Boundary element analysis of large amplitude of water motion of incident waves against permeable submerged breakwaters

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Abstract

We studied the wave reflection against oblique incident waves at a submerged breakwater. For this purpose, we adopted our experimental results and our boundary element analysis. First, we estimated the physical unknowns for a breakwater using the experimental results obtained by a wave generator in our laboratory. Second, we applied boundary element methods (Kanoh et al. [1]) to confirm the experimental results and to reduce the amount of necessary experiment. In our boundary element analysis, a small amplitude of water motion was assumed and the free-surface boundary was fixed. In this paper, a wave model based on fully nonlinear potential flow equations is applied to the study of the wave motion of incident waves against permeable submerged breakwaters by using the moving boundaries and the B-spline method for the free surface. While trying to reproduce the experimental results obtained in our laboratory by the boundary element method, we intend, in the future, to develop and improve the boundary element analysis of the problem.

1 Introduction

Submerged breakwaters offer shoreline protection by the induction of breaking and
the partial reflection-transmission of moderate and large incident waves. We consider that the protection from moderate incident waves mainly depends on the energy dissipation caused by the motion of the water which passes through the porous or perforated layer of the submerged breakwaters, while the protection from large incident waves depends on the breaking which occurs over submerged breakwaters. In order to present a design for an optimum submerged breakwater that is derived from our experimental results and our numerical analysis, we adopted the following methodology. The physical unknowns of energy dissipation of moderate and large incident waves against a permeable submerged breakwater were estimated using experimental results obtained by a wave generator in our laboratory. The moving boundaries and the B-spline method for the free surface were used in the boundary element method to express the moderate and large amplitudes of water motion of the nonlinear potential flow and to reproduce the energy dissipation in numerical solutions.

2 Experimental model simulation

In our experimental model, we studied the breaking and partial reflection-transmission of moderate and large incident waves against a permeable submerged breakwater, as shown in Figures 1 and 2. Our experimental model satisfies the following specifications: (1) It is a piston wave generator controlled by a computer. (2) It is placed in a channel that is 0.6m wide, 15m long, and 1m deep. (3) The submerged trapezoidal breakwater is permeable. (4) The submerged trapezoidal breakwater is followed by a constant slope and a wave absorber.
2.1 Dimensions and conditions

We adopted an experimental model of a perforated submerged breakwaters according to the dimensions and conditions shown in Table 1.

Table 1 : Experimental model dimensions [Scale of 1:25]

<table>
<thead>
<tr>
<th></th>
<th>Actual object</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water depth (h)</td>
<td>9.8(m)</td>
<td>27.2(cm)</td>
</tr>
<tr>
<td></td>
<td>10.8(m)</td>
<td>30.0(cm)</td>
</tr>
<tr>
<td>Submerged depth</td>
<td>1.98(m)</td>
<td>5.2(cm)</td>
</tr>
<tr>
<td>of crown (R)</td>
<td>2.98(m)</td>
<td>8.3(cm)</td>
</tr>
<tr>
<td>Wave steepness (H/L)</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>Wave period (T)</td>
<td>6 ~ 12(s)</td>
<td>1 ~ 2(s)</td>
</tr>
<tr>
<td>Wave length (L)</td>
<td>48.1 ~ 112.2(m)</td>
<td>133.6 ~ 448.8(cm)</td>
</tr>
<tr>
<td></td>
<td>49.3 ~ 117.2(m)</td>
<td>137.2 ~ 468.9(cm)</td>
</tr>
<tr>
<td>Wave height (H)</td>
<td>1.0 ~ 2.2(m)</td>
<td>2.7 ~ 6.2(cm)</td>
</tr>
<tr>
<td></td>
<td>1.0 ~ 2.3(m)</td>
<td>2.7 ~ 6.5(cm)</td>
</tr>
</tbody>
</table>
2.2 Experimental results

Figure 3 illustrates the pulse-height records of an incident wave and the corresponding transmitted wave obtained by the experimental model of an impermeable submerged breakwater. When the water depth is 38 cm (actual object size: 9.5 m), the submerged depth of crown is 8 cm (2 m), and wave period is 2 sec (12 sec), then the incident wave is observed as the sine wave of the wave height of 3.7 cm (0.93 m). The corresponding transmitted wave is changed to the nonlinear and irregular wave with a maximum wave height of 4.1 cm (1.03 m).

Figure 4 shows the relations among the transmission coefficient (Kt), the wave periods (T), and the submerged depth of crown (R) obtained by the experimental model. Here the wave periods have seven values ranging from 1.0 to 2.0 sec, and the submerged depth of crown is 5.2 or 8.3 cm. As the wave period diminishes, the transmission coefficient becomes smaller, i.e., the porous submerged breakwater has the effect of wave dissipation.

Figure 5 contains an illustration of the mechanism model of a wave dissipated by the porous submerged breakwater. We consider that the water motion through the porous plank is produced by the travelling wave trough and crest. An eddy loss occurs, resulting in effective dissipation by the porous submerged breakwater.

Figure 3: Experimental results of pulse-height analyzer for incident and transmitted waves: (a) pulse-height record of an incident wave; (b) pulse-height record of the corresponding transmitted wave.
Figure 4: Relations among the transmission coefficient (Kt), the wave periods (T), and the submerged depth of crown (R) obtained by the experimental model.

Figure 5: Wave-dissipation mechanism of the porous submerged breakwater.

Figure 6: Dimension of a numerical example.
3 Boundary element method analysis

3.1 Governing equations and boundary conditions

The time-dependent velocity potential $\phi(x, z, t)$ is used to describe two-dimensional flows in the vertical $(x, z)$ plane, as illustrated in Figure 6. The velocity $u \,(u, w)$ is written as

$$u = \nabla \phi \quad \text{or} \quad u = \frac{\partial \phi}{\partial x}, \quad w = \frac{\partial \phi}{\partial z} \quad (1).$$

The Laplace equation for the potential is adopted in the fluid domain $\Omega$ with boundary $\Gamma$:

$$\nabla^2 \phi = 0 \quad \text{in } \Omega \quad (2).$$

On the free surface $\Gamma_f$, the potential $\phi$ satisfies nonlinear kinematic and dynamic boundary conditions,

$$Dr / Dt = u = \nabla \phi \quad \text{on } \Gamma_f \quad (3)$$

$$D\phi / Dt = -gz + 1/2 \nabla \cdot \nabla \phi - \frac{P_a}{\rho} \quad \text{on } \Gamma_f \quad (4),$$

respectively (Grilli et al. [3]). Here $r$ denotes the position vector of a free-surface fluid particle, $g$ is the acceleration due to gravity, $P_a$ is the atmospheric pressure, and $\rho$ is the fluid density. These free-surface boundary conditions are integrated at time $t$ to establish both the new position and the relevant boundary conditions on the free surface at a subsequent time $t + \Delta t$. This integration is done, following the approach introduced by Dold & Peregrine [4], using Taylor expansions for both the position $r$ and the potential $\phi(r)$ on $\Gamma_f$. Hence the free-surface boundary conditions are expressed as

$$r(t + \Delta t) = r(t) + \Delta t(Dr/Dt) + \{(\Delta t)^2/2\} \cdot (D^2r/Dt^2) \quad (5)$$

for the free surface position and as

$$\phi(r(t + \Delta t)) = \phi(r(t)) + \Delta t(D\phi(r(t))/Dt) + \{(\Delta t)^2/2\} \cdot (D^2\phi(r(t))/Dt^2) \quad (6)$$

for the potential. The material derivative is defined as

$$D / Dt = \frac{\partial}{\partial t} + u \cdot \nabla \quad (7).$$

Detailed expressions of coefficients of Taylor series (5) and (6) are derived in the following: The kinematic free-surface boundary condition, Eq. (3) provides the first-order coefficient in the series (5). Applying the material derivative (7) to Eq.(3),
we obtain the general expression of the second-order coefficient in the series (5) as

$$D^2r/Dt^2 = Du/Dt = \partial \cdot u/ \partial t + u \cdot \nabla u$$  \hspace{1cm} (8)

In the same way, the dynamic free-surface boundary condition (4) provides the first-order coefficient in the series (6). The second-order coefficient in (6) is obtained by the material derivation of (4) as

$$D^2\phi/Dt^2 = D/Dt (D\phi/Dt) = -g \cdot Dz/Dt +1/2 \cdot D/Dt (\nabla \phi \cdot \nabla \phi) - D/Dt (P_a/\rho)$$  \hspace{1cm} (9)

Waves are generated by simulating a piston wavemaker motion on the right-hand ‘ocean’ boundary $\Gamma_{ro}$ of the computational domain. Motion $\bar{x}$ and normal velocity are specified over the paddle as

$$\bar{x} = x_p ; \nabla \phi \cdot n = \partial \phi/ \partial n = -u_p$$ \hspace{1cm} on $\Gamma_{ro}$  \hspace{1cm} (10)

The prescribed wavemaker motion $x_p$ and velocity $u_p$ are defined as

$$u_p = A \cdot \sin\omega t ; x_p = A/\omega (1 - \cos\omega t)$$  \hspace{1cm} (11)

Along the bottom and breakwater surfaces $\Gamma_b$ and other fixed boundaries $\Gamma_{rl}$, a no-flow condition is described as

$$\partial \phi/ \partial n = 0$$ \hspace{1cm} on $\Gamma_b$ and $\Gamma_{rl}$  \hspace{1cm} (12)

Along the porous plank surface $\Gamma_p$ of a submerged breakwater, potential flux condition is described as

$$\partial \phi/ \partial n = \{\phi_U - \phi_L\} h/\{d_0 V_v (\alpha - i \beta)\}$$  \hspace{1cm} (13)

where $\phi_U$ and $\phi_L$ are the potential of the upper and lower parts of the porous plank, respectively. Here $h$ is the water height, $i$ is the imaginary unit, and $d_0$ and $V_v$ are the depth and the vertical porosity of the plank layer, respectively. Then $\alpha$ and $\beta$ are defined as

$$\alpha = (1 + \mu_2)/V_v , \hspace{1cm} \beta = \mu_1/V_v$$  \hspace{1cm} (14)

where $\mu_1$ and $\mu_2$ are the coefficients of fluid resistance that are proportional to the fluid velocity and acceleration, respectively (Kanoh et al. [1]).

3.2 Boundary element expression

The discritized boundary element expression for analyzing Eq. (2) is written as

$$H\phi = Gq \hspace{1cm} \text{and} \hspace{1cm} H\dot{\phi} = G\dot{q}$$  \hspace{1cm} (15) and (16),
where $q = \frac{\partial \phi}{\partial n}$, and $\dot{\phi}$ and $\dot{q}$ are the time derivatives of $\phi$ and $q$, respectively (e.g. Kanoh [4]).

### 3.3 Results of numerical method

Regarding the numerical example shown in Figure 6, Figure 7 illustrates the relations among the transmission coefficient (K_t), the reflection coefficient (K_r), and the submerged width of crown (W) calculated by the numerical method. The numerical method was derived by Ijima (e.g. Okuzono [5]) from the theory based on Green’s function. SHG denotes the scalar quantity of the wave period ($SHG = (2\pi T)^2/g$), R is the submerged depth of crown, d is the depth of the porous plank, and $d_1$ is depth of the water chamber. Here R, d, and $d_1$ are also scalar quantities which are divided by the water depth h. As the submerged width of crown varies from 3 to 4, the transmission and reflection coefficients vary symmetrically with respect to the horizontal axis. They show the minimum or maximum value if the submerged width of crown is 3.6.

### 4 Results and discussion

Referring to the experimental and numerical solutions described above, we obtained the following results. (1) When the wave periods are small, a porous submerged breakwater has the effect of wave dissipation. (2) The numerical method solutions show that the transmission and reflection coefficients vary symmetrically.
with respect to the horizontal axis as the submerged width of crown varies from 3 to 4. The coefficients show the minimum or maximum value if the submerged width of crown is 3.6. (3) We consider that the water motion through the porous plank is produced by the travelling wave trough and crest and that an eddy loss occurs, resulting in the effective dissipation of the porous submerged breakwater. Using the experimental and numerical solutions, in future research we intend to improve the porous submerged breakwater such that it can have the effect of wave dissipation even if the wave periods are large.

5 Conclusion

In order to design suitably a permeable submerged breakwater, we estimated the physical phenomena for a breakwater using the results of our laboratory experiment. We then applied the numerical method to confirm the experimental results and to reduce the amount of necessary experiments. Using these experimental and numerical solutions, we found the mechanism of the effective dissipation of the porous submerged breakwater.

References


