Time domain calculation of the transient current distribution along a thin wire antenna array

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Abstract

Transient current distribution along an arbitrary thin wire antenna array is considered. The mathematical model is based on a set of coupled space-time integral equations of the Hallen type. The effect of inhomogeneous medium is taken into account by the space-time reflection coefficient appearing within the integral-equation kernels. Space-time current distributions along the wires are obtained by solving the integral equation set using the time-domain Galerkin-Bubnov Boundary Integral Equation Method (GB-BIEM). Numerical calculation is performed for the three and two wire antenna arrays horizontally located over a dielectric half-space, when the central wire is excited by a time-dependent Gaussian pulse voltage source.

1 Introduction

The analysis of the transient current distribution along an arbitrary thin wire antenna array horizontally located over a dielectric half-space is performed in this work. Space-time current distribution along wire antenna structures is significant in variety of applications such as electromagnetic interaction with biological tissue, mobile communications, ground-penetrating radar, target identification, printed circuit boards design, lightning protection design, etc.

Transient analysis of thin-wire structures is usually performed in the frequency domain, using inverse Fourier transform to obtain a time domain response [3, 4]. In this work the transient response of the wire structure is
obtained directly in a time domain, via Galerkin-Bubnov Boundary Integral Equation Method (TD GB-BIEM) based on the method described in [1].

Also, the analysis of a single thin wire is most frequently used as a showcase for a certain method. In this work, the applied GB-BIEM is extended to a case of an arbitrary thin wire antenna array. Geometry observed is constrained only by the request that wires have to be mutually parallel and located horizontally over a dielectric half-space.

The mathematical model is based on a set of coupled space-time integral equations of the Hallen type. The effect of inhomogeneous medium is taken into account by the space-time reflection coefficient (RC) appearing within the integral-equation kernels. Numerical calculation is carried out for the case of a lossless dielectric half-space, thus significantly reducing computational cost of the procedure [2].

Illustrative numerical results are presented for the antenna array consisting of three wires of identical length, radius and separation. General procedure presented is applicable to any number of wires, while length, radius, position and separation can be arbitrary chosen for each individual wire. Numerical results for a two-wire case are also presented and are found to be in a good agreement with the results published in [5].

2 Time domain formulation for a thin wire array

In the case of multiple wire antenna array configuration, it is more advantageous to describe structure geometry through coordinates of the wire end points (Figure 1). All the wires are located parallel to the dielectric plane at the height $z=h$, assuming dielectric plane is placed in $z=0$. Length of the $j$ antenna is $L_j=x_{L_j}-x_{Oj}$, while distance between wires $i$ and $j$ is given by $d_{ij}=|y_i-y_j|$.

![Figure 1: Geometry of the multiple wire array.](image-url)
If the array of M parallel wires is observed, all wires effecting observed wire should be taken into account, and space-time integral equation for a single wire can be expanded to a set of coupled space-time integral equations:

\[
\sum_{s=1}^{M} \int_{0}^{L_s} \frac{I_s(x', t - \frac{R_{vs}}{c})}{4\pi R_{vs}} \, dx' - \sum_{s=1}^{M} \int_{-\infty}^{0} \int_{-\infty}^{0} \frac{I_s(x', t - \frac{R_{vs}'}{c} - \tau)}{4\pi R_{vs}'} \, dx' \, d\tau
\]

\[
= F_{ov}(t - \frac{x - x_{ov}}{c}) + F_{lv}(t - \frac{x_{lv} - x}{c}) + \frac{1}{2Z_0} \int_{0}^{L_v} E_{x}^{inc}(x', t - \frac{|x - x'|}{c}) \, dx'
\]

where \( v, s = 1, 2, \ldots, M \) denote the index of the observed and source wire, respectively. It is assumed that \( x' \) (source points) are always located on the source antenna \( s \), and \( x \) (observation points) are on the observed antenna \( v \). Distances between observation point and source point are given by:

\[
R_{vs} = \sqrt{(x - x')^2 + d_{vs}^2 + 4h^2};
\]

\[
R_{vs} = \begin{cases} 
\sqrt{(x - x')^2 + d_{vs}^2} & : v \neq s \\
\sqrt{(x - x')^2 + a^2} & : v = s 
\end{cases}
\]

Unknown functions \( F_{ov}(t) \) and \( F_{lv}(t) \) are related to the multiple reflections of transient currents at the free wire ends and are given by:

\[
F_{ov}(t) = \sum_{n=0}^{\infty} K_{ov}(t - \frac{2nL_v}{c}) - \sum_{n=0}^{\infty} K_{lv}(t - \frac{(2n+1)L_v}{c})
\]

\[
F_{lv}(t) = \sum_{n=0}^{\infty} K_{lv}(t - \frac{2nL_v}{c}) - \sum_{n=0}^{\infty} K_{ov}(t - \frac{(2n+1)L_v}{c})
\]

where the auxiliary functions \( K \) are:

\[
K_{ov}(t) = \sum_{s=1}^{M} \int_{0}^{L_s} \frac{I_s(x', t - \frac{R_{vs}^{(0)}}{c})}{4\pi R_{vs}^{(0)}} \, dx' - \sum_{s=1}^{M} \int_{-\infty}^{0} \int_{-\infty}^{0} \frac{I_s(x', t - \frac{R_{vs}'^{(0)}}{c} - \tau)}{4\pi R_{vs}'^{(0)}} \, dx' \, d\tau
\]

\[- \frac{1}{2Z_0} \int_{0}^{L_v} E_{x}^{inc}(x', t - \frac{|x - x'|}{c}) \, dx'
\]
Space-time reflection coefficient $r_{vs}(\theta,t)$ is given by

$$r_{vs}(\theta',t) = A\delta(t) + \frac{4\beta}{1-\beta^2} \frac{e^{-\alpha t}}{t} \sum_{n=1}^{\infty} (-1)^{n+1} n A^n I_n(\alpha t)$$

where:

$$\beta = \frac{\sqrt{\varepsilon_r - \sin^2 \theta_{vs}^\prime}}{\varepsilon_r \cos \theta_{vs}^\prime}; \quad \theta_{vs}^\prime = \arctan \left( \frac{\sqrt{(x'-x)^2 + d_{vs}^2}}{2h} \right)$$

The set of N coupled space-time integral equations, given by eqn (1), can now be solved by taking into account boundary and initial conditions. In accordance with the well-known thin wire approximation, boundary conditions consist of request that each individual wire has zero current at its ends. Initial conditions require that the wire geometry should not be excited before the instant $t=t_0$.

## 3 Space-time numerical procedure

The set of eqns (1) derived in previous chapter can be solved using a certain direct time-domain numerical technique. Presented TD formulation is applicable to a real-ground case, however, before the numerical procedure is applied, it is useful to simplify the equation, thus greatly reducing computational cost of numerical procedure. The reflection coefficient $r(\theta,t)$ appearing inside IE kernel is space time dependent, thus its numerical solution is very computationally
expensive. Therefore, it makes sense observing two special cases where \( a = \delta \): wire array over a perfect ground and over a dielectric half-space. In these cases second term in eqn (7) vanishes, and reflection coefficient becomes only space dependant:

\[
    r(\theta, \tau) = A \delta(t) \tag{9}
\]

If the time domain convolution is performed in all terms with reflection coefficient and the property of the delta function is considered, two-dimensional integral in eqn (1) becomes one-dimensional:

\[
    \sum_{z=1}^{M} \int_{0}^{L_z} I_z(x', t - \frac{R_{xz}}{c}) \frac{dx'}{4\pi R_{xz}} - \sum_{z=1}^{M} \int_{0}^{L_z} r_{z}(\theta) \frac{I_z(x', t - \frac{R_{xz}}{c})}{4\pi R_{xz}} dx' = F_{0v}(t - \frac{x - x_{0v}}{c})
\]

\[
    + F_{Lv}(t - \frac{x_{Lv} - x}{c}) \frac{1}{2Z_0} \int_{0}^{L_v} E^{inc}_{x}(x', t - \frac{|x - x'|}{c}) dx'. \tag{10}
\]

Solutions of eqn (10) are obtained using TD GB BIEM technique. First, space discretisation is applied using standard finite element space discretisation procedure. Then, time discretisation is performed using weighted residual approach. Finally, time-marching technique is used and transient response of all wires is obtained using recurrence formula. Local approximation for the wire segment is given by:

\[
    I(x', t') = \{f\}^T \{I\} \tag{11}
\]

where \( \{I\} \) denotes space-time dependant solution vector, and \( \{f\}^T \) denotes vector containing a space-domain shape functions given by:

\[
    f_r(x') = \frac{x_{r+1} - x'}{x_{r+1} - x_r} \tag{12}
\]

\[
    f_{r+1}(x') = \frac{x' - x_r}{x_{r+1} - x_r}
\]

where \( r \) is the global node index of the observed boundary element.

Applying finite-element discretisation on eqn (10) yields local system of linear equations for the \( v \)th wire:
where \( i \) denotes index of the source element located on the \( s \)th wire, and \( j \) denotes index of the observed element located on the \( v \)th wire.

Global matrix system given by eqn (14) can be obtained by substituting eqns (2)-(8) into eqn (13). It is more convenient expressing global matrix using submatrices that describe spatial influence of \( s \)th wire on the \( v \)th wire.
where \{E\} denotes excitation vector and space-dependant matrices are:

\[
\begin{align*}
[A_\nu] &= \int \int \frac{1}{4\pi R_{\nu s}} \{f\} \{f\}_i^T \, dx' \, dx; \\
&= \int \int \frac{r_{\nu s}(\theta)}{4\pi R_{\nu s}} \{f\} \{f\}_i^T \, dx' \, dx \\
[B_\nu] &= \frac{1}{Z_0} \int \{f\}_i \{f\}_i^T \, dx' \, dx \\
[C_\nu] &= \int \int \frac{1}{4\pi R_{\nu s}^{(0)}} \{f\} \{f\}_i^T \, dx' \, dx; \\
&= \int \int \frac{r_{\nu s}(\theta)}{4\pi R_{\nu s}^{(0)}} \{f\} \{f\}_i^T \, dx' \, dx \\
[D_\nu] &= \frac{1}{Z_0} \int \{f\} \{f\}_i^T \, dx' \, dx \\
[E_\nu] &= \int \int \frac{1}{4\pi R_{\nu s}^{(L)}} \{f\} \{f\}_i^T \, dx' \, dx; \\
&= \int \int \frac{r_{\nu s}(\theta)}{4\pi R_{\nu s}^{(L)}} \{f\} \{f\}_i^T \, dx' \, dx
\end{align*}
\]

Expressions with summation from \(n=0\) to infinity pertain to reflections from the wire ends. Since observed time interval is finite, it is sufficient to take into account only those reflections appearing within given interval, thus replacing infinity with finite integer value. Shorter observed interval would yield smaller number of summands, and vice versa.

Final global matrix system has the same form as the eqn (14), except global matrices \([A], [A^*], [B], [C], [C^*], [D], [E], [E^*]\) are used instead of their appropriate submatrices. Global matrices are obtained by simply assembling submatrices for all observed and source wires \(o, s=1, 2, \ldots, M\).

After the space-domain discretisation is performed, the weighted residual approach is used for time-domain discretisation. Obtained global matrix system can be generally written as:

\[
\begin{bmatrix}
A
\end{bmatrix}\{I\} \bigg|_{t_r \rightarrow c} - \begin{bmatrix}
A^*
\end{bmatrix}\{I\} \bigg|_{t_r \rightarrow c} = \{g\}
\]

(16)

The time-domain solution on the \(i\)th finite element is given by:

\[
I_i(t) = \sum_{k=1}^{N_i} I_i^k T_k(t)
\]

(17)

where \(I_i^k\) are unknown coefficients, \(T_k\) are the linear time-domain shape functions and \(N_i\) is the total number of time samples. Using weighted residual approach on (16) leads to:
where $\theta_k$ denotes the set of time-domain weights. Using Dirac impulses for the test functions, time sampling is ensured and eqn (18) becomes:

$$
\int_{t_k}^{t_{k+\Delta t}} \left( A \{I\} \bigg|_{t_k} - \left( A^T \right)^\ast \{I\} \bigg|_{t_k} \right) dt = 0; \ k = 1, 2, ..., N_t
$$

(18)

If the space-time discretisation is performed so that, during one time increment $\Delta t$, the propagation along at most one space segment is considered, eqn (20), the transient current at present instant can be expressed using currents and excitations at all previous instances and excitation at the present instant.

$$
\Delta x \geq c \Delta t
$$

(20)

If condition given by eqn (20) is satisfied, the transient current for a $j$th space node and $k$th time node can be obtained via recurrence formula. If the members relating to the current at present instance $t_k$ in eqn (19) are separated, eqn (19) can be written as follows:

$$
A_{ij} I_j \bigg|_{t_k} + \left( A \{I\} \bigg|_{t_k} - \left( A^T \right)^\ast \{I\} \bigg|_{t_k} \right) \left( g \right)_{\text{all previous discrete instants}} = 0
$$

(19)

where overbar indicates the absence of diagonal members. The first term in eqn (21) pertains to the current at the $j$th space node and $k$th time node, i.e. present instance. Other terms are related to all previous instances. The recurrent formula for the transient current at $j$th space node can now be written as follows:

$$
I_j \bigg|_{t_k} = -\sum_{i=1}^{N} \frac{A_{ji} I_i \bigg|_{t_k} \left( R_{ax} \right) + A_{ji}^\ast I_i \bigg|_{t_k} \left( R_{ax} \right) \right) + g_j \left( g \right)_{\text{all previous discrete instants}}}{A_{jj}}
$$

(21)

where $N$ is total number of space elements, and $k=1, 2, ..., N_t$ is the index of the time node.

4 Numerical results

The illustrative numerical results are presented in this chapter. All of the analyzed configurations are excited by the Gaussian pulse voltage generator imposed on the center of the fed wire:
\[ V_g(t) = V_0 e^{-g(t-t_0)^2}; \quad V_0 = 1V, \quad g = 2 \cdot 10^9 s^{-1}, \quad t_0 = 2 ns \] (23)

Results obtained for the three wire configuration are used to illustrate numerical method discussed so far, but due to the lack of reference results for a multiple wires antenna, two wire configuration is also analyzed. Results for the two-wire configuration are compared, and found to be in a good agreement with the results published in [5].

Configuration 1 (Figure 2) consist of three wires located at the altitude \( h=1m \) above dielectric half-space (\( \varepsilon_r = 10 \)). All the wires have the same radius \( a=2mm \) and the same length of \( L=1m \), while the separation between the wires is \( d=0.5 m \).

Configuration 2 (Figure 3) has the same parameters as the configuration 1, except the number of wires is \( M=2 \).

![Figure 2: Transient current induced at the center of the passive wires (a) and active wire (b) of the configuration 1.](image)

![Figure 3: Transient current induced at the center of the active wire (a) and passive wire (b) of the configuration 2 against time comparison of FEIEM results published in [5].](image)
5 Conclusions

Time domain modeling of an arbitrary thin wire array above dielectric half-plane is presented in this paper. The mathematical model for a symmetrical two wire configuration, based on a space-time set of integral equations of the Hallen type, is extended to handle a case of multiple non-identical wires. The influence of dielectric half-plane is taken into account by the space-time reflection coefficient (RC) appearing within the integral-equation kernels. The coupled integral equation set is solved using time-domain GB-BIEM technique, and transient current distribution along all wires is obtained directly in time domain.

Two sets of results, for two different wire array configurations, are presented. Some results are compared with results obtained via frequency-domain procedures combined with Fourier transform, and found to be in good agreement with them.

References