Plane wave coupling to finite length cables buried in a lossy ground

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Abstract

The cable of a finite length below the ground surface is analyzed applying the scattering theory. The problem is formulated via the electric field integral equation (EFIE) for thin wires. The EFIE is solved by the Galerkin-Bubnov boundary element procedure and the current induced along the buried cable due to the transmitted plane wave excitation is obtained.

1 Introduction

Electromagnetic Coupling to cables buried in a lossy ground is one of the major causes of malfunction, or even destruction of telecommunication cable systems (equipment). This paper deals with the current induced in a finite length bare cable buried in a lossy half-space, due to a plane wave excitation.

It is worth emphasizing that the problem of plane wave coupling to overhead cables [1], [2] has been investigated to a greater extent than the buried cable systems [3].

On the other hand, almost all studies on this subject have been based on an approximate transmission line (TL) approach, pertaining to the infinite buried cables. Moreover the influence of the air-earth interface has often been neglected which implies the assumption that the cable is buried at a very large depth (the homogeneous case) [3].

However, for cases of low ground conductivity and when the finite length cables are buried near the air-earth interface the TL approximation is not valid. Consequently, for studies dealing with the finite length cables, a rigorous integral

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equation approach is required. Such study, using the rigorous Sommerfeld integral formulation for the treatment of the effect of the air-earth interface has been done in [4].

The method proposed in this paper follows the integral equation (IE) formulation [4], but on the other hand uses simplified approach via modified image theory [5], instead of time consuming Sommerfeld integral approach [4].

The analysis is carried out via the frequency domain electric field integral equation (EFIE) for thin wires (Pocklington equation). The effect of the lower lossy medium is taken into account via the transmission coefficient arising from the modified image theory. This coefficient appears as a part of the IE kernel.

The current along the buried cable is obtained by solving the corresponding integral equation via Galerkin-Bubnov Boundary Element Method (GB-BEM) [1], [2] and some illustrative numerical results are presented.

2 Integral equation formulation

The horizontal cable of length 2L and radius a, buried in a lossy medium at depth d is considered Fig 1. In accordance to the thin wire approximation and the modified image theory [5] the tangential component of the scattered electric field along the metallic wire surface is given by:

$$E_x = \frac{1}{j4\pi\omega\varepsilon_{eff}} \int_{-L}^{L} \left\{ \left[\frac{\partial^2}{\partial x^2} - \gamma^2 \right] g(x,x') \right\} I(x')dx' \tag{1}$$

where I(x') is the current distribution along the wire while g(x,x') denotes the corresponding Green function:

$$g(x,x') = g_0(x,x') - \Gamma g_i(x,x') \tag{2}$$

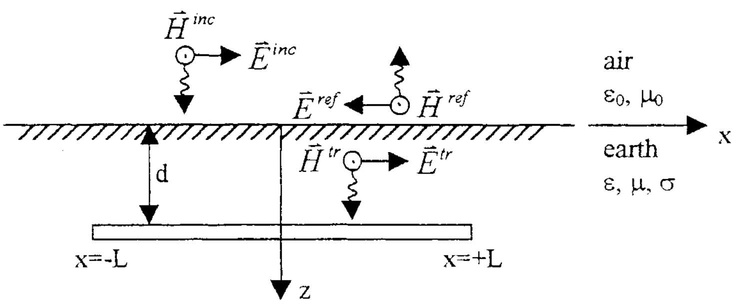

Figure 1: The geometry of the problem

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where Γ is the transmission coefficient obtained by the modified image theory [5]:

$$\Gamma = \frac{\varepsilon_{eff} - \varepsilon_0}{\varepsilon_{eff} + \varepsilon_0}, \varepsilon_{eff} = \varepsilon_r \varepsilon_0 - j\frac{\sigma}{\omega} \tag{3}$$

and g_0 and g_i are the Green functions of the homogeneous lossy medium for the source wire and its image, respectively:

$$g_0(x,x') = \frac{e^{-\gamma R}}{R}, g_i(x,x') = \frac{e^{-\gamma R^*}}{R^*} \tag{4}$$

where γ is the phase constant and R and R^* are the distances from the source wire and from its image to the observation point, respectively:

$$R = \sqrt{(x-x')^2 + a^2}, R^* = \sqrt{(x-x')^2 + 4d^2} \tag{5}$$

The total tangential electric field can be written as the sum of the transmitted plane wave electric field E_x^{tr}, and the related scattered field on the wire surface E_x^{sct}:

$$E_x^{tot} = E_x^{tr} + E_x^{sct} \tag{6}$$

For the perfectly conducting (PEC) wire the total tangential electric field vanishes:

$$E_x^{tot} = 0 \tag{7}$$

and the electric field integral equation (EFIE) for the horizontal wire buried in a lossy ground follows in the form:

$$E_x^{inc} = -\frac{1}{j4\pi\omega\varepsilon_{eff}} \int_{-L}^{L} \left[\frac{\partial^2}{\partial x^2} - \gamma^2 \right] g(x,x') I(x') dx' \tag{8}$$

Solving the integral equation (8) the equivalent current distribution is obtained.

3. The Galerkin-Bubnov boundary element procedure

In this work, the Galerkin Bubnov scheme of the boundary element method [1], [2] is applied resulting in the following variational equation:

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$$\sum_{i=1}^{n} I_i \left[\frac{1}{j4\pi\omega\varepsilon_{eff}} \left\{ -\int_{-L}^{L} \frac{df_j(x)}{dx} \int_{-L}^{L} \frac{df_i(x')}{dx'} g(x,x')dx'dx + \right.\right.$$

$$\left.\left. +\gamma^2 \int_{-L}^{L} f_j(x) \int_{-L}^{L} f_i(x') g(x,x')dx'dx \right\} \right] = \tag{9}$$

$$= -\int_{-L}^{L} E_x^{tr} f_j(x)dx, \; j = 1,2,\ldots,n$$

where n denotes the total number of basis functions, I_i are the unknown coefficients of the solution, and f_i and f_j are the basis and test functions, respectively. Performing the boundary element discretization of the domain of interest the following matrix equation arising from (9) is obtained:

$$\sum_{i=1}^{M} [Z]_{ji} \{I\}_i = \{V\}_j, \; j = 1,2,\ldots,M \tag{10}$$

where $[Z]_{ji}$ is the local matrix presenting the mutual impedance between the i-th source to the j-th observation wire segment:

$$[Z]_{ji} = -\frac{1}{j4\pi\omega\varepsilon_{eff}} \left\{ \int_{\Delta l_j}\int_{\Delta l_i} \{D\}_j \{D'\}_i^T g(x,x')dx'dx - \right.$$

$$\left. -\gamma^2 \int_{\Delta l_j}\int_{\Delta l_i} \{f\}_j \{f'\}_i^T g(x,x'),dx'dx \right\} \tag{11}$$

Matrices {f} and {f'} contain shape functions $f_k(x)$ and $f_k(x')$, while {D} and {D'} contain their derivatives (M is the total number of boundary elements, Δl_i, Δl_j is the width of i-th and j-th boundary element, respectively). Functions $f_k(x)$ are the Lagrange's polynomials defined by expression:

$$L_k(x) = \prod_{i=1}^{m} \frac{x - x_i}{x_k - x_i}, \; i \neq k \tag{12}$$

$\{V\}_j$ is the local voltage vector on the j-th observation segment along the wire:

$$\{V\}_j = -\int_{\Delta l_j} E_x^{tr} \{f\}_j \, dx \tag{13}$$

In this paper, the linear approximation over boundary element is used since this choice was shown [1], [2] to ensure the satisfactory convergence rate.

An important advantage of the presented method is the replacing of the second-order differential operator from the kernel thus avoiding the problem of quasi-singularity.

The linear equation system (10) is completed with associated boundary conditions for the zero current at the wire ends.

4 Numerical results

The excitation function tangential along the wire surface is a transmitted plane wave of the form:

$$E_x^{tr} = \Gamma E_0 e^{-\gamma z} \Big|_{z=d} \tag{14}$$

where E_0 is the field value at the ground level.

The cable is characterized by a length 2L=10m, with conductor radius a=0.005m, while the burial depth is h=0.3m. The magnitude of the plane wave at the ground level is chosen to be E_0=1V/m at the given frequency. The relative dielectric constant of the ground is ε_r=10.

Fig 2 and 3 show the real and imaginary part of the current induced along the buried wire for the various excitation frequency with the ground conductivity σ=0.001S/m. Next two figures present the influence of the ground conductivity to the induced current at the fixed frequency f=1MHz. Finally, the influence of the burial depth to the induced current at the fixed frequency f=1MHz can be observed on the figures 6 and 7.

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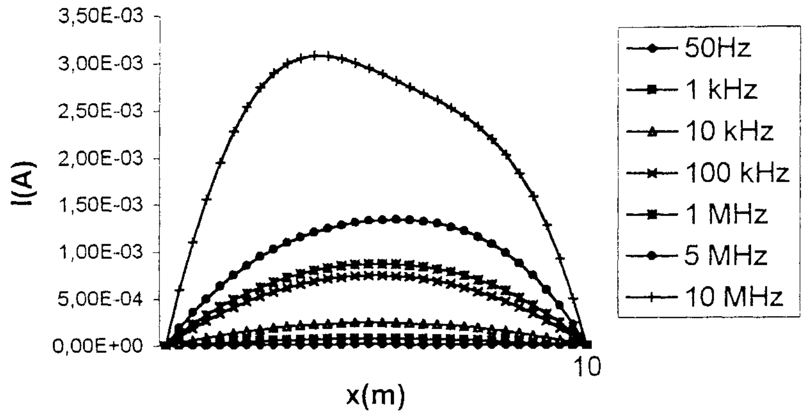

Figure 2: Real part of the induced current on the buried wire of length 2L=10m, radius a=5mm, burial depth h=0.3m, σ=0.001S/m, ε_r=10 for the various frequencies

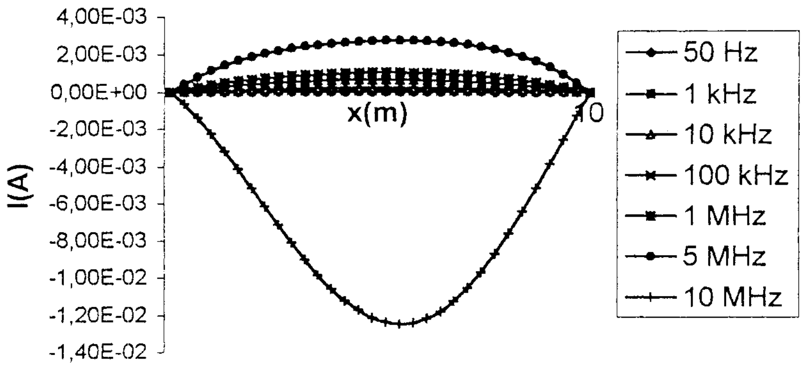

Figure 3: Imaginary part of the induced current on the buried wire of length 2L=10m, radius a=5mm, burial depth h=0.3m, σ=0.001S/m, ε_r=10 for the various frequencies

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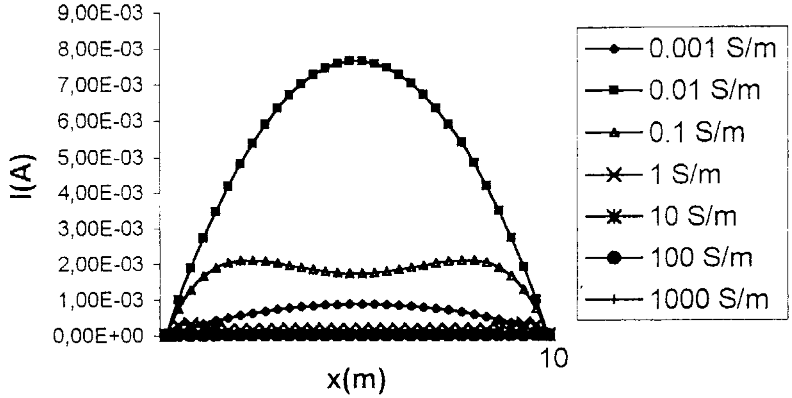

Figure 4: Real part of the induced current on the buried wire of length 2L=10m, radius a=5mm, buried depth h=0.3m, f=1MHz, ε_r=10 for the various conductivities

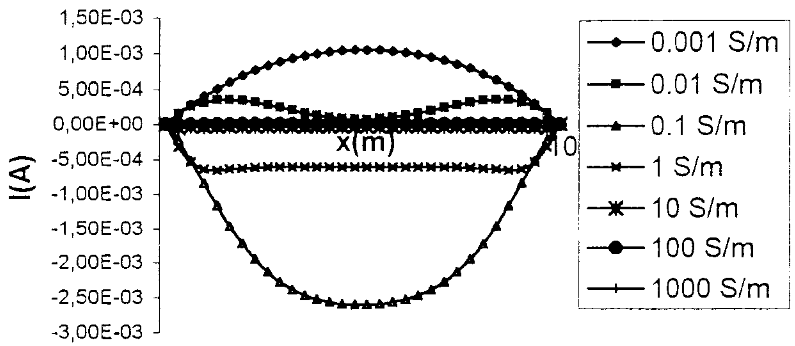

Figure 5: Imaginary part of the induced current on the buried wire of length 2L=10m, radius a=5mm, buried depth h=0.3m, f=1MHz, ε_r=10 for the various conductivities

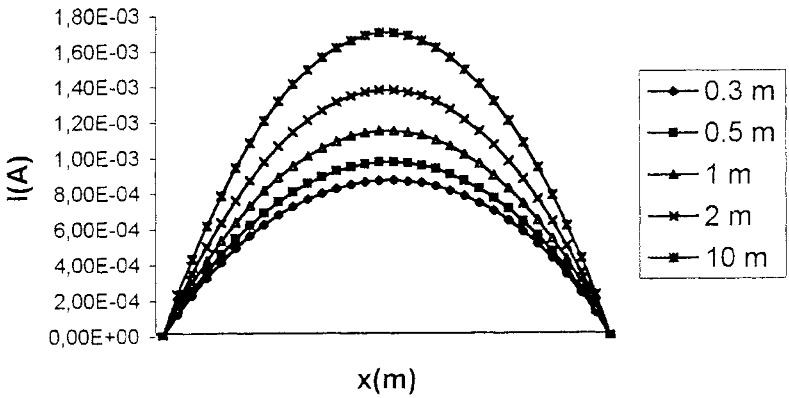

Figure 6: Real part of the induced current on the buried wire of length 2L=10m, radius a=5mm, f=1MHz, σ=0.001S/m, ε_r=10 for the various burial depths

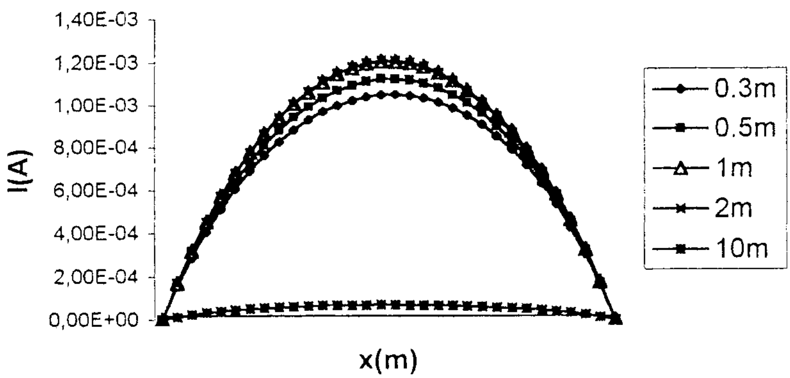

Figure 7: Imaginary part of the induced current on the buried wire of length 2L=10m, radius a=5mm, f=1MHz, σ=0.001S/m, ε_r=10 for the various burial depths

5 Concluding Remarks

The boundary element analysis of a cable buried in a lossy ground, illuminated by the plane wave excitation is presented in the paper. The electric field integral equation (EFIE) for the thin wire buried in a dissipative medium is solved by the Galerkin-Bubnov boundary element method (GB-BEM). The presence of lossy half-space has been taken into account via transmission coefficient obtained by the modified image theory.

The proposed numerical procedure ensures the satisfactory convergence rate. Moreover, the method discussed so far could be easily applied to the multiconductor underground cable systems.

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