Boundary element and experimental solutions to water motion of incident waves against quadrilateral breakwaters

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Abstract

We presented the boundary element analysis of large amplitude of water motion of incident waves against a permeable submerged breakwater. In our boundary element analysis, a wave model based on fully nonlinear potential flow equations was applied to the study of the wave motion of incident waves against trapezoidal submerged breakwaters by using the moving boundaries for the free surface. In order to present a design for an optimum submerged breakwater to dissipate the energy of incident waves, we propose an alternative quadrilateral submerged breakwater such that energy dissipation can be effectively caused by the motion of the water, which passes through the porous or perforated layer of the water chamber. We introduce the boundary conditions regarding the vertical and horizontal permeable layers of the water chamber. These boundary conditions and the effect of quadrilateral submerged breakwaters are estimated by using experimental models in our laboratory and our boundary element analysis in the paper.
1 Introduction

The protection offered by submerged breakwaters consists of the induction of breaking and the partial reflection-transmission of moderate and large incident waves. We consider the protection for moderate incident waves to depend mainly on the energy dissipation caused by the water motion which passes through the porous or perforated layer of the submerged breakwaters. We also consider the protection for large incident waves to depend on breaking which occurs over submerged breakwaters. The efficiency of energy dissipation of moderate incident waves against permeable and impermeable submerged breakwaters was estimated using experimental model simulations obtained by a wave generator in our laboratory. The moving boundaries and the B-spline method for the free surface were successfully used in the boundary element method to express the moderate amplitude of water motion of nonlinear potential flow. Our boundary element method was applied to reduce the amount of necessary experiments and reproduce energy dissipation in numerical solutions.

2 Experimental model simulation

In our experimental model, we studied the partial reflection-transmission of moderate incident waves against a permeable submerged breakwater as shown in Figures 1 and 2. The specifications of our experimental model were almost same as those described in our paper (Kanoh et al. [1]). However, the permeable and impermeable submerged quadrilateral breakwaters were alternatively adopted instead of the submerged trapezoidal breakwater. The permeable quadrilateral breakwater has several openings on its vertical walls and horizontal crown as shown in Figure 2.

![Figure 1: Concept of experimental model of quadrilateral submerged breakwater](image-url)
2.1 Dimensions and conditions

We adopted an experimental model of perforated quadrilateral submerged breakwaters according to the dimensions and conditions shown in Table 1.

![Figure 2: Drawing of a perforated quadrilateral submerged breakwater with openings: (a) horizontal crown plank; (b) vertical walls](image)

Table 1: Experimental model dimensions and conditions [on a scale of 1:25]

<table>
<thead>
<tr>
<th></th>
<th>Actual object</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water depth (h)</td>
<td>10.2 (m)</td>
<td>28.3 (cm)</td>
</tr>
<tr>
<td>Crown depth (R)</td>
<td>2.98 (m)</td>
<td>8.3 (cm)</td>
</tr>
<tr>
<td>Wave steepness (H/L)</td>
<td>0.005 - 0.03</td>
<td>0.005 - 0.03</td>
</tr>
<tr>
<td>Wave period (T)</td>
<td>6.0 - 12.0 (sec)</td>
<td>1.0 - 2.0 (sec)</td>
</tr>
<tr>
<td>Wave length (L)</td>
<td>48.6 - 114.1 (m)</td>
<td>135 - 317 (cm)</td>
</tr>
<tr>
<td>Wave height (H)</td>
<td>0.25 - 3.1 (m)</td>
<td>0.7 - 9.0 (cm)</td>
</tr>
</tbody>
</table>
2.2 Experimental results

Figures 3 and 4 illustrate the relations between the transmission coefficient (Kt) and the wave periods (T) obtained by the experimental model of permeable and impermeable quadrilateral submerged breakwater. Here the wave periods had the three kinds of values from 1.0 to 2.0 sec. As the wave period diminished, these transmission coefficients of the permeable breakwater became smaller than those of the impermeable one. Namely the submerged breakwaters of these porous type had the larger effect of wave dissipation.

Figure 3 : Relations among the transmission coefficient (Kt) and the wave periods (T) of permeable quadrilateral submerged breakwaters : (a) both the crown plank and walls are permeable ; (b) the crown is permeable, the walls are impermeable.
3. Boundary element method analysis

3.1 Governing equations and boundary conditions
Using the time-dependent velocity potential \( \phi(x, z, t) \), we adopt the Laplace equation in the fluid domain \( \Omega \) with boundary \( \Gamma \) to describe two-dimensional flows in the vertical (x, z) plane as illustrated in Figure 5.

\[
\nabla^2 \phi = 0 \quad \text{in } \Omega \quad (1)
\]

On the free surface \( \Gamma_f \), the potential \( \phi \) satisfies nonlinear kinematic and dynamic boundary conditions,

\[
Dr/Dt = u = \nabla \phi \quad \text{on } \Gamma_f \quad (2)
\]
\[
\frac{D\phi}{Dt} = -gz + \frac{1}{2} \nabla \phi \cdot \nabla \phi - \frac{P_a}{\rho} \quad \text{on } \Gamma_f
\] (3)

respectively (Kanoh et al. [1]). Here \( r \) denotes the position vector of a free surface fluid particle, \( g \) is the acceleration due to gravity, \( P_a \) is the atmospheric pressure, \( \rho \) is the fluid density. Hence the free surface boundary conditions are expressed as

\[
r(t + \Delta t) = r(t) + \Delta t(Dr/Dt) + \{(\Delta t)^2/2\}*(D^2r/Dt^2)
\] (4)

for the free surface position, and for the potential we have

\[
\phi(r(t + \Delta t)) = \phi(r(t)) + \Delta t*(D\phi(r(t))/Dt) + \{(\Delta t)^2/2\}*(D^2\phi(r(t))/Dt^2)
\] (5)

Waves are generated by simulating a piston wave-maker motion on the right hand ‘Ocean’ boundary \( \Gamma_{ro} \) of the computational domain. Motion \( x \) and normal velocity are specified over the paddle as,

\[
x = A/\omega (1 - \cos \omega t) ; \nabla \phi \cdot n = \partial \phi / \partial n = -A \sin \omega t \quad \text{on } \Gamma_{ro}
\] (6)

Along the bottom and impermeable breakwater surfaces \( \Gamma_b \), and other fixed boundaries \( \Gamma_{rl} \), a no-flow condition is prescribed as,

\[
\partial \phi / \partial n = 0 \quad \text{on } \Gamma_b \text{ and } \Gamma_{rl}
\] (7)

Along the horizontal crown plank \( \Gamma_C \) or vertical wall boundary \( \Gamma_w \) of a submerged breakwater, we propose the following two kinds of potential flux conditions:

1. The potential flux condition along the porous plank, which was described in our papers (Kanoh et al. [1], [2]), was expanded to the horizontal plank and vertical wall boundaries. Specifically, we have

\[
\partial \phi / \partial n = \{ \Phi_{out} - \Phi_{in} \} / \{ d_0 V_n (1 + \mu_2 - i \mu_1) \}
\] (8)

where \( \Phi_{out} \) and \( \Phi_{in} \) are the time-independent potential of the outer and inner part of the boundary in the normal direction. Here \( i \) is the imaginary unit, and \( d_0 \) and \( V_n \) are the depth and the porosity of the crown or wall layer. Where \( \mu_1 \) and \( \mu_2 \) are the coefficients of fluid resistance that are proportional to the fluid velocity and acceleration, respectively (Kanoh et al. [2]).

![Figure 6: Rigid parts and openings of horizontal crown plank or vertical wall](image)
(2) Boundary elements and sub-domains were adopted as shown in Figure 6 such that the rigid parts of the plank or wall and their openings were applicable to the fixed boundary and free to water motion, respectively. Here the rigid parts were boundary elements where a no-flow condition was held and the openings were the boundaries of sub-domains where the water motion was kept free.

3.2 Boundary element expression

Discritized boundary element expression for analyzing eqn (1) is written as,

\[ H\phi = Gq, \quad q = \frac{\partial \phi}{\partial n}, \quad H\phi' = Gq' \]  

where \( \phi' \) and \( q' \) are the time derivatives of \( \phi \) and \( q \), respectively (e.g. Kanoh [1]).

3.3 Results of boundary element method

Based on the numerical example shown in Figure 5, Figure 6 illustrates the free surface and velocity vectors at 2.0 and 2.6 sec after the wave-maker piston started. Comparing the results of the permeable and impermeable submerged breakwaters, we observed that the wave dissipated by porous breakwater reduced the height of the incident wave. In analyzing the water motion against the permeable and impermeable breakwaters, the B-spline technique was applied to restrain efficiently the oscillation of the free surface and make the shape very smooth. Using the numerical results of 2.0 and 2.6 sec, we recognized that the wave trough and crest caused the water to flow in and out in different directions through the openings of the crown planks and walls along the porous submerged breakwater. Namely, at 2.0 sec, the water flowed out from the openings of the vertical walls and flowed into those of the horizontal crown plank. On the other hand, at 2.6 sec, the opposite water motion was observed. These water motions introduced the eddy loss and the effective dissipation of the porous submerged breakwater.

Regarding the same numerical example as shown in Figure 5, Figure 8 shows the relations between the transmission coefficient (K_t) and wave periods (T) calculated by our boundary element method. Where we adopted the three kinds of submerged breakwaters. Namely the crown and walls of the first kind were impermeable, the crown of the second one was permeable and both the crown and walls of the third one were permeable. In case the wave height (H) varied the value from 1.0 to 0.5m, the transmission coefficients (K_t) of the permeable breakwater changed the values, whereas those of the impermeable one were almost same each other. As the wave period (T) varied the values from 2 to 1, the transmission coefficients of the permeable breakwater reduced the values. Especially in case H = 1m, the value of K_t varied widely from 0.75 to 0.31. Referring the matters described above we considered that there was rather large difference between the impermeable and permeable quadrilateral breakwaters for the effect of wave dissipation.
Figure 7: Boundary element solutions of permeable quadrilateral submerged breakwaters: (a) 2.0 sec; (b) 2.6 sec

Figure 8: Relations between the transmission coefficient (Kt) and wave period (T) obtained by our boundary element method.
4 Results and Discussion

From the experimental and boundary element solutions described above, we obtained the following results: (1) The porous-type quadrilateral submerged breakwater had a larger effect on wave dissipation than the trapezoidal one estimated by us (Kanoh et al. [1]). (2) Our boundary element solutions show clearly that the traveling wave trough and crest produced the water motion through the openings of the horizontal porous planks and vertical walls, and these water motions through the openings were sometimes in different directions. (3) We consider that these water motions produced the eddy loss, resulting in the effective dissipation of the porous quadrilateral submerged breakwater. By combining the boundary element and the weighted finite difference methods proposed by us (Kanoh et al. [3]), in future research we intend to improve the hybrid method such that it can handle a large amplitude of wave motion and the turbulent flow that is produced by wave motion and wave breaking.

5 Conclusion

In order to shorten the crown-width of a permeable submerged breakwater and make it economically predominant over an impermeable one, we estimated the water motion through a porous quadrilateral submerged breakwater using the experimental results obtained in our laboratory. We then applied the boundary element method to confirm the experimental results and reduce the number of necessary experiments. Using these experimental and numerical solutions, we tried to find the mechanism of the effective dissipation of a porous quadrilateral submerged breakwater.

References


