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Similarity theory and model of diffusion in turbulent atmosphere at large scales

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Abstract

The turbulent diffusion model is submitted. The model is based on the similarity theory of random processes and is derived from "the first principles". The problem about turbulent diffusion of an impurity above an industrial region at specific emission is considered. The general solution of the problem in special case when the emission is a periodically function of time is obtained.

1 Introduction

The theory of turbulent diffusion is a basic for the air pollution transport modeling. At the moment the models of turbulent transport are based mainly on the Monin-Obukhov theory of similarity [1] and hypothesis that the eddy diffusivity of an impurity is proportional to the eddy viscosity (Russell *et al.* [2]; Pugliese *et al.* [3]). In turn the turbulent eddy viscosity is determined from the empirical formulas (Businger *et al.* [4]) or from $k-\epsilon$ model (Pugliese *et al.* [3]), or from the turbulent kinetic energy model (Moraes *et al.* [5]). But in general case the problem of closures is not decided, since the parameters of an atmospheric turbulence depend on several scales and non-local parameters (Zilitinkevich [6]). The next important problem of air pollution modelling is that the interaction of a wind with a geometrically complex urban surface makes turbulent intermixing effect of pollutants going from various sources. Thus, the eddy diffusivity of an impurity depends on geometric parameters in a case of complex terrain (Raupach *et al.* [7]; Hoydysh & Dabberdt [8]). The realistic model of turbulent diffusion should reflect the interaction of a wind with the urban landscape and the contributions of various scales, for example, from buildings and trees, and from adjoining industrial regions (Oke [9]).

In this paper the turbulent diffusion model is submitted. The model is based on the similarity theory of random processes (Trunev [10,11]) and is derived from "the first principles". The turbulent diffusion parameters are determined for the whole region and thus meteorological, roughness and non-local parameters are included in the model automatically.

2 Input equations and similarity theory

We shall consider an air flow containing a scalar impurity. Air is assumed as viscous, heat-conducting, incompressible gas in rather slow turbulent motion. Thus, the model of an air flow is the following:

$$\nabla.\tilde{\mathbf{v}} = 0 \tag{1}$$

$$\frac{\partial \tilde{\mathbf{v}}}{\partial t} + (\tilde{\mathbf{v}}.\nabla)\tilde{\mathbf{v}} + \frac{\nabla \tilde{p}}{\tilde{\rho}} = \nu \nabla^2 \tilde{\mathbf{v}} + \frac{\mathbf{g}}{\tilde{\rho}}(\tilde{\rho} - \rho_e)$$

$$\frac{\partial \tilde{T}}{\partial t} + (\tilde{\mathbf{v}}.\nabla)\tilde{T} = \frac{\nu}{\mathbf{Pr}}\nabla^2 \tilde{T}$$

$$\frac{\partial \tilde{\phi}}{\partial t} + (\tilde{\mathbf{v}}.\nabla)\tilde{\phi} = D\nabla^2 \tilde{\phi}$$

where $\tilde{\rho}$ is the air density, $\tilde{\mathbf{v}}$ is the flow velocity vector, ν is the kinematic viscosity, $\tilde{p}$ is the pressure except the hydrostatic atmospheric pressure p_e, $\mathbf{g}$ is the gravity acceleration vector, ρ_e is the equilibrium density, $\tilde{T}$ is the temperature, $\mathbf{Pr}$ is the Prandtl number, $\tilde{\phi}$ is the mass concentration of an impurity, D is the molecular diffusion coefficient. The hydrostatic equation and the standard Boussinesq approximation for the density fluctuations can be written as

$$\nabla p_e = \mathbf{g}\rho_e(p_e, T_e), \quad \rho - \rho_e = -\rho_e \beta_e(\tilde{T} - T_e)$$

where $\beta_e = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p$ (for ideal gas $\beta_e = 1/T_e$).

The coordinate system should be determined in such a way that the Z-axis is directed opposite to the vector of the gravity acceleration. The relief of a ground surface is described by a function $z = r(x, y)$ - see Figure 1.

The usual application of the similarity theory in hydrodynamics concerns only to mean values (Monin & Obukhov [1]; Prandtl [12]), whereas the actual flow parameters are obviously random parameters. Similarity theory of random process (STRP) in a turbulent flow was developed in [10-11]. The main assigning of the new STRP is to allocate among all solutions of the hydrodynamic equations those solutions, which are random functions and satisfy to a similarity principle. Such functions in general case can be presented as

$$\bar{\mathbf{G}}(t, \eta, r, h, h_t, h_x, h_y) = \lim_{\delta V \to dV,} \frac{1}{\delta V}\int_{\delta V} \tilde{\mathbf{G}}(t, x, y, \tilde{\eta})dxdydz \tag{2}$$

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where $\bar{\mathbf{G}} = (\bar{\rho}, \bar{\mathbf{v}}, \bar{p}, \bar{T}, \bar{\phi}), \tilde{\mathbf{G}} = (\tilde{\rho}, \tilde{\mathbf{v}}, \tilde{p}, \tilde{T}, \tilde{\phi})$, dV_s is the volume of a statistical cell, $dV_s = \Delta V f_s(r, h, h_t, h_x, h_y) dr dh dh_t dh_x dh_y$, $\Delta V = \Delta x \Delta y dz$ is the volume of averaging, $f_s = f_s(r, h, h_t, h_x, h_y)$ is the multiple density of a probability distribution function, r, h, h_t, h_x, h_y are the random parameters characterizing the roughness geometry, $\tilde{\eta} = z/h(x,y,t), \eta = z/h$, δV is an arbitrary volume put in ΔV, and containing dV_s, as a whole. The surface $z = h(x,y,t)$ is the dynamical roughness. In a laminar flow this surface corresponds to the equation

$$h_t + \tilde{u} h_x + \tilde{v} h_y - \tilde{w} = 0$$

In a turbulent flow this surface can be described by the random continuous parameters h, h_t, h_x, h_y, characterizing the height, the transference speed, and the inclination of a surface elements. The subregion of the flow dV_s in the general case is a multiply connected domain and its separate parts in the form of dust are distributed in the volume ΔV. Statistical moment of the first order (mean value) of a random function $\bar{\mathbf{G}}$ is given by

$$< \bar{\mathbf{G}} > (t, z) = \int \bar{\mathbf{G}}(t, z/h, r, h, h_t, h_x, h_y) f_s dr dh dh_t dh_x dh_y$$

Note, that the real flow parameters can be written as a sum of the random functions $\bar{\mathbf{G}}(t, z/h, r, h, h_t, h_x, h_y)$ and indeterminate values $\tilde{\mathbf{G}}'(t, x, y, z)$ which named "turbulent noise": $\tilde{\mathbf{G}}(t, x, y, \tilde{\eta}) = \bar{\mathbf{G}}(t, z/h, r, h, h_t, h_x, h_y) + \tilde{\mathbf{G}}'(t, x, y, z)$. Therefore, the accuracy of this theory is determined by the value of a turbulent noise, which here will not be taken into account. But the intensity of a turbulent noise here is much less, than in the usual theories of a turbulence, where this value is determined by a difference between actual and mean flow parameters: $\mathbf{v}' = \tilde{\mathbf{v}} - < \mathbf{v} >$. A measure of their relation is the ratio of the volume of a statistical cell dV_s to the volume of the whole area ΔV. Thus, the given theory is exact in relation to the usual theories of a turbulence, which are based on the empirical formulas as for example the Prandtl theory or $k - \varepsilon$ model. It is clear, that the mean flow parameters are obtained by average of actual flow parameters in a subregion $\Delta V = \Delta x \Delta y dz$, but is not direct, and with an intermediate step, on which random functions $\bar{\mathbf{G}}(t, z/h, r, h, h_t, h_x, h_y)$ should be obtained. It is similar to the Bolzman equation which was deduced from equations of molecular dynamics as an intermediate step between micro- and macro-dynamics or hydrodynamics.

3 Turbulent flow model

The equations for the random functions $\bar{\mathbf{G}}$ can be derived from the equations (1) written in the curvilinear coordinate system $(t, x, y, \eta(t, x, y))$. In detail

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how the hydrodynamic equations can be presented in arbitrary curvilinear coordinate system explained by Pulliam & Steger [13]. Than we can separate out the solutions satisfying the similarity principle (2). Additionally we use the following hypotheses:

$$\lim_{\delta V \to dV_s} \frac{1}{\delta V} \int_{\delta V} \tilde{v}_i(x,y,\tilde{\eta},t)\tilde{v}_k(x,y,\tilde{\eta},t)dxdydz = \bar{v}_i\bar{v}_k + \theta\delta_{ik} \tag{3}$$

$$\lim_{\delta V \to dV_s} \frac{1}{\delta V} \int_{\delta V} \tilde{\mathbf{v}}(x,y,\tilde{\eta},t)\tilde{T}(x,y,\tilde{\eta},t)dxdydz = \bar{\mathbf{v}}\bar{T}$$

$$\lim_{\delta V \to dV_s} \frac{1}{\delta V} \int_{\delta V} \tilde{\mathbf{v}}(x,y,\tilde{\eta},t)\tilde{\phi}(x,y,\tilde{\eta},t)dxdydz = \bar{\mathbf{v}}\bar{\phi}$$

for $i, k = 1,2,3$, where $3\theta/2$ is the turbulent kinetic energy in a small volume dV_s, δ_{ik} is Kronecker symbol, $\delta_{ik} = 0$, when $i \neq k$, $\delta_{ik} = 1$, when $i = k$.
With this assumptions the model of the turbulent air flow with random parameters can be written as follows:

$$\frac{\partial \bar{w}}{\partial \eta} - \eta \frac{\partial \bar{\Phi}}{\partial \eta} = 0 \tag{4}$$

$$\frac{\partial \bar{\mathbf{v}}}{\partial t} + \frac{W}{h}\frac{\partial \bar{\mathbf{v}}}{\partial \eta} + \frac{\mathbf{N}}{\bar{\rho}h}\frac{\partial \bar{P}}{\partial \eta} = \frac{\nu}{h^2}\frac{\partial}{\partial \eta}(1+n^2\eta^2)\frac{\partial \bar{\mathbf{v}}}{\partial \eta} - \frac{\nu n^2 \eta}{h^2}\frac{\partial \bar{\mathbf{v}}}{\partial \eta} +$$

$$+\frac{\nu \mathbf{N}}{h^2}\frac{\partial \bar{\Phi}}{\partial \eta} + \frac{\mathbf{g}}{\bar{\rho}}(\bar{\rho} - \rho_e)$$

$$\frac{\partial \bar{T}}{\partial t} + \frac{W}{h}\frac{\partial \bar{T}}{\partial \eta} = \frac{\nu}{\mathbf{Pr}h^2}\frac{\partial}{\partial \eta}(1+n^2\eta^2)\frac{\partial \bar{T}}{\partial \eta} - \frac{\nu n^2 \eta}{\mathbf{Pr}h^2}\frac{\partial \bar{T}}{\partial \eta}$$

$$\frac{\partial \bar{\phi}}{\partial t} + \frac{W}{h}\frac{\partial \bar{\phi}}{\partial \eta} = \frac{D}{h^2}\frac{\partial}{\partial \eta}(1+n^2\eta^2)\frac{\partial \bar{\phi}}{\partial \eta} - \frac{D n^2 \eta}{h^2}\frac{\partial \bar{\phi}}{\partial \eta}$$

Where $W = \bar{w} - \eta\bar{\Phi}$, $\bar{\Phi} = h_t + h_x\bar{u} + h_y\bar{v}$; $\bar{P} = \bar{p} + \theta$; $n = \sqrt{h_x^2 + h_y^2}$; $\mathbf{N} = (-\eta h_x, -\eta h_y, 1)$; $W_p = W - \tau_p \mathbf{g}$.
Due to a symmetry of a system (4) it is possible to receive the expression for an integral of pressure:

$$\frac{\mathbf{N}^2}{\bar{\rho}h}\frac{\partial \bar{P}}{\partial \eta} = \frac{2\nu \mathbf{N}^2}{h^2}\frac{\partial \Phi}{\partial \eta} - \frac{\mathbf{g}}{\bar{\rho}}(\bar{\rho} - \rho_e) - \frac{\partial W}{\partial t} \tag{5}$$

where $\mathbf{N}^2 = 1 + n^2\eta^2$.
The hydrodynamic part of the system (4) in a case of the steady flow was considered in [11]. Analytical expressions of parameters in a turbulent steady flow over a rough surface were obtained. Was established, that the momentum, heat and mass flux generated by turbulent motion with vertical velocity W is relatively small and it can be neglected in a case of steady flows. In such case the system (9) is resulted in a linear type and its analysis is much simplified. For the turbulent diffusion problem at large scales the similar simplifications are possible.

4 Model of turbulent air flow at large scales

We define the new variables: $\Psi = h_y\bar{u} - h_x\bar{v}$, $\zeta = \lambda\,\mathrm{Arsh}(z/\lambda)$, where $\mathrm{Arsh}(x) = \ln(x + \sqrt{1+x^2})$, $\lambda = h/n$. It was established in [3] that $\Psi \sim \zeta, \Phi \sim \eta^{-1}$ at $\eta \gg 1$. Thus Ψ is the main part of the solution at large scales. Let $\Psi = \Psi(t,\zeta)$, $\phi = \phi(t,\zeta)$, then the system of equations (4) is transformed into following form

$$\frac{\partial\Psi}{\partial t} + \frac{W}{\sqrt{1+z^2/\lambda^2}}\frac{\partial\Psi}{\partial\zeta} = \nu\frac{\partial^2\Psi}{\partial\zeta^2} \tag{6}$$

$$\frac{\partial\bar{T}}{\partial t} + \frac{W}{\sqrt{1+z^2/\lambda^2}}\frac{\partial\bar{T}}{\partial\zeta} = \frac{\nu}{\mathbf{Pr}}\frac{\partial^2\bar{T}}{\partial\zeta^2}, \quad \frac{\partial\bar{\phi}}{\partial t} + \frac{W}{\sqrt{1+z^2/\lambda^2}}\frac{\partial\bar{\phi}}{\partial\zeta} = D\frac{\partial^2\bar{\phi}}{\partial\zeta^2}$$

If the value W is limited the terms containing W in the system of equations (6) decreases rather quickly as z^{-1} at $z \gg \lambda$. On other hand at $\eta \sim 1$ we have $W(t,0) = W(t,1) = 0$, thus, for non-stationary flows, it is possible to neglect influence of vertical mass flux by velocity fluctuations generated. In this approximation the equations of turbulent diffusion at large scales looks like a usually diffusion equation:

$$\frac{\partial\Psi}{\partial t} = \nu\frac{\partial^2\Psi}{\partial\zeta^2}, \quad \frac{\partial\bar{T}}{\partial t} = \frac{\nu}{\mathbf{Pr}}\frac{\partial^2\bar{T}}{\partial\zeta^2}, \quad \frac{\partial\bar{\phi}}{\partial t} = D\frac{\partial^2\bar{\phi}}{\partial\zeta^2} \tag{7}$$

In a general case we can include in the equations (7) a horizontal momentum, heat and mass flux. Put $\gamma = \arctan(h_y/h_x)$ and let us averaged the equations (7) over γ. Used Ψ we can compute the horizontal velocity by the following way:

$$U_x = \frac{1}{n^2}\int_0^{2\pi} h_y\Psi f_s d\gamma, \quad U_y = -\frac{1}{n^2}\int_0^{2\pi} h_x\Psi f_s d\gamma$$

where $\mathbf{U} = (U_x, U_y)$ is the horizontal flow velocity vector. Taken into account a horizontal pressure gradient and the Coriolis forces we can written the model of turbulent air flow as follows

$$(\nabla.\mathbf{U}) = 0 \tag{8}$$

$$\frac{\partial\mathbf{U}}{\partial t} + (\mathbf{U}.\nabla)\mathbf{U} + 2[\mathbf{\Omega}.\mathbf{U}] + \frac{\nabla_0 P}{\bar{\rho}} = \nu\frac{\partial^2\mathbf{U}}{\partial\zeta^2}$$

$$\frac{\partial\bar{T}}{\partial t} + (\mathbf{U}.\nabla)\bar{T} = \frac{\nu}{\mathbf{Pr}}\frac{\partial^2\bar{T}}{\partial\zeta^2}, \quad \frac{\partial\bar{\phi}}{\partial t} + (\mathbf{U}.\nabla)\bar{\phi} = D\frac{\partial^2\bar{\phi}}{\partial\zeta^2}$$

where $\mathbf{\Omega}$ is the Earth rotation velocity vector, $\nabla_0 P$ is the horizontal pressure gradient.

The function ζ has a following property: $\lim_{\lambda\to\infty}\zeta(\lambda, z) = z$. Therefore, when parameter λ increases ad infinitum model (7) (and models (4,8) too) is resulted in usual classical model, the finite values of parameter λ correspond to various realizations of turbulent flow. The typical value λ calculated from the mean velocity profile for an isothermal turbulent flow over a smooth surface is $\lambda \sim \lambda_0 = 7.71\nu/u_*$, where u_* is the dynamical velocity [11]. For a rough surface in a case of large-scale roughness the additive dynamical roughness model is applied. In this model $\lambda \sim \lambda_r = r_*\lambda_B$, where r_* it the typical roughness height, λ_B is the pitch to height ratio or Bettermann's roughness density parameter [11,14]. For a stable or unstable stratification λ depends on the Monin-Obukhov scale (Businger *et al.* [4]) $\lambda \sim \lambda_T = u_*^3\rho_e c_p T_e/\kappa g \mid H_0 \mid$, where c_p it the specific heat at constant pressure of the gas, κ is Von Karman's constant, H_0 is the heat flux from the ground to the air. The main parameter of this theory λ is not evidently dependent on the choice of the scale h. Hence, the model (8) is true for the atmosphere turbulent flow above natural large scale roughness.

5 Air pollution above an industrial region

We shall consider the problem about turbulent diffusion of an impurity at specific emission. As well known the emission for a big industrial region is a periodical function depend on a daily, weekly and yearly human activity. For simplification let us conjecture that the emission has one maximum during day and one minimum during night, thus

$$q(t) = q_0 + q_1 \cos\omega(t - t_m)$$

where $q_0 = .5(q_{max} + q_{min})$, $q_1 = .5(q_{max} - q_{min})$, q_{max}, q_{min} is the maximal and minimal value of emission respectively, $\omega = 2\pi/t_d, t_d = 24\text{h}$, t_m is the maximal emission time. The boundary conditions for the last equation (8) are follow from the divergence form of the last equation (4) and given by

$$z = r: \quad -w_0\bar{\phi} - D(1 + r^2/\lambda^2)\frac{\partial\bar{\phi}}{\partial z} = q(t); \quad t_d^{-1}\int_0^{t_d}\bar{\phi}dt = c_0 \tag{9}$$

where w_0 is the deposition velocity. The general solution of the problem for a periodical function $q(t)$ is given by

$$\bar{\phi} = c_0 - \frac{q_0 + w_0c_0}{D\sqrt{1 + r^2/\lambda^2}}(\zeta - \zeta_r) + \frac{q_1 e^{-k(\zeta-\zeta_r)}\cos\psi(t,\zeta)}{\sqrt{(w_0 - kD_r)^2 + k^2D_r^2}} \tag{10}$$

$$\psi = \omega(t - t_m) - k(\zeta - \zeta_r) - \alpha$$

where $\zeta_r = \lambda\,\mathrm{Arsh}(r/\lambda)$, $k = \sqrt{\omega/2D}$, $D_r = D\sqrt{1 + r^2/\lambda^2}$, α is a positive root of the equation $\tan\alpha = kD_r/(kD_r - w_0)$. Using the expression (10) we

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can calculate the mean value

$$\phi(t,z) = \int_0^\infty dr \int_0^\infty \bar{\phi}(t,z,r,\lambda) f_s(r,\lambda) d\lambda \tag{11}$$

In general case the mean amplitude ϕ can be presented as a linear combination of amplitudes, dependent on one scale:

$$\phi(t,z) = c_1\bar{\phi}_1(t,z,r_*,\lambda_0) + c_2\bar{\phi}_2(t,z,r_*,\lambda_r) + c_3\bar{\phi}_3(t,z,r_*,\lambda_T) \tag{12}$$

where c_i are the weight factors, $\sum_i c_i = 1$, $\bar{\phi}_i$ are the "typical" amplitudes given by (10) at $r = r_*$, and at $\lambda = \lambda_0, \lambda_r, \lambda_T$ respectively.

6 Results and discussion

The expressions (10-12) were applied to the aerosol turbulent transport problem. In this case was established, that among three addends in the right part (12) the greatest contribution is given by amplitude with the least value of parameter λ. It is stipulated by large value of the Schmidt number for aerosols, $\mathbf{Sc}_p = \nu/D \gg 1$, were the particles diffusivity due to thermal fluctuations is $D = kT/3\pi\mu d_p$, k is the Boltzmann constant, d_p is the particle aerodynamical diameter. For usual atmospheric conditions the least value has a main turbulent length scale $\lambda \sim \lambda_0 \approx 7.71\nu/u_*$. Two other turbulent length scales r_*, λ_T are much more than λ_0. Thus, let $\lambda = \lambda_0$. In this case it is possible to simplify the equation (11) using the asymptotic formula $k(\zeta - \zeta_r) \approx k\lambda \ln(z/r)$ at $z \geq r_* \gg \lambda_0$. Then the mean particle numerical density can be written as

$$N_p = N_0 - \frac{q_0 + w_0 N_0}{Dr_*}\lambda_0^2 \ln \bar{z} + N_1 \bar{z}^{-\delta} \cos\psi \tag{13}$$

$$\psi = \omega(t - t_m) - \delta \ln \bar{z} - \arctan[kr_* D/(kr_* D - w_0\lambda_0)]$$

where N_0 = is the aerosol particle numerical density averaged over t_d-period, $\bar{z} = z/r_*$, $\delta = k\lambda_0$, $N_1 = q_1\lambda_0/\sqrt{(w_0\lambda_0 - kr_* D)^2 + (kr_* D)^2}$.
We can calculate the effective particle vertical velocity over ground w_0 from the assumption that $N_p = 0$ at $z \geq H_e$, where the height H_e is about 10-100 km for atmospheric aerosols. Neglected by the last therm in the right part of (13) at $z = H_e \gg r_*$ we have:

$$w_0 = w_D - q_0/N_0 \tag{14}$$

where $w_D = Dr_*/\lambda_0^2 \ln(H_e/r_*)$ is the deposition velocity in a zero emission case. For spherical particles at the normal conditions and at $H_e/r_* = 2.10^5$ we have $w_D \sim .05 u_*^2 r_*/d_p$, where $w_D, u_*[m/s], r_*[m], d_p[\mu]$. It is in agreement

with experimental results (Garland [15], Sehemel [16]).
Let $N_1 = \epsilon N_0$, where the nondimensional parameter ϵ is given by

$$\epsilon = \epsilon_0 \frac{q_0}{N_0}[(w_D - q_0/N_0 - kr_* D/\lambda_0)^2 + (kr_* D/\lambda_0)^2]^{-\frac{1}{2}} \tag{15}$$

$\epsilon_0 = q_1/q_0$ is the maximal value of parameter ϵ. Then the equation (13) can be written as

$$N_p/N_0 = -\ln(z/H_e)/\ln(H_e/r_*) + \epsilon \bar{z}^{-\delta} \cos\psi \tag{16}$$

At $z = r_*$ we have from the equation (16) $N_p/N_0 = 1 - \epsilon\cos\psi$, therefore $\epsilon = (N_{max} - N_{min})/(N_{max} + N_{min})$, where N_{max}, N_{min} is the maximal and minimal value of the mean particle numerical density over ground respectively. In considered case $w_D \ll kr_* D/\lambda_0$. Thus we have only one scale $kr_* D/\lambda_0$ for the emission intensity. Neglected by the value w_D we have

$$N_0 = \frac{q_0 \lambda_0 F}{kr_* D} \approx \frac{7.71 q_0 \nu F}{u_* r_* \sqrt{\omega D}}; \quad F = \frac{(\epsilon_0/\epsilon)^2 - 1}{1 + \sqrt{2(\epsilon_0/\epsilon)^2 - 1}} \tag{17}$$

In a case of a traffic emission q_0 is direct proportionality to the traffic volume. Therefore $N_0 \sim$ traffic volume/wind speed. It is in agreement with experimental data (Jenkins *et al.* [17]). Note, that the meteorological parameters u_*, T in the equations (13-17) may not to be constants, but we can use statistical values such as weekly or monthly meteorological parameters. On Figure 2,a the normalized emission $q(t)/q_0$ (marked line) and the normalized partiles numerical density $N_p(t, r_*)/N_0$ (solid line) depending on time at $z = r_*$, $\epsilon = .6, \epsilon_0 = .8$ are shown. On Figure 2,b the vertical profiles of normalized particles numerical density $N_p(t_i, z)/N_0$ at $t_i = 3, 6, ..., 24$h and at $H_e/r_* = 2.10^5$, $r_* = 1.1$m, $u_* = 0.1$m/s, $d_p = 2.5\mu$, $T = 278$K are shown. From the Figure 2,b we can see, that the aerosol numerical density varies periodically in the time and damps with height increasing. The damping parameter is $\delta = k\lambda_0 \approx 1.8\ 10^{-4}\sqrt{\mathbf{Sc}_p}/u_*$. Thus, the damping scale $z_0 \sim r_* \exp(1/\delta) \approx r_* \exp(5556 u_*/\sqrt{\mathbf{Sc}_p})$ is a very sensitive to the dynamical velocity and particles size variations (in a case of spherical particles $z_0 \sim r_* \exp(0.59 u_*/\sqrt{d_p})$ where u_*[cm/s], $d_p[\mu]$).
Note, that the equations (10-16) can be applied to the turbulent heat transfer by replacement $D \to \nu/\mathbf{Pr}$, $\phi \to T$. In general case the terms $\bar{\phi}_2, \bar{\phi}_3$ in the equation (12) are important. Thus, the turbulent diffusion parameters of an impurity depend on the roughness scale r_*, on the roughness density parameters such as the pitch to height ratio λ_B, and on the meteorological parameters: dynamical velocity u_*, temperature T, and Monin-Obukhov scale λ_T.

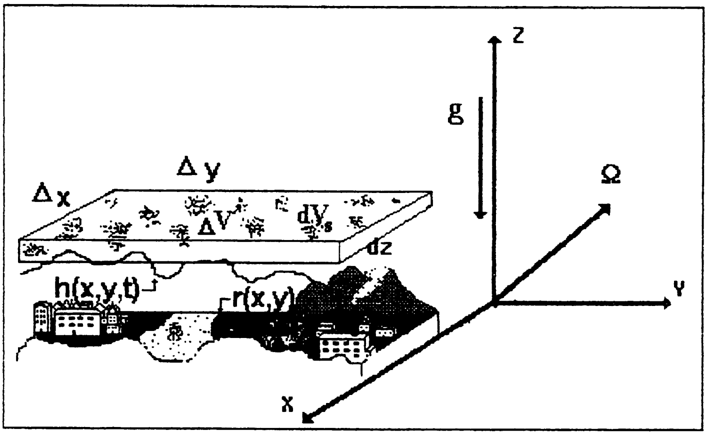

Figure 1: The volume of averaging $\Delta V = \Delta x \Delta y dz$ and multiply connected domain dV_s (black points inside ΔV)

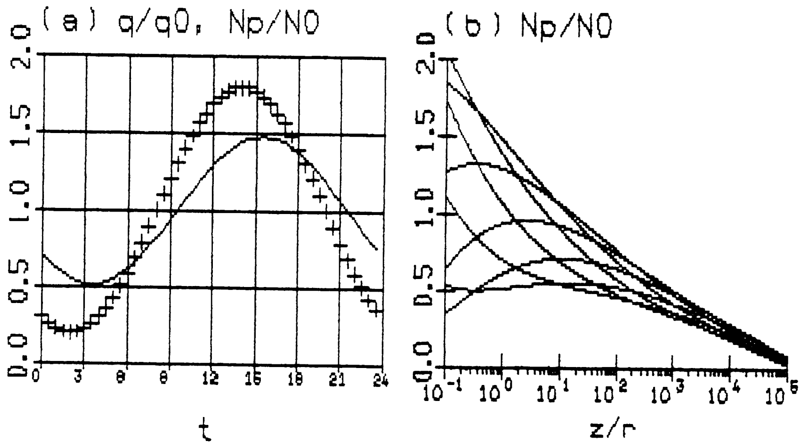

Figure 2: (a) - temporal evaluation of the normalized emission $q(t)/q_0$ (marked line) and the normalized partiles numerical density $N_p(t, r_*)/N_0$ (solid line) at $z = r_*$; (b) - the vertical profiles of the normalized particles numerical density $N_p(t_i, z)/N_0$ at $t_i = 3, 6, ..., 24$h.

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