Combined application of BEM and cellular automata to some shape optimization in acoustic fields

M. Tanaka¹, T. Matsumoto¹ & Y. Arai²

¹Department of Mechanical Systems Engineering, Shinshu University, 4-17-1 Wakasato, Nagano City, 380-8553 Japan
²Research Associate of Shinshu University, Japan

Abstract

This paper concerns a combined use of BEM and cellular automata to some shape optimization in the acoustic fields. Attempt is made to find novel optimal shapes of the sound-insulating wall for reduction of noise from auto-highways. The optimization procedure is briefly explained, while emphasis is placed on demonstrating versatility of the present optimization procedure and also applicability to some topological shape optimization of the sound-insulating wall under several important situations.

1 Introduction

This paper presents a combined use of BEM and cellular automata [1] [2] [3] [4] to some shape optimization in the acoustic fields. Cellular automata have been widely applied to various problems, and expected as a useful method even for optimization. In application of cellular automata to optimization problems, no sensitivity analysis is required. Therefore, it can be expected that the solution is rather easily obtained using a simple local rule and also a simple transition rule. In this paper, we shall apply a cellular automaton, combining the boundary element method, to finding novel optimal shapes of the sound-insulating wall for reduction of noise from auto-highways.

A two-dimensional model of the auto-highway with sound-insulating wall, in which a wall stands perpendicularly to the infinite horizontal plane of the ground and a point source of noise is located at the central point on the road. Details of
the cellular automaton applied together with the boundary element method are discussed in authors’ separate paper [5]. In this paper, the optimization procedure is briefly explained, while emphasis is placed on demonstrating versatility of the present optimization procedure and also applicability to some topological shape optimization of the sound-insulating wall under several important situations.

2 Boundary element analysis of acoustic fields

Under the assumption of time-harmonic vibration with an infinitesimal amplitude, the acoustic fields are governed by the non-homogeneous Helmholtz equation as follows [6]:

$$\nabla^2 p(x) + k^2 p(x) + f(x) = 0$$  \hspace{1cm} (1)

where $p$ is the sound pressure, $f$ the source term, and $k$ the wave number.

The boundary element method [7] for the acoustic fields governed by the above Helmholtz equation (1) is briefly explained. The integral expression of eqn (1) multiplied with the fundamental solution $p^*(x, y)$ over the whole domain of interest is twice integrated by parts. The boundary integral equation can eventually be obtained in the regularized from if the uniform potential condition is taken into account, and it can be expressed as follows [7]:

$$\left[ \int_{\Gamma} \left\{ q^*(x, y) - Q^*(x, y) \right\} d\Gamma(x) \right] p(y) + \int_{\Gamma} q^*(x, y) \left\{ p(x) - p(y) \right\} d\Gamma(x) = i\omega \rho \int_{\Gamma} p^*(x, y) v(x) d\Gamma(x) + Ip^*(x^s, y)$$  \hspace{1cm} (2)

where $Q^*(x, y)$ is the normal derivative of the fundamental solution for the Laplacian operator, and $I$ the intensity of point sound source. It is interesting to note that the standard Gaussian numerical quadrature can be applied to numerical computation of discretized set of equations, because the regularized integral equation is employed in this study. A semi-infinite acoustic field of two dimensions is treated in this study, but eqn(2) can be used even for such problems without any difficulty, because the Sommerfeld radiation condition is satisfied at infinity by the fundamental solution of the Helmholtz equation.

The boundary integral equation (2) can yield a set of equations for the sound pressure $p$ and the particle velocity $v$, if the boundary $\Gamma$ is discretized into boundary elements. Application of the boundary conditions provides the solution of the problem [7]. As has been well known, the solution of the external problem is influenced by fictitious eigenvalues of the internal problem. To circumvent this difficulty, an appropriate number of the boundary integral equations with the source points located in the external domain are supplemented into the system of equations, and the solution is obtained by using the least-square method [7].
3 Application of cellular automata

In the cellular automaton (CA), the domain of interest is discretized into a number of uniform cells. Some quantity which represents the cell state is assigned to each cell, and then is modified by a transition rule under the condition that a local rule for cells is satisfied [1] [2]. An optimal solution can thus be found in an iterative manner. A concrete procedure of the CA applied to a two-dimensional problem will be briefly explained in the following.

Triangular or rectangular cells can be used for the two dimensional problems. In this study, rectangular cells are employed and the domain of investigation is discretized into cells of a uniform size. The Moore neighbor cells are defined so that the target cell is surrounded by neighbor eight cells. Then, we introduce a local rule determining a mutual relation between the neighbor cells for the current state of iterative computation. In this study, a particular portion of the sound-insulating wall is altered to reduce the sound pressure level (SPL) of a mean sound pressure of several evaluation points in the acoustic field. Only the domain of investigation is discretized into uniform rectangular cells as shown in Fig. 1. The fictitious cells are introduced so that the local rule can be applied even if the target cell is located at the corners of the domain.

![Figure 1: Feasible region divided into uniform square cells.](image)

3.1 Local rule

A local rule for the target cell is determined by considering some constraints on the wall shape. In this study, we shall impose the constraint that the cavity should not exist in the wall, and that only one disconnect part of the wall is allowed to exist in the wall shape. Under these assumptions, the wall shapes which are illustrated in Fig. 2 can not be permitted. It is assumed that the state of each cell can be ‘Alive’ or ‘Dead’. It is checked whether the Moore neighbor cells follow the local rule or not, by assuming that the target cell changes from the current state to its opposite state. If the neighbor cells follow the local rule, no change in the cell state is assumed and the next cell is examined. If the neighbor cells do not follow the local rule, the assumed state of cell is adopted and boundary element analysis is carried out.
under a modified shape of the wall. In this study, we also assume that there can be cells which always remain, called ‘Remaining Cell’.

Type A : Upper and lower cells as well as right and left cells are all ‘Alive’.

![Figure 2: Unallowable wall shapes.](image)

3.2 Method of evaluation

As an evaluation value for optimization, we shall take the averaged value of absolute sound pressures obtained at several points of evaluation and its sound pressure level (SPL) in dB.

3.3 Transition rule

We shall employ a transition rule to determine whether one cell is added or removed to find a more suitable shape of the insulating wall. In what follows, such a transition rule is explained.

![Procedure 1 and Procedure 2: Evaluation procedure.](image)

**Step 1** In ‘Procedure 1’ of Fig. 3, the target cell of cell 1 is assumed to be ‘Alive’ (See ‘Procedure 2’ in the figure.) This step can be examined even if the
target cell is located at the boundary of domain under consideration, because fictitious cells are assumed to be along the boundary.

Step 2 Apply the local rule to the target cell of ‘Alive’ to check whether the Moore neighbor cells follow the local rule. If the local rule holds for the Moore neighbor of the target cell, the target cell should be ‘Dead’. Then, we proceed the target cell to the next one.

Step 3 If the local rule does not hold in the step 2, then boundary element analysis is carried out by adding one cell.

Step 4 Perform the same approach from step 2 to step 3 at the cell 2 to 34 in turn.

Step 5 The best evaluation value and the corresponding data of wall shape are stored as the new value for the next loop of computations from Step 2.

Step 6 If the evaluation value does not change anymore, it is considered that the optimal shape is obtained.

It is interesting to note that BE analysis is carried out in the following manner. The boundary curve of the sound-insulating wall is created by a spline function using the center points of cell sides located on the boundary, and the created curve is divided into boundary elements with quadratic interpolation functions. The details of this procedure are explained in authors’ separate paper [5].

![Figure 4: Analysis model.](image)

4 Numerical results and discussions

To demonstrate the versatility of the proposed method for optimal shape design of the sound-insulating wall, we now consider a two-dimensional model shown in Fig. 4 in which the sound source is located at point A. It is assumed that the infinite horizontal plane and the sound-insulating wall are subject to the rigid condition in which particle velocity \( v = 0 \). Furthermore, we assume that the intensity of point source A is \((2.0, 1.0)\) [Pa], the sound speed \( C_0 = 340 \) [m/s], and density of mass \( \rho = 1.2 \) [kg/m\(^3\)]. It is noted that symmetry with respect to the axis \( x_1 \), the infinite horizontal plane, is taken into account, whereas symmetry with respect to the axis
Figure 5: Initial wall shape and initial cell condition for the region surrounded by dashed line.

Figure 6: Final wall shape for pure sound of 50 [Hz].

$x_2$ is not considered. Four evaluation points of sound pressure are located on the horizontal ground as shown in Fig. 4. Assuming that the shape of sound-insulating wall can change in the area hatched in the figure, this area is divided into square cells of a uniform size \(0.2 \text{[m]} \times 0.2 \text{[m]}\). We now assume the initial wall shape including the disconnect part of a roof, as shown in left side of Fig. 5. The square region surrounded by a dashed line in left side of Fig. 5 is divide into uniform cells as shown in right side of Fig. 5. We first consider the cases where the noise source emanates a pure sound of 50 [Hz] and also 63 [Hz].

The optimal shapes of the wall found by the present procedure are shown in Fig. 6 for 50 [Hz] and Fig. 8 for 63 [Hz], respectively. The convergence properties with respect to the mean SPL are illustrated in Fig. 7 for 50 [Hz] and Fig. 9 for 63 [Hz], respectively. It can be seen that optimal shapes of the wall are successfully found.
in which a drastic reduction is made for the mean SPL.

Next, we shall show the results obtained for a simple noise model which includes two components of 50 [Hz] and 63 [Hz] with an identical intensity. In this case, we twice carry out boundary element analysis of the acoustic field for the two components of sound, and then superpose the results obtained for each frequency. The optimal shape is shown in Fig. 10, which has been found from the initial shape
Figure 9: Change in mean SPL for pure sound of 63 [Hz].

Figure 10: Final wall shape for noise of 50 [Hz] and 63 [Hz].

shown in Fig. 5. The convergence property is illustrated in Fig. 11 for the SPL of the averaged value of absolute sound pressures.
Figure 11: Change in mean SPL for noise of 50 [Hz] and 63 [Hz].

5 Concluding remark

The boundary element method (BEM) has been combined with a cellular automaton (CA) to find new optimal shapes of the sound-insulating wall under various conditions. The novel shapes of the wall have been successfully found by the proposed procedure.

It may be concluded that the present optimization procedure would provide a means to find some topological optimal shapes for reducing the averaged sound pressure level at several measuring points. The paper has been focusing shape optimization of the sound-insulating wall, a similar approach could be applied to a number of optimization problems in acoustic fields as well as other field problems.

The cellular automata could provide a number of optimization procedures, if the local rule and the transition rule are changed. Therefore, as future work in this direction, it is recommended to find some novel shapes of the sound-insulating wall from the various view points, such as economical, effective, beautiful, etc.

References


