Regularization procedures for damage problems

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Abstract

In this paper, a boundary element method for a category of nonlocal damage simulations is presented. The formulation and also the boundary element calculation code put together in the course of this investigation have turned out to be very simple and effective, yielding trustworthy data about the strains and stresses in damage-softening models. In the numerical interpretation, a finite-grid is applied to assess the damage magnitudes, including an example by the way of an illustration.

1 Introduction

Damage is the progressive physical procedure by which materials break. The damage mechanics is the investigation, via mechanical variables, of the systems included in this deterioration when the materials are subject to loading. At the microscale plateau this is the microstress accumulation in the vicinity of defects or interfaces and the breaking of links that damage the material.

At the mesoscale level of the representative volume element, this is the growth and the agglutination of microcracks or microvoids that together initiate a crack. At the macroscale level this is the propagation of that crack. The two first phases can be studied through damage variables of continuum damage mechanics defined at the mesoscale level. The third stage is usually investigated employing fracture mechanics with variables limited at the macroscale plateau.

When studying engineering materials, such as metals and alloys, composites and polymers, ceramics, rocks, concrete, and wood, it is very astonishing to
observe how such materials that have various physical textures, are alike in their qualitative mechanical behaviour.

All demonstrate elastic characteristics, yielding, some mode of plastic or irreversible deformation, anisotropy caused by strain, cyclic hysteresis loops, damage by continuous loading or by fatigue, and crack growth under static or dynamic loads. This means that the general mesoscopic attributes may be interpreted by a few energy mechanisms which are similar for all these materials. This is the main reason to render explaining material behaviour to good effect with continuum damage mechanics and the thermodynamics of irreversible processes that simulate the materials by virtue of phenomenological aspects.

According to [1], this paper covers a boundary element formulation for a specific class of nonlocal damage models, namely on the basis of the regularization procedure suggested originally in [2].

2 Nonlocal concepts of damage simulation

The mesh dependence typical for a local simulation is, in general, evidently inadmissible. Like this, some sort of regularization is necessary when a damage model is to be applied. Hereafter some options from the literature are described. Specifically, the eventuality of nonlocal damage models is contemplated. That is the basic hypothesis of the nonlocal damage regularization, namely mesh dependence and ill-posedness of the problem, are due to localization in the damage and strains rather than the existence of the symptoms of the mathematical ill-posedness of the problem. Hence, the idea is to arrange the constitutive law in such a manner that localization (or damage discontinuities) is eliminated or at least limited. This is seen to neglect the local effect principle. Damage is supposed to depend not only on the state of the particular volume element but also on the condition in an enclosed zone limiting the specific point considered. It is possible to put a nonlocal damage simulation in the following form:

\[ \sigma_{ij} = (1 - D)H_{ijkl}\varepsilon_{kl} \quad \text{state law} \]
\[ f = \bar{\varepsilon}_{eq} - K(D) \quad \text{damage function} \]

where \( \bar{\varepsilon}_{eq} \) means an average of \( \varepsilon_{eq} \) in the vicinity of the volume element. It may be observed that only the damage evolution law is adapted with regard to the local simulating. The way in that the average of equivalent strain is determined makes to various nonlocal models. In [3], a relation was suggested, as follows:

\[ \bar{\varepsilon}_{eq} = \frac{1}{V_r(x)} \int \int \int \alpha(s - x)\varepsilon_{eq}(x)dv(s); \quad V_r(x) = \int \int \int \alpha(s - x)dv(s) \]

where \( \alpha \) is an empirical weighting function and \( x, s \) are the co-ordinate vectors and \( V \) is the volume of the body.
In this simulation, any strain concentration passes to the vicinity which will prevent damage localization.

Another nonlocal variant is formed by a "grid damage model". This may be accounted for a simplification, namely of the philosophy afore-mentioned and it is inspired in the observation that many materials have a typical volume when damage is practically even. In grid simulations, a regular grid cell is located on the solid under consideration and the dimensions of the cell are equal to a specific material parameter $\lambda$. Inside each cell the damage state is supposed to be constant and depends on the average of the equivalent strains $\varepsilon_{\text{eq}}$ in the cell. This may be expressed by the relations

$$\sigma_{ij} = (1 - D)E_{ijkl} \varepsilon_{kl}$$  \text{state law}

$$D(x) = D^k \text{ if } x \in V^k$$  \text{(3)}

where $V^k$ means the domain of the $k$-th cell of the grid, $D^k$ is the damage of the cell, $f^k$ stands for the damage function of the cell and $\bar{\varepsilon}_{\text{eq}}^k$ is the equivalent strain of the cell, namely

$$\bar{\varepsilon}_{\text{eq}}^k = \frac{1}{V^k} \int \int \int_{V^k} \varepsilon_{\text{eq}}(x) \, dv$$  \text{(4)}

It should be emphasized that the grid is independent of the discretization employed for the numerical solution of the problem. In [4], a finite element approach was employed for this kind of model is presented and the convergence of the solution with mesh refinement was investigated.

### 3 Formulation and realization of the BEM

Let an isotropic elastic damageable solid satisfy the constitutive law (3). For each cell of the structure, two problems are taken into consideration:

(a) The real problem which comprises a cell under the boundary conditions set by the neighbouring cells and when the cell concerned is situated on the body surface, the external loading.

(b) The fictitious problem that embraces the same cell of an elastic damageable material with a uniform condition for the damage which equals $D^k$ and subject to a unitary force. From the reciprocal theorem of the BEM formulation, for each cell $k$ of the grid it follows:

$$w_i(P) = \int_{V^k} \left[ U_{ij}(P, Q; D^k) U_{ij}(Q) - T_{ij}(P, Q; D^k) u_j(Q) \right] dS \quad P \in V^k \quad Q \in S^k$$  \text{(5)}

where $V^k$ constitutes the $k$-cell of the grid and $S^k$ its boundary. In the standard boundary element method, displacements or tractions are known on the boundary. This is not the case for the cells where neither the displacements nor the tractions are known, when a surface of the cell does not tally with the
confines of the solid. However, the boundary conditions may be replaced by the compatibility and equilibrium equations that express the continuity of displacements and tractions along the boundary $S_i^k$ between two adjoining cells $k$ and $l$:

$$u_i^k(Q) = u_i^l(Q) \quad \forall Q \in V^k \cap V^l = S_i^k$$  \hspace{1cm} (6)

where the symbols + or - indicates a quantity in the cell $k$ or $l$, respectively.

It has to be emphasized that Eq. (5) is dependent on the cell damage since this variable alters the elastic characteristics of the material, i.e., the elastic constant $G$ in one expression of the "Kelvin Solutions" must be substituted for $(1-D^k)G$ when employed for a damaged cell $k$.

Accordingly, Eq. (5), the boundary conditions, the compatibility and equilibrium equations and the damage evolution law for each cell (16) relate a boundary element formulation for a grid-damage model.

$$f^k = \bar{\varepsilon}_{eq}^k - K(D^k)$$

$$\begin{cases} 
\dot{D}^k = 0 & \text{if } f^k < 0 \quad \text{or} \quad \dot{f}^k < 0 \\
\dot{D}^k > 0 & \text{if } f^k = 0 \quad \text{or} \quad \dot{f}^k = 0 
\end{cases}$$  \hspace{1cm} (7)

To solve the problem numerically, a standard step by step process is introduced. Subsequently, the discretized form of the damage evolution law reads:

$$\begin{cases} 
\dot{f}^k(e, D^k) = 0 & \text{if damage is vigorous in cell } k \\
\Delta D^k = 0 & \text{alternatively}
\end{cases}$$  \hspace{1cm} (8)

where $\Delta D^k$ constitutes the increment of the damage in the course of the step. Concurrently, the terms $\varepsilon$ and $D^k$ are the magnitudes of these variables at the end of the step. For the damage evolution law defined by means of the equivalent strain, thereafter discretization consists, simply in the calculation of $\text{Max} \bar{\varepsilon}_{eq}$ when considering $\varepsilon_{eq}$ only at the end of the steps, instead of taking into account all of the strain history.

After discretizing, we obtain for each step and cell, see Fig. 1:

$$[A^k(D^k)] [a^k] = [B^k(D^k)] [b^k]$$  \hspace{1cm} (9)

Considering the compatibility and equilibrium Eqs (6) via a possible conglomeration of matrices $[A^k]$ and $[B]$ of each cell into the global matrices $[A]$ and $[B]$, the following system of equations become:

$$[A(D)] [a] = [B(D)] [b]$$  \hspace{1cm} (10)
The nonlinear system of equations modelled by (8, 10) may be calculated through a usual direct iteration algorithm when the computation of the body states that \((U(x); D(x))\) at a time \(t_i\) is given by means of the following routine:

(a) The damage field \(D(x) = D^j\) (at the zero iteration) is considered as being the field at time \(t_{i-1}\).

(b) The displacement field \(U(x) = U^j\) (at the iteration \(j\)) is calculated with the solution of (9) employing the damage field at the iteration \(j-1\).

(c) The damage field \(D(x) = D^j\) (at the iteration \(j\)) is determined, after applying the displacement field at the iteration \(j\), from the solution of (8).

(d) Convergence of the algorithm is tested. When no convergence is verified, a new iteration is necessary by returning to step (b).

For the variant of damage evolution law with \(\varepsilon_{eq}\), the step (c) is straightforward – the maximum equivalent strain is calculated, considering the set of values at the end of each step only. Otherwise, a predictor-corrector algorithm should be applied. This lies in the determination of the damage function \(f(D^k, \varepsilon_{eq})\) through the damage value at the outset of the step and the equivalent strain at the end of the step. With this function being negative, or zero, next the damage remains constant during the step, in some another way, the damage at the end of the step is determined by putting the damage function equal to zero.

Furthermore, the model suggested by Mazars [5] was adjusted to the grid scheme described by equations (3)-(4), and comprehended in the computer program, when applying the relations:

\[
\varepsilon_{eq} = \sqrt{\left(\varepsilon_1\right)^2 + \left(\varepsilon_2\right)^2 + \left(\varepsilon_3\right)^2} \quad K^{-1}(z) = \alpha_i F_i(z) + \alpha_c F_c(z) \tag{11}
\]

where

\[
F_i(z) = 1 - \frac{\varepsilon_0 (1 - A_i)}{z} \frac{A_i}{\exp[B_i(z - \varepsilon_0)]};
\]

\[
F_c(z) = 1 - \frac{\varepsilon_0 (1 - A_c)}{z} \frac{A_c}{\exp[B_c(z - \varepsilon_0)]};
\]

and \(\alpha_i, \alpha_c, \varepsilon, A_i, A_c, B_i\) and \(B_c\) are material constants.
The characteristics of the concrete used in modelling are, as follows:

- Initial shear modulus \( G_i = 13,174.5 \) MPa
- Initial Poisson’s ratio \( \nu_i = 0.215 \)
- Parameters for the traction damage evolution law: \( A_t = 0.7858 \), \( B_t = 8857.39 \)
- Parameters for the compression damage evolution law: \( A_c = 1.0267 \), \( B_c = 230.71 \)
- Damage threshold strain \( \varepsilon_{th} = 0.000129 \)

Example 1.
The characteristics of an L-shaped plate, subject to horizontal displacements on the left hand side, whereas the right hand side of the plate is assumed to be clamped, is investigated. Sixteen interval subregions are employed to discretize the domain while it was necessary to introduce 32 linear boundary elements and 21 nodal points, see Fig. 2.

![Figure 2: The L-shaped plate subject to horizontal displacements.](image)

Nine internal points were applied in the subregions near the singular point (internal corner) as the highest magnitudes of the damage are expected at the aforesaid cells. In the residue of the cells, four internal points were used. The
damage distribution at 60 % of the total imposed displacement is depicted in Fig. 3, demonstrating a plain concentration of high damage values in zones close to the singular point. Fig. 4 indicates the plot of the equivalent strains.

**Figure 3:** Damage distribution in the L-shaped plate: 60 % of the total displacement.

**Figure 4:** Equivalent strains in the L-shaped plate: 60 % of the total imposed displacements (values are multiplied by $10^5$).
It should be recorded that a sensitivity analysis was performed to estimate the outcome independent of the boundary discretization. The damage magnitudes obtained are in fair agreement with those obtained via the mesh demonstrated in Fig. 2, with the variances being lesser than 5%.

4 Conclusion

Nonlocal; grid-damage calculations using the FEM require comprehensive adaptations of the common codes.

For now, highly specialized programs realize these analyses. Further, the current algorithms are not too effective, being very costly by virtue of computer storage demands. On the contrary, the BEM seems to be the most useful technique for grid-damage analyses. It is demonstrated in this paper that the BEM leads to a very simple statement for the calculation of this class of problems and it is of great importance that any existing BEM program may be modified, with a small computational labour, to take into account grid-damage simulations. It transpired that the application of continuum damage mechanics requires the use of some regularization method. The realization of these approaches in common structural analysis programs is in consequence a priority when damage models are to be employed in industrial applications.

Nonlocal and grid simulations belong to the most major regularization charts but their implementation in FE codes demands extensive modifications of the programs and the memory of a illustrously larger amount of particulars relevant to local simulations. On the other hand, for the BEM, simple preprocessors can arrange the grid by means of the geometrical data of the structure. Sophistications of the method can be performed very easily, by enhancing the number of the mesh nodes on the grid, and the number of integration points in the cells. Overall, for the specific examples considered it is clear that the BEM indicates considerable advantages over other numerical methods. It is evident that a program for the three-dimensional case could easily be obtained from the current codes.

In the general 3D situation, the advantages of the suggested BE formulation over the FE techniques would be presumably extended.

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References


