BEM for natural convection in non-Newtonian fluid saturated porous cavity

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Abstract

The main purpose of this work is to present the use of the Boundary Element Method (BEM) in the analysis of the flow transport phenomena of non-Newtonian fluids in porous medium domain. To solve the stated problem the Boundary Domain Integral Method (BDIM) for the numerical solution of unsteady incompressible Newtonian fluid flow in porous medium is extended in order to analyse the effects of the available non-Newtonian viscosity. The governing equations are transformed by the velocity-vorticity variables formulation, therefore the computation scheme is partitioned into kinematic and kinetic part. To evaluate the presented approach the Carreau model which is adequate for many non-Newtonian fluids is considered, representing the fluid that saturate the vertical porous cavity heated from the side. Numerical results representing the flow and heat transfer characteristics of the fluid within the porous cavity will be presented.

1 Introduction

Many fluids encountered in engineering applications exhibit non-Newtonian behaviour characterised by a non-linear relationship between shear stress and shear rate. A few parametric viscosity models are available in the literature representing this relationship. When the temperature of the saturating fluid phase in a porous medium is not uniform, certain flows induced by buoyancy effects may occur. These flows, commonly called free or natural convection movements, depend on density differences due to temperature gradients and boundary conditions. Natural convection is one of the most frequently studied transport phenomena in porous medium, and is therefore used also in our test case.
2 Mathematical formulation

The configuration described in the present investigation is shown in Fig. 1. The solution domain is a two-dimensional square cavity with a side length $D$. The cavity is filled with a material consisting of a solid and a fluid phases called porous medium. The solid phase is homogeneous, isotropic and non-deformable, while the fluid is single phase and its density is taken not to depend on pressure variations, but only on variations of the temperature. The thermophysical properties of the fluid and the solid phases of the porous medium are taken to be constant except for the density variation, which is handled according to the Boussinesq approximation. Furthermore, the solid phase is made of spherical particles, while the porosity and permeability of the medium are assumed to be uniform throughout the cavity. The top and bottom walls are subjected to insulated boundary conditions while the vertical walls are maintained at a different temperature. The left wall is isothermally heated and the right one is isothermally cooled.

\[
\frac{\partial T}{\partial y} = 0
\]

\[
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\]

Fig. 1: Geometry and boundary conditions for the porous cavity.

The point governing equations for mass, momentum and energy are transformed to the macroscopic level using the volume-average technique over the suitable representative elementary volume, which has to be determined such, that irrespective of its position in porous medium, it always contain both a solid and a fluid phases. The averaging method, however, requires information on numerous empirical transport coefficients that are supplemented by experimental findings and analytical solutions for some rather simplified cases (Bear and
Bachmat [1]). Therefore, it is of great importance to take into account all above-mentioned assumptions, relating the properties of the discussed porous medium. The obtained macroscopic equation representing the basic conservation balance of mass is the continuity equation:

$$\frac{\partial v_i}{\partial x_i} = 0,$$

(1)

where $v_i$ is volume-averaged velocity, and $x_i$ the $i$-th coordinate.

The momentum equation represents the basic conservation balance of momentum:

$$\frac{1}{\phi} \frac{\partial v_i}{\partial t} + \frac{1}{\phi^2} \frac{\partial v_j v_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial P}{\partial x_j} + F \frac{v_f}{K} v_i + v_{\text{eff}} \frac{\partial^2 v_i}{\partial x_j \partial x_j},$$

(2)

where $\phi$ is porosity, $v_f$ the fluid kinematic viscosity, $v_{\text{eff}}$ the effective kinematic viscosity, $K$ permeability of porous medium, $\partial P/\partial x_j$ pressure gradient in the flow direction, $\rho$ the fluid density, $g_i$ gravity. $F$ is the normalised density-temperature variation function taken as $F = (\rho - \rho_0)/\rho_0 = -\beta_T (T - T_0)$, with $\rho_0$ denoting the reference fluid mass density at temperature $T_0$ and $\beta_T$ being the thermal volume expansion coefficient of the fluid. The coefficient $K$ is independent of the nature of the fluid but it depends on the geometry of the medium. In general, it is a second-order tensor, but for the case of an isotropic medium it can be taken as a scalar quantity. It is possible to calculate $K$ in terms of geometrical parameters, at least for the case of a simple geometry. For example, in the case of beds of particles the hydraulic radius theory of Charman-Kozeny leads to the relationship $K = \frac{d_p^2 \phi^3}{150(1-\phi)^2}$, where $d_p$ is the solid particle diameter. With his experimental investigations Ergun has found that the constant 150 in expression for $K$ may be replaced by 180, and several other authors determined similar values (Chen and Hsiao [2]).

The momentum equation (2), commonly known as Brinkman equation consists of two viscous terms. The first is the usual Darcy term, and the second is analogous to the Laplacian term that appears in the Navier-Stokes equations for pure fluid (Nield and Bejan [3]). The Laplace term is commonly called Brinkman term or Brinkman extension that expresses the viscous resistance or viscous drag force exerted by the solid phase on the flowing fluid at their contact surfaces. The effective viscosity $v_{\text{eff}}$ depends on the geometry of the porous medium. It may have a different value than the fluid viscosity $v_f$, therefore parameter $\Lambda$ denoting viscosity ratio, is introduced:

$$\Lambda = \frac{v_{\text{eff}}}{v_f}.$$  

(3)
A detailed averaging process for an isotropic medium yields for the parameter \( \Lambda \) the expression \( \Lambda = 1/\phi T^* \), where \( T^* \) is called tortuosity of the medium (Bear and Bachmat [1]). Since \( \Lambda \) depends on the geometry of the medium, its value is often approximated by \( \Lambda = 1/\phi \) (Jecl et al. [4]) but in cases with a high value of porosity a reasonable approximation is \( \Lambda = 1 \). In several articles authors have disagreed on whether to include the Laplacian term or not, and it has been confirmed that for many practical purposes there is no need to include it. But if it is important to satisfy the no-slip boundary condition, when one wishes to compare flows in porous medium with those in pure fluids, or to match solutions in a porous medium and in an adjacent viscous fluid, then the Laplacian term is indeed required. The Brinkman equation has a parameter \( K \), therefore this equation is essentially an interpolation scheme between the Navier-Stokes and Darcy equations. Namely, the Brinkman equation reduces to a form of the Navier-Stokes equation as \( K \to \infty \) and to the Darcy equation as \( K \to 0 \).

Now we introduce the non-Newtonian fluid that saturates our porous medium domain. Many practical simulations of process fluid flows in complex geometries are still based on the use of generalised Newtonian fluid models (GNF) due to their simplicity and applicability. Focusing on incompressible fluids and neglecting the elastic properties of the fluid, the simple viscous constitutive model may be applied for the relationship between the stress tensor \( \sigma_{ij} \) or shear stress \( \tau_{ij} \) and shear rate \( \dot{\varepsilon}_{ij} \) in the form \( \sigma_{ij} = -p \delta_{ij} + 2\mu(\dot{\gamma})\dot{\varepsilon}_{ij} \), where \( \delta_{ij} \) is Kronecker function and \( \mu \) is dynamic viscosity which is assumed to be strain rate dependent, \( \mu = \mu(\dot{\gamma}) \). Obviously, the viscous stress tensor \( \tau_{ij} \) for GNF is given by the following constitutive hypothesis \( \tau_{ij} = 2\mu(\dot{\gamma})\dot{\varepsilon}_{ij} \), where the shear strain rate or magnitude of the shear strain rate tensor is \( \dot{\gamma} = (2\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij})^{1/2} \), and the shear rate being \( \dot{\varepsilon}_{ij} = 1/2 \left( \partial v_i / \partial x_j + \partial v_j / \partial x_i \right) \). A few parametric viscosity models are available in the literature, for example the “power law”, “the Carreau-Yasuda model”, the “Eliss model”, while the constitutive model applied in this paper is the Carreau model:

\[
\mu(\dot{\gamma}) = \mu_\infty + (\mu_0 - \mu_\infty) \left[ 1 + (\dot{\gamma}/\dot{\gamma}_c)^{\nu-1} \right]^{\nu/2},
\]

where \( \mu_0 \) and \( \mu_\infty \) are the zero and infinite shear rate viscosities, and \( \dot{\gamma}_c \) a time constant. The dynamic viscosity \( \mu \) is related with the kinematic viscosity \( v_f \) in the momentum equation (2) by \( v_f = \mu / \rho \). If the viscosity is further partitioned into constant and perturbed parts so that \( v_f = \bar{v}_f + \tilde{v}_f \) the Brinkman extension in momentum equation is divided into two parts and the equation (2) is now:
Energy equation represents the basic conservation balance of energy:

\[ \frac{\partial}{\partial t} \left[ \phi \left( \rho c_f \right) + (1-\phi) \rho_s c_s \right] T + \left( \rho c_f \right) \frac{\partial v_i}{\partial x_j} \frac{T}{x_j} = \frac{\partial}{\partial x_j} \left( \lambda_e \frac{\partial T}{\partial x_j} \right), \]  

(6)

where \( \rho_s \) and \( \rho \) are the solid and fluid densities respectively, \( c_s \) and \( c_f \) the solid and fluid specific heats at constant pressure respectively, \( T \) stands for temperature, and \( \lambda_e \) represents the effective thermal conductivity of the saturated porous medium. The effective thermal conductivity of a porous medium can be conventionally approximated with the classic mixing rule \( \lambda_e = \phi \lambda_f + (1-\phi) \lambda_s \), where \( \lambda_f \) and \( \lambda_s \) are the fluid and the solid thermal conductivity (Nield and Bejan [3]). Unfortunately, the mixing rule could give rise to considerable error especially when \( \lambda_s / \lambda_f >> 1 \). Therefore another possibility to compute the effective thermal conductivity emerges as a combination of the conductivities of two constituents, a stagnant component \( \lambda_m \) and a dispersion component \( \lambda_d \) as \( \lambda_e = \lambda_m + \lambda_d \). The stagnant component based on the experimental findings is

\[ \frac{\lambda_m}{\lambda_f} = (1-\sqrt{1-\phi}) - \frac{2}{1-\theta} \left[ \frac{(1-\theta)B}{(1-\theta B)^2} \ln(\theta B) + \frac{B+1}{2} + \frac{B-1}{1-\theta B} \right], \]

with \( B = 1.25 \left[ \frac{1-\phi}{\phi} \right]^{10} \) and \( \theta = \frac{\lambda_f}{\lambda_s} \), whereas the dispersion conductivity is

\[ \frac{\lambda_d}{\lambda_f} = 0.5 \left[ \left| v \right| \frac{d_p}{v_f} \right] \frac{v_f}{a_f}, \quad \frac{\lambda_d}{\lambda_f} = 0.1 \left[ \left| v \right| \frac{d_p}{v_f} \right] \frac{v_f}{a_f}. \]

The modulus of the velocity vector is \( \left| v \right| = \sqrt{v_x^2 + v_y^2} \) with \( v_x \) and \( v_y \) being the filtration velocity in x and y directions respectively, and \( a_f \) the thermal diffusivity of the fluid (Amiri [5]). The thermal diffusivity of the porous medium is defined as \( a_p = \lambda_e / \rho c_f \), and will be similarly as the kinematic viscosity, partitioned into constant and perturbed parts \( a_p = a_p + \tilde{a}_p \). Introducing the heat capacity ratio \( \sigma \):

\[ \sigma = \frac{\phi (\rho c_f) + (1-\phi) (\rho c_s)}{\rho c_f}, \]

(7)

the heat energy equation (6) can be rewritten in the following form:
\[
\sigma \frac{\partial T}{\partial t} + \frac{\partial \nu_j T}{\partial x_j} = \tilde{\alpha}_p \frac{\partial^2 T}{\partial x_j \partial x_j} + \frac{\partial}{\partial x_j} \left( \tilde{\alpha}_p \frac{\partial T}{\partial x_j} \right).
\]  
(8)

With above formulated set of conservative equations (1), (5) and (8), commonly named modified Navier-Stokes equations for porous medium, we can solve in principal any transport phenomena in porous medium if the appropriate hydrodynamic and thermal boundary conditions are precisely defined. In accordance with the problem description, the initial and boundary conditions are:

\[
\begin{align*}
\bar{v}_x &= \bar{v}_y = 0, & \bar{T} &= 0 \quad \text{for} \quad t = 0 \\
\bar{v}_x &= \bar{v}_y = 0 \quad \text{for} \quad x = 0, D \quad \text{and} \quad y = 0, D \\
\frac{\partial \bar{T}}{\partial y} &= 0 \quad \text{for} \quad y = 0, D \\
\bar{T} &= \bar{T}_H = 1 \quad \text{for} \quad x = 0 \\
\bar{T} &= \bar{T}_C = 0 \quad \text{for} \quad x = D
\end{align*}
\]  
(9)

3 Numerical scheme and procedure

Boundary Domain Integral Method (BDIM) represents an extension of the classical BEM for solving complicated diffusion-convective problems. In BDIM, the obtained modified Navier-Stokes equations (1), (5) and (8), are further transformed with the use of the velocity-vorticity (VVF) variables formulation. With the vorticity vector \( \omega_i = e_{ij} \frac{\partial v_k}{\partial x_j} \), representing the curl of the velocity field, the computational scheme is partitioned into its kinematic and kinetic part so that the continuity and momentum equations are replaced by the equations of kinematics and kinetics (Škerget et al. [6]). As the computational results of the present work are limited to the two-dimensional case, all the subsequent equations will consequently be written for the case of planar geometry only. Applying the curl operator directly to the vorticity defined above, using the continuity equation and with addition of the relaxation parameter \( \alpha \), the kinematic can be formulated in the form of the parabolic kinematic equation:

\[
\frac{\partial^2 v_i}{\partial x_j \partial x_j} - \frac{1}{\alpha} \frac{\partial v_i}{\partial t} + e_{ij} \frac{\partial \omega}{\partial x_j} = 0.
\]  
(10)

The kinetics is governed by the vorticity transport equation obtained as a curl of the momentum equation (5). Introducing the new variable \( \tau = t/\phi \), the so called modified vorticity time step, only as a necessary mathematical step allowing us to use the VVF on our momentum equation, the vorticity transport equation is now:
The last term of equation (11) represents a contribution from non-linear material properties:

\[ f_j = \phi^2 \Lambda \left( \frac{\partial \tilde{v}_f}{\partial x} \left( \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} \right) - 2 \frac{\partial \tilde{v}_f}{\partial y} \frac{\partial v_x}{\partial x}, 2 \frac{\partial \tilde{v}_f}{\partial x} \frac{\partial v_y}{\partial y} - \frac{\partial \tilde{v}_f}{\partial y} \left( \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} \right), 0 \right) \]

For the same reason as with vorticity kinetics, we introduce new modified temperature time step \( \tau_f = t/\sigma \) also into energy equation which permits us to rewrite equation (8) in the form:

\[ \frac{\partial T}{\partial \tau_f} + \frac{\partial T}{\partial x_j} \frac{\partial T}{\partial x_j} = a_p \frac{\partial^2 T}{\partial x_j \partial x_j} + \frac{\partial}{\partial x_j} \left( a_p \frac{\partial T}{\partial x_j} \right) \]  

Equations (10), (11) and (12) represent the leading non-linear set of equations to which the weighted residuals technique of the BDIM has to be applied (Škerget et al. [6]). Each of this equation can be represented with the parabolic diffusion-convective equation, the accurate integral representation of which is of key importance to the success of the BDIM. For all equations, the time derivative of a field function \( u \) (velocity, vorticity, temperature) is approximated by the three level second order implicit scheme:

\[ \left( \frac{\partial u}{\partial t} \right)^{n+1} = 3 u^n - 4 u^{n-1} + \frac{u^{n-1}}{2 \Delta t} \]  

with \( n \) denoting the time step number.

The velocity equation (10) rewritten as a nonhomogenous parabolic diffusion equation is of the form:

\[ \bar{\alpha} - \frac{\partial^2 v_i}{\partial x_j \partial x_j} - \frac{\partial v_i}{\partial t} + b = 0 \]  

with the following corresponding integral representation:

\[ c(\xi) v_j(\xi, t^{n+1}) + \bar{\alpha} \int_{\Gamma} v_j \frac{\partial u^*}{\partial n} dt d\Gamma = \bar{\alpha} \int_{\Gamma} v_j \frac{\partial u^*}{\partial n} dt d\Gamma + \]

\[ + \bar{\alpha} e_{ij} \int_{\Gamma} \omega u^* n_j dt d\Gamma - \bar{\alpha} e_{ij} \int_{\Omega} \omega \frac{\partial u^*}{\partial x_j} dt d\Omega + \int_{\Omega} v_{j-1} u^{n-1} d\Omega. \]

Parameter \( c(\xi) \) denotes the fundamental solution related coefficient depending
on the position of the source point, $\Gamma$ is the boundary of the domain $\Omega$. $u^*$ is the parabolic diffusion fundamental solution:

$$u^* = \frac{1}{4\pi \alpha \tau} \exp \left(-\frac{r^2}{4\alpha \tau} \right),$$  \hspace{1cm} (16)$$

with $\tau = t^{n+1} - t^n$ and $r$ being the distance from the source point $\xi$ to the reference point $s$. Assuming constant variation of the velocity within the individual time increment, the time integrals in equation (15) may be evaluated analytically. After discretization of the computational domain, assembly of all influence matrices, and incorporation of boundary and initial conditions, the following matrix form of kinematic integral equation (15) appears:

$$[H][v_j] = [G]\left(\frac{\partial v_j}{\partial n}\right) + \epsilon_g\left(\omega n_j\right) \exp \left(-\frac{r^2}{4\alpha \tau} \right) + [G]\left(\omega n_j\right) + [B]\left[v_i^{n-1}\right].$$  \hspace{1cm} (17)$$

Here $[H]$, $[G]$, $[D]$, $[B]$ are matrices composed of integrals, representing the integration, taken over all individual boundary elements and all the internal cells. The formulations of the integral representation for the vorticity kinetics and temperature kinetics are based on a nonhomogeneous elliptic diffusion-convective equation with a constant reaction term (Škerget and Jecl [7]):

$$\frac{\bar{\kappa}}{k} \frac{\partial^2 u}{\partial x_j \partial x_j} - \frac{\partial \vec{v}_j u}{\partial x_j} - \beta u + b = 0,$$  \hspace{1cm} (18)$$

where $u$ is taken as vorticity $\omega$ and temperature $T$, respectively and $\bar{\kappa}$ being defined considering the conservation laws and constitutive hypothesis. Finally, the following resulting integral representation for vorticity and temperature kinetics is obtained:

$$c(\xi)\phi(\xi) + \phi^2 \Lambda \bar{v}_f \int_{\Gamma} \frac{\partial u^*_\omega}{\partial n} d\Gamma =$$

$$= \int_{\Gamma} \left( \phi^2 \Lambda \bar{v}_f \frac{\partial \omega}{\partial n} - \phi \frac{\partial e_j g_j F n_j + f_j n_j}{\partial x_j} \right) u^*_\omega d\Gamma - \int_{\Omega} \frac{\phi^2 \bar{v}_f}{\bar{\kappa}} \omega u^*_\omega d\Omega +$$

$$+ \int_{\Omega} \left( \bar{v}_j \omega - \phi^2 \Lambda \bar{v}_f \frac{\partial \omega}{\partial x_j} - \phi \frac{\partial e_j g_j F - f_j}{\partial x_j} \right) \frac{\partial u^*_\omega}{\partial x_j} d\Omega - \frac{1}{2\Delta \tau^v} \int_{\Omega} \left( 4\omega - \omega^{n-1} \right) u^*_\omega d\Omega,$$  \hspace{1cm} (19)$$

$$c(\xi)T(\xi) + \bar{a}_p \left( \int_{\Gamma} \frac{\partial T}{\partial n} d\Gamma = a_p \left( \frac{\partial T}{\partial n} - T v_n \right) u^*_T d\Gamma +$$

$$+ \int_{\Omega} \left( \bar{v}_j T - \bar{a}_p \frac{\partial T}{\partial x_j} \right) \frac{\partial u^*_T}{\partial x_j} d\Omega + \frac{1}{2\Delta \tau^T} \int_{\Omega} \left( 4T^n - T^{n-1} \right) u^*_T d\Omega.$$  \hspace{1cm} (20)$$
The system of discretized equations (2) (3) and (22) (28) (31) is solved by coupling with the corresponding matrix form of Volterra and Fredholm transport now reads:

\[
\begin{align*}
\{ \frac{d}{dx} \} \{ d \} + \{ f \} - \{ \omega \} + \{ \frac{d}{dx} \} \{ d \} + \{ f \} - \{ \omega \} & = 0 \\
\{ \frac{d}{dx} \} \{ d \} + \{ f \} - \{ \omega \} + \{ \frac{d}{dx} \} \{ d \} + \{ f \} - \{ \omega \} & = 0 \\
\end{align*}
\]

The corresponding matrix form of Volterra and Fredholm transport now reads:

\[
\begin{align*}
\{ \frac{d}{dx} \} \{ d \} + \{ f \} - \{ \omega \} + \{ \frac{d}{dx} \} \{ d \} + \{ f \} - \{ \omega \} & = 0 \\
\{ \frac{d}{dx} \} \{ d \} + \{ f \} - \{ \omega \} & = 0 \\
\end{align*}
\]

The corresponding matrix form of Volterra and Fredholm transport now reads:
4 Numerical results

To check the validity of the proposed numerical procedure the problem of natural convection in a square porous cavity heated from the side is investigated. The description of the problem is shown on Fig. 1., with the initial and boundary conditions given by equation (9). Whenever we consider the Brinkman term in momentum equation (2), we have to deal with the parameter called Darcy number $Da$ (Nield and Bejan [3]). The Darcy number is defined as the ratio between the permeability and the characteristic length multiplied by the viscosity ratio:

$$Da = \Lambda \frac{K}{D^2}.$$  \hspace{1cm} (24)

In our case the viscosity ratio is equal to the reciprocity of the porosity $\Lambda = 1/\phi$ (Jecl et al. [4]), where we have chosen the porosity to be $\phi = 0.5$. The other important governing parameter for the present problem is the Rayleigh number for the porous medium (Nield and Bejan [3]):

$$Ra^* = \frac{g \beta_T K D \Delta T}{\nu_f a_p}.$$  \hspace{1cm} (25)

where $\Delta T$ is the temperature difference between hot and cold walls and all the other parameters have been defined earlier. In order to illustrate the typical numerical results the heat capacity ratio in the heat energy equation (8) needs to be defined and it has been taken as $\sigma = 1$. A uniform computational mesh of $10 \times 10$ subdomains was used. Time step is $\Delta t = 0.001$, while the convergence criterion is determined to be $\varepsilon = 5 \times 10^{-6}$. Non-Newtonian model was chosen to be Carreau model with the following parameters: $\mu_0 = 1.01$, $\mu_\infty = 5.9E-04$, $\chi = 0.81$ and $n = 0.364$.

The effects of the Darcy number and the Rayleigh number on the flow and thermal behaviours of the porous cavity are studied. A numerical model based on the presented theoretical work and chosen parameters is, at this moment, in the phase of evaluation and testing, therefore the simple test case is presented here. The complete analysis will likely serve to confirm the fact that, when using the Brinkman momentum equation, the effect of the second viscous term in equation (2), Laplace or Brinkman term, becomes negligible for $Da < 0.0001$ (Lauriat and Prasad [8]). It also turns out that using the Brinkman extension the no-slip boundary condition on the impermeable walls bounding the porous medium is satisfied, what however is not the case when using the Darcy law.

Time evolution of the velocity field at different time steps is presented in Fig. 2. and the time evolution of the temperature field at the same time steps is shown on Fig. 3 for $Ra^* = 100$, $Da = 10^{-1}$ and $\Delta T = 1$. 
Fig. 2: Vector velocity fields at different time steps (0.01, 0.3 and 0.7 s) and $Ra^* = 100$, $Da = 10^{-1}$, $\phi = 0.5$, $\Delta T = 1$ for Carreau fluid.

Fig. 3: Temperature fields at different time steps (0.01, 0.3 and 0.7 s) and $Ra^* = 100$, $Da = 10^{-1}$, $\phi = 0.5$, $\Delta T = 1$ for Carreau fluid.

More test cases with detailed final results will be presented at the conference.

5 Conclusion

The problem of natural convection in porous medium saturated with non-Newtonian fluid is investigated utilising a Boundary Domain Integral Method (BDIM). The Brinkman equation is used as the starting momentum equation, and the Carreau model, which is adequate for many non-Newtonian fluids, is considered representing the fluid that saturates the vertical porous cavity heated from the side. The solution is based on the velocity-vorticity formulation of constitutive equations, which allows separation of the computational scheme into its kinematic and kinetic part. Parabolic diffusion fundamental solution is used for the kinematic part of fluid motion, while elliptic diffusion-convective fundamental solution is employed for the kinetic part. The subdomain technique is applied, where each subdomain is being constructed of four discontinuous 3-node quadratic boundary elements, and one continuous 9-node corner continuous quadratic internal cell. The proposed numerical procedure is studied for the case
of natural convection in square porous cavity heated from the side. The characteristics of the flow and temperature fields in the cavity are analyzed for different parameters. The results indicated that the BDIM can be efficiently used for solving the transport phenomena in porous medium.

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