh et al. [1] or Boström et al. [2]). In this case a numerical model for a generic geometry is established and solved by the boundary element method (BEM). In identification problems, the BEM was first used numerically for defects defined by simple geometrical entities by Bezerra and Saigal [3] or Mellings and Aliabadi [4]. Many other authors made important contributions in the search procedure during the last decade, as Bonnet [5], Burczyński et al. [6] or Stavroulakis [7]. For the solution of the IP we choose the procedure developed by Gallego, Rus and Suárez [8, 9, 10], based on the minimization of the discrepancy between measured and predicted response.

The issue of the modelization of the transducers has never been explicitly studied with much detail. There is very little knowledge about the mechanical coupling between the transducer and the specimen, and this needs to be improved in this project. Some guidelines are given in the nondestructive testing handbook [11], mainly about basic theoretical considerations (parts 2 and 3). Some experimental results are shown by Marty et al. [12] or Kimoto and Hirose [13]. A transfer function is also introduced to take into account the transducer-specimen system, as introduced by Schmerr [14].

The main disadvantages to a large transducer include signal distortion, cutting-off of certain frequency components, and the near field effect, which are referred to as the **aperture effects** in general. The benefits of using point sources and point receivers to NDE have been addressed by Sachse [15] or Ying [16]. For this experiment a triangular wedge of aluminum is introduced, whose analysis and design is addressed by Rus et al. [17].

2 NDE system model

In order to build a model we differentiate several parts in the system for the nondestructive testing (Fig. 1): the **waveform generator**, which generates the signals for the emitters; the **transducers**, transmitters, which are in charge of emitting a wave into the specimen; and receivers, who will register some data about the propagation of it; the **specimen**, a block of material to be analyzed; and an **oscilloscope**, aimed at recording the signals from the receivers.

From the point of view of the traveling information, among the numerous ones, the following steps are of particular interest: Input signal \( s_{m}^{(I)}(t) \). This is the signal emitted from the wave generator. In a general case, there may be one for each transmitter \((m)\). Transmitter signal \( s_{m}^{(T)}(t) \). This is the signal sent between the
emitter and the specimen. Receiver signal $s^{(R)}_{mn}(t)$. This is the signal sent between the specimen and the receiver. In general, for each receiver $(n)$ there will be a different signal for each transmitter $(m)$. Output signal $s^{(O)}_{mn}(t)$. This is the signal recorded at the oscilloscope. The relationships between these signals are the following (* means convolution), in time domain:

$$s^{(O)}_{mn}(t) = h^{(RO)}_{m}(t) \ast s^{(R)}_{mn}(t)$$
$$s^{(R)}_{mn}(t) = h^{(TR)}_{mn}(t) \ast s^{(T)}_{m}(t)$$
$$s^{(T)}_{m}(t) = h^{(IT)}_{m}(t) \ast s^{(I)}_{m}(t)$$

(1)

The hypothesis for the transducers are that (1) they are modelled as prescribed displacements or tractions, (2) the mechanical and electrical systems are assumed to be linear time shift invariants which can be reduced and modeled as a set of transfer functions $h(t)$, (3) an uniformly weighted amplitude and phase is considered over the interface surface, and (4) only the normal component of the displacement or traction is considered:

$$s^{(T)}_{m}(t) = q_i(x, t)n_i \quad \text{(transmitter)}$$
$$s^{(R)}_{mn}(t) = u_i(t)n_i = \int_{\Gamma_n} u_i(\Gamma, t) d\Gamma n_i \quad \text{(receiver)}$$

(2)

(3)

3 Procedure for the forward problem

3.1 Measurement data

In order to eliminate the uncertainties from the shape of the excitation as well as some coherent noise, a procedure is designed based on the convolution and comparison between the damaged and a sane specimen. The linear relationships obtained between the recorded output signal at receiver $n$, $s^{(O)}_{nm}(t)$, and the induced
input signal for transmitter $m s_m^{(I)}(t)$ in equations 1 can be grouped into $g_{mn}(t)$,

$$s_{mn}^{(O)}(t) = g_{mn}(t) \ast h_{mn}^{(TR)}(t)$$  
$$g_{mn}(t) = s_{n}^{(I)}(t) \ast h_{m}^{(IT)}(t) \ast h_{n}^{(RO)}(t)$$  

This takes into account the complete performance of the NDE system in one single step. $h_{mn}^{(TR)}(t)$ is the response of the specimen in terms of averaged displacements over the transducer due to a pulse traction at the emitter.

### 3.2 Real and theoretical models

The relationship between a real and an ideal model is described to assert some hypothesis of the model. The real and ideal models are governed by the following reduced relationships (we introduce the hat to denote theoretical values):

$$s_{mn}^{(O)}(t) = h_{n}^{(RO)}(t) \ast h_{mn}^{(TR)}(t) \ast h_{m}^{(IT)}(t) \ast s_{m}^{(I)}(t)$$  
(real)  

$$s_{mn}^{(O)}(t) = \hat{h}_{n}^{(RO)}(t) \ast \hat{h}_{mn}^{(TR)}(t) \ast \hat{h}_{m}^{(IT)}(t) \ast \hat{s}_{m}^{(I)}(t)$$  
(ideal)

In order to establish a relationship between the real and theoretical magnitudes $f(t)$ and $\hat{f}(t)$, we assume a nonlinear form $f(t) = z(\hat{f}(t) + n(t) \ast \zeta(t))$, where an unknown gain scaling parameter $z$ is introduced since the overall amplitude of the output signal or input signal may vary depending on variations on the pressure on the transducers, as well as an arbitrary choice of magnitude of measurement.

### 3.3 Boundary element method

In modeling and predicting the propagation of the ultrasonic waves in the specimen, we use the boundary element method (BEM) because of its clear advantages over the finite element or other discrete methods. First, the BEM does not require re-meshing of the body domain at each iteration. This not only reduces the computational time but also eliminates small but important perturbations due to the changes of the mesh. Second, by reducing the dimension of the problem by one, the fine meshes required by high frequency become affordable through the BEM. We make use of the singular formulation of the boundary integral equation

$$c_k^i(x) u_k(x) + \int_{\Gamma} \left[ p_k^i(y; x) u_k(y) - u_k^i(y; x) p_k(y) \right] d\Gamma(y) = 0 ,$$

This equation obviously relates the displacements $u_k$ and the tractions $p_k$ exclusively at the boundaries. If we use complex presentation of fundamental harmonic solutions for $p_k^i$ and $u_k^i$, then solving this equation yields fundamental harmonic solutions for a single frequency $\omega$. In the equation, $c_k^i$ is a geometry-dependent constant, and the integral has the sense of Cauchy’s Principal Value. In implementation, we use the classical conforming discretization scheme with quadratic elements, 8-point Gauss integration after regularization and displaced collocation strategy. The implementation details are developed in Rus [18]. This equation is used for both boundary and internal points [19, 20]. The voids are modeled as
The discretization is made using a maximum size of 2 mm in order to obtain an accuracy of the order of 1%.

Once the response in terms of displacements, tractions and measurements are obtained in the frequency domain over a sampling from 0 to 4 MHz every 20 kHz (chosen to capture a time of 50 µs and the performance of a broadband transducer of 500 kHz), they are processed by a Fourier transform to obtain the time domain response.

4 Procedure for inversion

4.1 Parametrization

The parametrization can be defined within the subject of inverse problems as a characterization of the sought information (i.e. defect size and position) with a reduced set of \( g \) variables \( (p_g) \). The issue of parametrization is complicated due to the relationship with many considerations of the inverse problem. From the conceptual point of view, it can be seen as the most powerful means of regularization of inverse problems, since it provides a priori information in the form of strong hypothesis in the possible forms of the sought defects. In this case, a simple parametrization is used, consisting of two parameters: depth and the diameter of the side-drilled hole defect type.

4.2 Residual

A residual vector \( \mathbf{R} \) is defined as the discrepancy between ideal measurements and theoretical predictions. Here the previously presented concept of parametrization is introduced. The prediction is dependent on a set of \( g \) parameters \( p_g \) (a superscript \( r \) denotes the case of the real flaw). The information to be used for the definition of the residual is not the classical output signal \( s^{(O)}(t) \) but the response of the model \( h^{(TR)}(t) \) or \( H^{(TR)}(\omega_j) \) (the convention of capital letters for frequency domain is adopted). It is moreover analyzed in the frequency domain.

\[
\hat{H}^{(TR)}_{mnj}(p_g, z) = \frac{1}{z^{(TR)}} H^{(TR)}_{mnj} + N^{(TR)}_{mnj} * \zeta_j
\]  (7)

The approach is understood in the following sense, and a residual vector \( \mathbf{R} \) in terms of the discrepancy \( D \) as in equation 9 (here we have introduced a filter \( W_j \) aimed at weighting more important frequencies and eliminating others, of which we can make powerful use):

\[
\hat{H}^{(TR)}_{mnj}(p_g) = \hat{H}^{(TR)}_{mnj}(p_g, z) + D_{mnj}(p_g, z) = W_j \left[ \hat{H}^{(TR)}_{mnj}(p_g) - \frac{1}{z} H^{(TR)}_{mnj} \right]
\]  (8)
Table 1: Configuration of the experimental setup.

<table>
<thead>
<tr>
<th>Wave generator</th>
<th>Oscilloscope</th>
</tr>
</thead>
<tbody>
<tr>
<td>Waveform Generator</td>
<td>HP 33120A</td>
</tr>
<tr>
<td>Repetition rate</td>
<td>50 Hz</td>
</tr>
<tr>
<td>Burst cycle</td>
<td>1</td>
</tr>
<tr>
<td>Pulse height</td>
<td>500 mV</td>
</tr>
<tr>
<td>Frequency</td>
<td>500 kHz</td>
</tr>
<tr>
<td>Burst shape</td>
<td>spike pulse</td>
</tr>
<tr>
<td>RF Power Amplifier</td>
<td>ENI 2100L</td>
</tr>
<tr>
<td>Amplification</td>
<td>+50 dB</td>
</tr>
<tr>
<td>Frequency range</td>
<td>10 kHz-12 MHz</td>
</tr>
</tbody>
</table>

4.3 Cost functional

We define a cost functional $J$ in terms of the former residual $R$ in a least squares sense. This definition is meaningful from the statistical point of view, as well as from a space theory, since it minimizes distances in an euclidean sense. The cost functional is hence defined as in Eqn. 10 (where $T$ stands for the transpose in vectorial notation and $\overline{R}$ means the conjugate of the complex magnitude $R$):

$$J = \frac{1}{2} R^T \overline{R} = \frac{1}{2} ||R||^2 \quad \Rightarrow \quad J(p_g, z) = \frac{1}{2} R_{mnj}(p_g, z) \overline{R}_{mnj}(p_g, z) \quad (10)$$

5 Experimental results

5.1 Methodology

The previous formulation is used in the analysis of a subsurface defect in an aluminum specimen. The identification is intended in 2D. For that purpose, a sufficiently wide specimen is used, together with a strip-like defect and line transducers. These are made using trapezoidal aluminum wedges between the piezoelectric and the specimen, in order to transform the contact area of the former to a line source or line receiver. The design of this wedge is based on the criteria of a work by Rus et al. [17]. The information of the electronic setup is given in table 1, and the geometry of the specimen as well as some pictures of the specimen and the experimental setup are given in figure 2.

5.2 Results

Figure 3 shows an example of the comparison between the experimental and synthesized signals. Notice that the signals for the case without and with defect are
very similar because the main part of it consists of a surface wave, and the reflection from the defect is softer and difficult to see. Fig. 4 shows the evolution of the cost functional $J$ with respect to the defect parameters. In both cases, the cost functional shows a minimum for the value of the parameter very close or corresponding to reality (relative diameter $\frac{D_{\text{predicted}}}{D_{\text{real}}} = 1$, and relative depth...
Figure 4: Variation of the cost functional with respect to the diameter (left) and depth (right).

\[
\frac{d_{\text{predicted}} - d_{\text{real}}}{d_{\text{real}}} = 0.
\]

The case of the diameter is less sensitive, which we can see in the figure from the fact that the range of values for \( J \) (ratio around 1.2:1) is smaller than for the depth (ratio around 2.5:1). This explains that the presence of errors from any source alters the prediction.

6 Conclusions

A method for evaluation of defects using frequency domain information has been developed and tested experimentally. Its main characteristics are: (1) the inversion technique successfully overcomes the predominance of surface waves. (2) The noise problem is dealt with by a strong regularization scheme such as the parametrization, besides not leaving aside any of the information gathered during the measurement. (3) The BEM is used as a very convenient tool for solving the forward problem of modeling the wave propagation in the specimen. Besides the computational speed, the most important characteristic is the lack of need for a domain mesh, which eliminates numerical perturbations that strongly affect the search procedures. (4) A calibration in both amplitude and time domain, in addition of the presence of very superabundant measurements allows to work with unknown parameters, such as the exact wavespeed, the actual pressure applied to the transducers, their response, and the input signal.

Acknowledgements

We would like to gratefully acknowledge the Fulbright Foundation and the Ministerio de Educación Cultura y Deporte for the postdoctoral fellowship FU2002-0442 supporting this work.
References


