Estimating eddy diffusivities coefficients from spectra of turbulence
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Abstract

The behaviour of spectra of turbulence in the surface layer is used to estimate the vertical eddy diffusivity coefficient in order to describe the dispersion of pollutant emitted by a thermoelectric power plant located in the south of Brazil. The simulation of the transport of the material is done with an analytical solution of advection–diffusion equation. The meteorological inputs for this model were obtained in field campaigns conducted in the region. The data set analysed comprised twenty-four hours of turbulence measurements. The results of the model are compared with results obtained by the same model but using different parameterization for the eddy diffusivity coefficient.

1 Introduction

Heat, mass and momentum transfers are important for characterisation of the planetary boundary layer (PBL). In the past, studies of these turbulent transport in the atmosphere have led to hypothesis concerning the eddy diffusivity coefficient $K$. From the mathematical point of view this coefficient arises from representing the total turbulent flow in terms of the mean flow as required by the closure problem.

The mixing efficiency $K$ of the atmosphere must be dependent upon general characteristics of the turbulent flow field. Following this concept Kolmogorov [8] and Prandtl [10] related the eddy diffusivity to the turbulent kinetic energy as follow
Air Pollution

\[ K = c_o \cdot l \cdot E^{1/2} \]  \hspace{1cm} (1)

where \( E \) is the turbulent kinetic energy, \( l \) is a characteristic length scale and \( c_o \) is a constant taken as 0.4.

On the other hand turbulence energy spectra give an appropriate representation of the eddy size ranges active during mixing process. Parameterizations of the eddy diffusivity coefficients with results from spectral analysis has become popular through methods advocated by Pasquill [9] and Hanna [3,4]. For example one way of parameterizing diffusivities is in terms of turbulent dissipation rate (\( \varepsilon \)) estimated from the inertial subrange of spectra (Berkowicz and Prahm [1]). Another frequently used way is through the spectral maximum frequency.

In this paper we use a portion of the data collected during fields experiments in the south of Brazil to study the local spectral characteristics of the surface layer. From this spectral characteristics we estimate the length scale of the energy-containing eddies. Using the same data set we are able to know the turbulent kinetic energy and, as a consequence, the eddy diffusivity as hypothesised by Prandtl-Kolmogorov. In addition we use the \( K^* \)'s obtained from turbulence measurements in the KAPPAG model (Tirabassi et al [12]) in order to study the dispersion of SO2 emitted by a thermoelectric power plant located in the region and compare the results with those obtained by the same model using the classical similarity schemes.

2 The Candiota Program and Site Description

The data used in this study are part of a larger set obtained during 1994/1995 Candiota experiments conducted by brasilian Micrometeorological Laboratories from Universidade Federal de Santa Maria and Universidade de São Paulo.

The Candiota Program is a large co-operative project which aims to address key issues related to the acid rain problem in Southern Brazil. This project is sponsored by Companhia Estadual de Energia Elétrica (CEEE) of Rio Grande do Sul state and five fields campaigns was conducted to study the local atmospheric circulation and the turbulent characteristics of the planetary boundary layer in Candiota city where the CEEE's thermal power plant is located. The Candiota power plant is remote from other large sources of SO2. It is located at 31° 28' S, 53° 40' W, 250 above mean sea level, in the south of Brazil near Uruguay's border. Its chimney is about 150 meters height. The capacity of the power plant is 400 MW and its daily coal consumption is about 3.02 x 10^3 tones. It is estimated that 700 g of SO2 are emitted per second from the stack of the plant. The estimated emission of SO2 is based on the typical sulphur content of 2% of Brazil coal.

The terrain in this part of the country is flat and moderately heterogeneous. The area surrounding the site is mostly savannah and agricultural land used for farming and raising livestock. The steepest gradient (2%) is to the Northeast. There was no change in the surface during the experiments. Figure 1 shows a coarse view of the Candiota terrain.
Figure 1: Topography of the Candiota region. At the centre the thermoelectric power plant.

3 Instrumentation and Data Set

Instrumentation for the Candiota experiments included one-axis anemometer, two Gill anemometers, fine platinum wire thermometer and Krypton hygrometer mounted on a 12 m micrometeorological tower.

Slow-rise radiosondes ascents made every 2 h during observational periods gave informations about vertical profiles of wind, temperature, humidity and pressure. From these data we were able to determine the height of the atmospheric boundary layer and its variation as a function of time. Throughout the day theodolite-tracked balloons measured vertical profiles of wind in the interval between the radiosondes launched.

The data set analysed here comprised of twenty four 13.66 min runs belong to a continuous day-night period. Each test was taken in the beginning of the hour and consisted of simultaneous velocities and temperatures fluctuations measurements at 10 m on the tower. The day in question (10th Feb. 1995), hereafter abbreviated 100295 was characterised by moderate, slightly anti-cyclonic westerly flow near near a weak quasi-stationary. The table below presented some parameters from each run.
Table 1: wind speed (m/s), stability parameter and turbulent kinetic energy (m²/s²) for each analyzed run.

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<th>hour</th>
<th>U</th>
<th>z/L</th>
<th>TKE</th>
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4 Spectra of Velocity and the Length Scales

The spectra were computed using the fast-Fourier technique. The available bandwidth was covered in two stages. The higher range was obtained by dividing each 14-min recorded into 8 consecutive blocks of 1,024 data points and constructing a composite spectrum by averaging the 8 separate spectra. The composite spectra was then smoothed by averaging spectral estimates over 22 frequency bands.

For the lower range a new time series was generated from the original series by subjecting it to a 2 non overlapping block average. The spectrum computed from this series has inherently more scatter than the composite spectrum but the agreement between the two where they overlap is very good. The higher range was, therefore, treated as our basic spectrum with estimates from the lower range used only to extend its bandwidth to 0.0024 Hz.

The logarithmic spectra of vertical velocity component for some selected runs are presented in Figure 2. These runs corresponding to stable, near neutral and convective conditions.

![Figure 2: Spectra of vertical velocity from 10 m level on the tower.](image)
Whereas the shape of the velocity spectra is quite similar under widely varying conditions, the magnitudes vary greatly. In general the spectra of velocity exhibit the $a^{-5/3}$ power law. Also it is observed that the spectral intensity increase with increasing shear and instability. Another interesting behaviour observed in Figures 1 is an orderly progression of the spectral peak in the direction of increasingly smaller $f$ as $z/L$ varies from +0.6 to -0.1.

The twenty four composite spectra were used to estimate the characteristic length scale to be used in equation (1). Several schemes for estimating the characteristic length scale of the energy-containing eddies can be found in the literature (Tennekes and Lumley [11], Kaimal [7]). One of the most commonly is the wavelength ($\lambda_m$) corresponding to the peak of the logarithmic power spectrum which is reasonably easy to estimate. This length scale is used widely in the interpretation of the atmospheric spectra and become important in diffusion computation since the works of Pasquill [9] and Hanna [3,4].

Figure 3 presents plots of $(f_m)_w$ vs. $z/L$. Kaimal et al [6] showed from the Kansas data a systematic shift in the spectral peak with respect to changes on $z/L$. The results here presented show approximately the same behaviour.

$$K = c_o \cdot l \cdot E^{1/2}$$ (1)
where $E$ is the turbulent kinetic energy, $l$ is a characteristic length scale and $c_o$ is a constant taken as 0.4. The spectral peaks for vertical velocity spectra vary from approximately 2.5 under stable conditions to 0.2 under convective conditions. The spectra of $w$ allow us to define a length scale $l = z / (n_m) = U / (n_m)$, where $n_m$ are the values of frequency $n$ where the spectra $nS_w(n)$ are at their maximum.

The figure 4 shows the day evolution of the eddy diffusivity estimated from the spectra of turbulence. At the same figures it is showed the eddies coefficients used by the KAPPAG model and derived with Similarity theory.

**Figure 4:** Eddies coefficients derived from equation (1) and originally used by the KAPPAG model.

### 6 The KAPPAG model

Tirabassi et al. [12] attempted to fill the gap by suggesting a computer routine (KAPPAG) based on analytic solution of the advection-diffusion equation for power law vertical profiles of wind and eddy coefficient and on a Gaussian formula for lateral diffusion.

It is a routine based on an analytical solution of the advection-diffusion equation in which vertical diffusion for power-law vertical profiles of both wind speed and eddy diffusivity. The KAPPAG simulates lateral diffusion using a Gaussian solution as suggested by Huang [5]. The performance of the model has been assessed with success against experimental data and it has been shown that KAPPAG is able to represent concentrations in boundary layers whose vertical exchange coefficient go to zero at the top.

KAPPAG is very flexible in the definition of the meteorological inputs. The minimum information required, besides emission data, is the wind speed measured at some height, the vertical atmospheric stability (either a Pasquill-Gifford stability class or the Monin-Obukhov length) and the horizontal atmospheric stability. The
package can extrapolate this minimum information to compute full vertical profiles. However, if vertical meteorological profiles are available, they can be used directly by the package. In this study we used the wind speed measured at 10 m, the vertical atmospheric stability (classified according the Monin-Obukhov length) and the horizontal atmospheric stability (according the Pasquill-Gifford stability class).

The model output is a statistical summary of the concentrations computed at each receptor, during each time step, and due to each source. Partial and total concentrations are computed for hourly and multi-hour averages. Highest and second-highest values are also evaluated.

7 Results and Discussion

The objective of this paper has been to compare the KAPPAG model results using different methods to calculate eddy diffusivities coefficients.

The KAPPAG model as originally proposed by Tirabassi el at [12] computed the vertical diffusion coefficient through similarity theory, using the formula

\[ K = \frac{\kappa U h}{\Phi_h \Phi_m} \]

where \( \kappa \) is the von Karman constant, \( \Phi_h \) is the nondimensional temperature gradient, \( \Phi_m \) is the nondimensional wind speed and \( U \) is the wind speed at height \( h \). The routine utilises the Dyer [2] parameterization for \( \Phi_h \) and \( \Phi_m \).

The surface concentration pattern from the Candiota source are showed in figures 5 and 6 for two different parameterizations of the vertical diffusivity.

![Figure 5: SO2 ground level concentration. 24 hour average. The source is located at the centre of the square. Results from the KAPPAG model with eddy diffusivities from Similarity theory.](image-url)
From the above figures some relevant differences must be considered. At first the significant differences in the predictions can be explained largely by the differences in their predictions of stability and associated dispersion coefficient (see figure 4). The measurements indicated during daytime the $K$’s values are associated with unstable B and C conditions while the values calculated by equation (2) indicated A and B unstable conditions.

The maximum ground level concentration estimated by KAPPAG with coefficient given by equation (1) is five times greater than the estimated by coefficient given by equation (2). In the first simulation (figure 6) the maximum ground level concentration is about 74 $\mu g / m^3$ at 500 meters from the source. In the second case (figure 5) the maximum ground level concentration is about 15 $\mu g / m^3$ at 12 km from the source. This difference can also be explained by the differences in the stability. Because downdrafts cover more than half the area of the horizontal plane over the bulk of the mixed layer depth, a majority of material released by an elevated source starts descending and continuous to descend for a significant time.

Finally must be stressed that the Protection Environmental Agency of Rio Grande do Sul State (FEPAM) has only one environmental station in the region. It is located in the point (-15,000;-30,000) in figures 5 and 6. In this point the maximum ground level concentration is 1 $\mu g / m^3$ in both models and agree with experimental measurements. To prove how more physics in the model will be necessary for experimental data comparison a new project is now being implemented. In this project 8 different points of observations will be installed and continuous micrometeorological data will be obtained.

**Figure 6**: same as figure 4 but with eddies coefficients estimated from spectra of turbulence.
References


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