Acoustic insulation provided by a single elastic wall dividing a tunnel calculated via BEM

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Abstract

The Boundary Elements Method (BEM) is used to compute the sound insulation provided at low frequencies by an elastic wall that divides an air-filled tunnel. In the simulated model, a 3D source is placed in one section of the tunnel, while the response (acoustic pressure) is computed inside both tunnel sections. The BEM is formulated in the frequency domain. The insulation conferred by the wall is characterised, identifying the location of insulation dips in the frequency domain with those dips related to both its own natural dynamic vibration modes and those related to the natural vibration modes of the tunnel. This method models the surfaces of the tunnel and the separating wall completely, and takes full account of the coupling between the fluid (air) and the elastic medium. This model is also used to assess how the dimension of the rooms, the rigidity and thickness of the wall affect acoustic insulation. The transmission loss results obtained via the BEM are then compared with those provided by simplified analytical models such as the Mass Law.

1 Introduction

Airborne wall sound insulation is a classic problem in acoustics. The first publications on it appeared at the beginning of the twentieth century (e.g. Rayleigh [1]).

Different researchers propose different simplified methods to predict the sound insulation conferred by a single panel above, below, and in the vicinity of the coincidence effect (e.g. Cremer [2]; Sewell [3]; Sharp [4]; Beranek and Vér [5]).
To predict airborne sound transmission by single partitions at low frequencies, Osipov et al. [6] used an infinite plate model, a baffled plane model and a room-plate-room model.

The measurement of the sound insulation provided by a partition construction element separating two compartments, for low frequencies (below 400 Hz), is not an easy task. For typical European transmission rooms with a volume of 50-70 m³, the reproducibility of sound insulation measurements is not satisfactory at frequencies below 100 Hz (e.g. Roland et al. [7]; Pedersen et al. [8]). It should be noted that no standard takes enough account of the fluctuating nature of noise, and fails to make a sufficient correction for large fluctuations (e.g. Mathys [9]).

Well-established numerical techniques, such as the finite element and finite difference methods, have failed because the domain being analysed has to be fully discretized, and very fine meshes are needed to solve excitations at high frequencies.

However, the Finite Element Method has been used by Osipov et al. [10] to study the effect of room dimension on the sound insulation of a separating wall at low frequencies. Maluski et al. [11] also used the Finite Element Method to predict the sound insulation between adjacent rooms at low frequency, and to compare the results with experimental data. Their results show that the sound insulation provided by a separating wall, at low frequencies, is strongly dependent on the modal characteristics of the sound field within each compartment.

Sgard et al. [12] computed the low frequency diffuse field transmission loss through double-wall sound barriers with elastic porous linings. The different layers of the sound barrier have been modelled making use of the Finite Element Method coupled to a variational boundary element method to account for fluid loading. The diffuse field is assumed to be a combination of uncorrelated freely propagating plane waves with equal amplitude, no two of which are travelling in the same direction.

In this work, the Boundary Elements Method (BEM) is used to compute the sound insulation provided by an elastic wall separating two air-filled tunnels at low frequencies. The responses are obtained inside the two tunnels at low frequencies, when a 3D source is excited in one of the tunnels.

This technique is used because it only requires the discretization of boundary of the tunnels and automatically satisfies the far field conditions.

2 Problem formulation

Twin rectangular tunnels, separated by a concrete wall, are driven along the z direction in an elastic infinite medium. The lining of the tunnel is assumed to be concrete, with density \( \rho \), allowing a shear wave velocity of \( \beta \) and a compressional wave velocity of \( \alpha \). The lining is assumed to have similar properties as those of the surrounding material. Since the aim of the present model is to calculate the airborne sound insulation conferred by a wall, dividing
two tunnels, this simplification is not relevant because the quantity of energy crossing the lining of the tunnel is very small compared with the energy that goes through the separating wall. The fluid (air) inside the tunnels permits a compressional wave velocity \( a_a \) and has a density \( \rho_a \). A dilatational point source is placed in one of the tunnels at position \((x_0, y_0, z_0)\), oscillating with a frequency \( \omega \). The pressure incident field is expressed by

\[
P_{\text{inc}} = \frac{A e^{-i \omega t - i \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}}}{\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}}
\]

in which \( A \) is the wave amplitude and \( i = \sqrt{-1} \).

The solution can be obtained as a summation of two-dimensional problems, for varying effective wavenumbers since the geometry of the tunnels does not change along the \( z \) direction (e.g. Bouchon et al. [13]),

\[
k_{a_a} = \sqrt{\frac{\omega^2}{a_a^2} - k_z^2}, \quad \text{Im} k_{a_a} < 0
\]

where \( k_z \) is the axial wavenumber after Fourier transformation of the problem in the \( z \) direction. In this frequency wavenumber domain the incident field is given by,

\[
\hat{p}_{\text{inc}}(\omega, x, y, k_z) = -\frac{i A}{2} H_0^{(2)}(k_{a_a} \sqrt{(x-x_0)^2 + (y-y_0)^2})
\]

in which the \( H_n^{(2)}(...) \) are second Hankel functions of order \( n \). This is often referred to in the literature as a 2½D problem, for the reason that the geometry is 2D and the source is 3D.

The BEM only requires the discretization of the inner surfaces of the tunnels, since the lining is assumed to be surrounded by material with identical properties. The system of equations required for the solution is arranged so as to impose the continuity of the normal displacements and normal stresses, and null shear stresses along the boundary of the fluid-filled tunnels. This system of equations requires the calculation of the following integrals along the appropriately discretized boundary of the borehole,

\[
H_{ij}^{(s)kl} = \int_{C_i} H_{ij}^{(s)}(x_k, x_l, n_i) dC_l \quad (i, j = 1, 2, 3)
\]

\[
H_{ai}^{(s)kl} = \int_{C_i} H_{ai}^{(s)}(x_k, x_l, n_i) dC_l
\]

\[
G_{ij}^{(s)kl} = \int_{C_i} G_{ij}^{(s)}(x_k, x_l) dC_l \quad (i = 1, 2, 3; \ j = 1)
\]

\[
G_{ai}^{(s)kl} = \int_{C_i} G_{ai}^{(s)}(x_k, x_l) dC_l
\]

(4)
in which $H_{ij}^{(s)}(x_k, x_l, n_l)$ and $G_{ij}^{(s)}(x_k, x_l)$ are respectively the Green’s tensor for traction and displacement components in the elastic medium, at point $x_l$ in direction $j$ caused by a concentrated load acting at the source point $x_k$ in direction $i$; $G_{aij}^{(a)}(x_k, x_l)$ are the components of the Green’s tensor for displacement in the fluid medium, at point $x_l$ in the normal direction, caused by a pressure load acting at the source point $x_k$; $H_{aij}^{(a)}(x_k, x_l, n_l)$ is the unit outward normal of the $l$th boundary segment $C_l$; the subscripts $i, j = 1, 2, 3$ denote the normal, tangential and $z$ directions, respectively. These equations are conveniently transformed from the $x, y, z$ Cartesian coordinate system by means of standard vector transformation operators. The required two-and-a-half dimensional fundamental solution (Green’s functions) and stress functions in Cartesian co-ordinates, for the elastic and fluid media can be found at Tadeu and Kausel [14].

The integrations in equation (4) are performed analytically for the loaded element (e.g. Tadeu et al. [15-16]). A Gaussian quadrature scheme is used when the element to be integrated is not the loaded element.

3 Numerical applications

Figure 1 illustrates the geometry of the twin concrete rectangular tunnels, and the concrete separating wall, driven along the $z$ direction in elastic medium. The lining of the tunnel is assumed to be thick and to have material properties analogous to those of the surrounding elastic medium, as described above. The number of boundary elements, modelling the tunnels’ surfaces, increases with the frequency excitation of the harmonic source. The ratio between the wavelength of the incident waves and the length of the boundary elements is at least 10. The minimum number of boundary elements used to model each tunnel is 80. Notice, that the length of boundary elements that models the wall is at least 8 times smaller than its thickness, given the small distance between the two faces of the separating wall.

In the numerical examples, the material properties of the air filling the tunnel $(\rho_a = 1.22 \text{ kg/m}^3, \quad c_a = 340 \text{ m/s})$, and the concrete $(\sigma = 3499 \text{ m/s}, \quad \beta = 2245 \text{ m/s}, \quad \rho = 2500 \text{ kg/m}^3)$, are kept constant.

The calculations were performed for a source in the centre of the tunnel, as illustrated in Figure 1a. Note, that the BEM algorithm makes use of the geometry symmetry of the problem to improve its efficiency.
The response was obtained for pairs of receivers when analyzing the response at particular positions, while a grid of receivers was used to compute the average sound insulation provided by the wall separating the two tunnels (see Figure 1b).

A selection of results illustrating the main findings is given below. First, to illustrate how the sound pressure level within the tunnels modifies when subjected to the incidence of cylindrical waves with different spatial sinusoidal variation along the z direction, a separating wall 0.20 m thick is used. The peaks and troughs in the Fourier amplitude spectra occur at particular frequencies and in definite frequency intervals. They are related to the transversal vibration modes of the wall and to the eigenfrequencies of the tunnels. The average sound insulation over a grid of receivers, provided by the separating wall, is then obtained for different wall thicknesses.

3.1 Incidence of Cylindrical Waves of $k_z = 0$

Next, it is assumed that the 0.20 m thick wall is subjected to the incidence of cylindrical waves of $k_z = 0$. This corresponds to waves arriving at the receivers with a 90° inclination in relation to the z axis.

Figure 2 displays responses obtained when the source is placed on the axis of the first tunnel in the frequency range from 0.5 Hz to 400.0 Hz. Figure 2a illustrates the sound pressure level responses, in a dB scale, obtained at the pair of receivers (1, 2), placed 0.2 m from the separating wall at its mid height, in the two tunnels. Receiver 1 is placed inside the first tunnel together with the source, while receiver 2 is placed in the second tunnel. A source situated on the axis of the tunnel only excites modes which do not exhibit null pressure at the centre of this tunnel [(2q,2r) with $q = 0,1,...$ and $r = 0,1,...$]. However, part of the energy generated in the first tunnel, is transmitted to the second tunnel. In the second tunnel, this energy is observed as though generated by an off-centre source placed at mid height of both tunnels, which excites additional modes.
Notice that the modes \( (q,2r-1) \) with \( q = 0,1,... \) and \( r = 0,1,... \) are not excited, since the source is on the horizontal symmetry axis, for which these modes registered zero pressure. The dynamic process does not stop here, since the energy in this second tunnel is also transmitted to the first tunnel, and there it can be seen as an off-centre source, exciting the additional modes. The results in Figure 2a agree with this interpretation: at receiver 1, the sound pressure field exhibits enhanced peaks in the vicinity of the first excited modes \( f_{10} = 85.0 \text{ Hz}, \ f_{12} = 141.7 \text{ Hz}, \ f_{14} = 242.1 \text{ Hz}, \ f_{20} = 113.3 \text{ Hz}, \ f_{22} = 204.3 \text{ Hz}, \ f_{04} = 226.7 \text{ Hz}, \ f_{24} = 283.3 \text{ Hz}, \ f_{40} = 340.0 \text{ Hz}, \ f_{42} = 358.4 \text{ Hz} \) and \( f_{26} = 380.1 \text{ Hz} \), while additional peaks are clearly visible on the sound pressure response recorded on the second tunnel, which correspond to the excitation of the modes \( f_{1,0} = 85.0 \text{ Hz}, \ f_{12} = 141.7 \text{ Hz}, \ f_{14} = 242.1 \text{ Hz}, \ f_{30} = 255.0 \text{ Hz}, \ f_{32} = 279.1 \text{ Hz}, \ f_{34} = 341.2 \text{ Hz}, \) and \( f_{16} = 350.5 \text{ Hz} \); small peaks in the vicinity of these later modes are seen in the responses for the first tunnel, as the result of energy being transmitted from the second tunnel to the first.

The difference between the sound pressure level registered at receivers 1 and 2 is illustrated in Figure 2b. As expected, low insulation is observed in the vicinity of the eigenmodes of the tunnel. Figure 2b also includes the sound transmission loss predicted by the Theoretical Mass Law. It can be observed that the computed sound pressure difference between the receivers 1 and 2 exhibits pronounced peaks and dips not predicted by the simplified model. These differences are particularly important at lower frequencies (\(<50 \text{ Hz}\)) for which the BEM solution indicates higher insulation values than that expected by the Mass Law.

Figure 2c shows the average sound insulation computed from the response obtained over a grid of 96 receivers, equally spaced 0.25 m along the vertical and the horizontal directions, placed in both tunnels (see Figure 1b). This plot is smoother than the computed pressure level difference between receivers 1 and 2. However, it is still very discordant from the Mass Law predicted insulation. The computed response appears to be highly dependent on the excited modes of the tunnels, showing poor insulation in the vicinity of the corresponding eigenfrequencies. As for the receivers 1 and 2, at very low excitation frequencies the BEM model anticipates higher insulation than that obtained by the Theoretical Mass Law. The results do not reveal a clear dip in insulation related to the coincidence effect. For larger tunnels, or in cases where single panel walls are inserted in an unbounded medium, this effect would be more relevant. The constructive interference among the reflected fields would be weaker, leading to an enhanced contribution due to the propagation of the guided waves along the wall, for which the coupling between the solid and the fluid would be even more important.
Figure 2: Responses obtained when the Source ($k_z = 0$) is excited in the presence of a concrete wall 0.20 m thick: a) Sound pressure level at receivers 1 and 2. b) Sound pressure difference between receivers 1 and 2 (—) versus the theoretical mass law prediction (——). c) Average sound insulation (——) versus the theoretical mass law prediction (——).
The sound pressure level obtained at receivers 1 and 2 when the separating wall is 0.05 m, 0.10 m and 0.20 m thick, for the frequency range from 0.5 Hz to 400.0 Hz, is displayed in Figure 3. The responses recorded at receiver 1 do not appear to change significantly as the thickness of the wall varies. However, the sound pressure level registered at receiver 2 changes markedly when the wall thickness changes. As expected, the lowest sound pressure level at receiver 2 is obtained when the wall is 0.20 m thick, while the highest response is obtained when the wall is 0.05 m thick. The Theoretical Mass Law predicts a constant increase in sound insulation of about 6 dB for each doubling of mass, which does not occur with the BEM solution. Indeed, the sound pressure difference among the various responses computed at receiver 2 oscillates strongly throughout the frequency domain.

The peaks of the sound pressure level computed at the receivers 1 and 2 are still related to the eigenmodes of the tunnels. The existence of additional enhanced sound pressure levels can also be observed in the vicinity of specific frequencies not associated with the normal modes of the tunnels. These enhanced responses are generated by the vibration of the wall, and change according to its thickness.

4 Conclusions

The Boundary Element Method was formulated and used to calculate the sound insulation provided by a separating wall inside two tunnels, for low frequencies
(< 400.0 Hz). This model avoids the limitations in the thickness of the wall, as occurs with the Kirchhoff and Mindlin theories, and takes the coupling between the solid wall and the fluid (air) fully into account. The sound pressure results appeared to be highly dependent on the vibration modes of the tunnels. The type of modes excited inside the tunnels is defined by the source position.

Simplified methods, like the Theoretical Mass Law, are not able to anticipate the oscillations in the insulation provided by a wall, generated by the bending modes of the wall and by the air vibration modes inside the tunnels, which appear to fully define its acoustic behavior. As the thickness of the wall decreases, that is, as the mass of the wall diminishes, the differences between the insulation predicted by the simplified model and that given by the BEM model are more marked. The predicted insulation is lower than the one computed by the BEM model, particularly, at very low frequencies. In the low frequency range, the simplified model should be applied with caution.

When a wall inserted between two rooms is weaker than the surrounding walls, the observed insulation features in this work are expected to be similar to those registered in buildings compartments.

References


