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Similarity theory for turbulent flow over natural rough surface in pressure and temperature gradients

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Abstract

The similarity theory and the model of a turbulent flow over a natural large scale roughness in pressure and temperature gradients are presented in the paper. The general analytical results such as the logarithmic mean velocity and temperature profiles for a turbulent flow over smooth and rough surfaces, the temperature and velocity profiles for a turbulent natural convection over a large scale roughness and the velocity defect law for the Ekman layer over a rough surface have been obtained.

1 Introduction

The similarity theory was widely applied to solve the turbulent flow problems including atmospheric problems (Prandtl [1], Monin & Obukhov [2], Zilitinkevich & Monin [3]). The usual application of the similarity theory concerns only the averaged parameters but the analysis of the random processes does not mention. The more general theoretical results can be received if to apply the similarity theory to the analysis of the random processes such as fields of velocity, pressure and temperature in a turbulent flow. The similarity theory of the random processes in the turbulent boundary layer has been developed, and the closed equation system without any indeterminate variables, like Reynolds stress, for an isothermal flow has been obtained by Trunev [4].

The similarity theory and the model of a turbulent flow over a natural large scale roughness in pressure and temperature gradients are developed in the paper. The buoyancy forces, the planetary rotation and the random parameters characterizing the roughness density and geometry are taken into account in the model.

2 General equations

The general model of the turbulent natural convection in the atmosphere is the following (see Landau & Lifshitz [5])

$$\nabla.\tilde{\mathbf{v}} = 0 \tag{1}$$

$$\frac{\partial \tilde{\mathbf{v}}}{\partial t} + (\tilde{\mathbf{v}}.\nabla)\tilde{\mathbf{v}} + 2[\mathbf{\Omega}\tilde{\mathbf{v}}] + \frac{\nabla \tilde{p}}{\rho} = \nu\nabla^2\tilde{\mathbf{v}} + \frac{\mathbf{g_e}}{\rho}(\rho - \rho_e)$$

$$\frac{\partial \tilde{T}}{\partial t} + (\tilde{\mathbf{v}}.\nabla)\tilde{T} = \frac{\nu}{Pr}\nabla^2\tilde{T}$$

where $\tilde{\mathbf{v}}$ is the flow velocity vector, $\mathbf{\Omega}$ is the Earth rotation velocity vector, $\tilde{p}$ is the pressure except hydrostatic pressure, ρ is the density, $\mathbf{g_e}$ is the gravity acceleration vector, $\tilde{T}$ is the temperature, ν is the kinematic viscosity, Pr is Prandtl number. The hydrostatic equation can be written as

$$\nabla p_e = \mathbf{g_e}\rho_e(p_e, T_e) \tag{2}$$

The standard Boussinesq approximation for the density fluctuations is

$$\rho - \rho_e = -\rho_e\beta_e(\tilde{T} - T_e) \tag{3}$$

where $\beta_e = -\frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_p$ (for ideal gas $\beta_e = 1/T$).
The coordinate system should be determined in such a way that the Z-axis is directed opposite to the vector of the gravity acceleration, the X-axis is directed from the west to the east and Y-axis is directed to the north, then $\mathbf{\Omega} = \Omega(0, \cos\varphi_e, \sin\varphi_e)$, where φ_e is the latitude. The relief of a ground surface is described by a function $z = r(x, y)$. Boundary conditions are set as follows:

$$\text{when } z = r(x,y): \quad \tilde{\mathbf{v}} = 0, \quad \tilde{T} = T_g \tag{4}$$

$$\text{when } z = H_e: \quad \tilde{\mathbf{v}} = U_0(\cos\gamma_e, \sin\gamma_e, 0), \quad \tilde{T} = T_{e0}$$

where T_g is the surface temperature, H_e is the boundary layer height, U_0 is the wind velocity at the height H_e , γ_e is the angle between the wind direction and X-axis, T_{e0} is the temperature at the height H_e.

3 Similarity theory

To select the solutions satisfying the similarity principle, the flow velocity vector, the pressure and the temperature should be written as

$$\tilde{\mathbf{v}} = \tilde{\mathbf{v}}(x, y, z/h(x,y,t), t) \tag{5}$$

$$\tilde{p} = \tilde{p}(x, y, z/h(x,y,t), t), \quad \tilde{T} = \tilde{T}(x, y, z/h(x,y,t), t)$$

where the surface $z = h(x, y, t)$ is the dynamical roughness (see Trunev [4]). In a laminar flow this surface corresponds to the equation

$$h_t + \tilde{u}h_x + \tilde{v}h_y - \tilde{w} = 0$$

In a turbulent flow the above surface can be described by random continuous parameters h, h_t, h_x, h_y, characterizing the height, the transference speed, and the inclination of a surface elements. If we consider fully rough surfaces, we can use the additive dynamical roughness model:

$$h(x, y, t) = r(x, y) + \tilde{h}(x, y, t) \tag{6}$$

where $\tilde{h}(x, y, t)$ is the height of the viscous sublayer over the surface $z = r(x, y)$. Let us express from (5) the random processes which satisfies the similarity principle. For this purpose, the area of the sufficient value $\Delta S = \Delta x \Delta y$ should be determined at the XY plane. Let $\Delta V = \Delta x \Delta y dz$ is the volume of averaging, dV_s is the volume of a statistical cell,

$$dV_s = \Delta V f_s(r, h, h_t, h_x, h_y) dr dh dh_t dh_x dh_y,$$

f_s is the multiple density of a probability distribution function. The subregion of the flow dV_s in the general case is a multiply connected domain and its separate parts in the form of dust are distributed in the volume ΔV. The random process to be found, satisfying the similarity principle is

$$\bar{\mathbf{v}}(\eta, t, r, h, h_t, h_x, h_y) = \lim_{\delta V \to dV_s} \frac{1}{\delta V} \int_{\delta V} \tilde{\mathbf{v}}(x, y, \tilde{\eta}, t) dx dy dz \tag{7}$$

$$\bar{p}(\eta, t, r, h, h_t, h_x, h_y) = \lim_{\delta V \to dV_s} \frac{1}{\delta V} \int_{\delta V} \tilde{p}(x, y, \tilde{\eta}, t) dx dy dz$$

$$\bar{T}(\eta, t, r, h, h_t, h_x, h_y) = \lim_{\delta V \to dV_s} \frac{1}{\delta V} \int_{\delta V} \tilde{T}(x, y, \tilde{\eta}, t) dx dy dz$$

where $\tilde{\eta} = z/h(x, y, t), \eta = z/h$, δV is an arbitrary volume put in ΔV, and containing dV_s, as a whole. Statistical moment of the order m of a random function $\bar{v}_i$ is to be derived from the expression:

$$U_i^m(z) = \int v_i^m(z/h, r, h, h_t, h_x, h_y) f_s dr dh dh_t dh_x dh_y \qquad i = 1, 2, 3$$

We have considered a case when one surface $z = h(x, y, t)$ is enough for similarity flow parameters modeling. However it is well known, that there is the thermal boundary layer together with the dynamic boundary layer (see Cebeci & Bradshaw [6]). Therefore, it is necessary to consider the temperature similarity concerning some surface $z = h_T(x, y, t)$ in the heat transfer problems . The similarity theory for this purpose can be applied by analogy with (5-7).

4 Turbulent flow model

In accordance with the similarity theory developed above one must substitute the expressions (5) for $\tilde{\mathbf{v}}, \tilde{p}$ and $\tilde{T}$ in (1) and separate out the solutions satisfying the similarity principle (6). In this case it is possible to save the structure of the classical Navier-Stokes hydrodynamic equations using the following hypotheses:

$$\lim_{\delta V \to dV_s} \frac{1}{\delta V} \int_{\delta V} \tilde{v}_i(x,y,\tilde{\eta},t)\tilde{v}_k(x,y,\tilde{\eta},t)dxdydz = \bar{v}_i\bar{v}_k + \theta\delta_{ik} \tag{7}$$

$$\lim_{\delta V \to dV_s} \frac{1}{\delta V} \int_{\delta V} \tilde{\mathbf{v}}(x,y,\tilde{\eta},t)\tilde{T}(x,y,\tilde{\eta},t)dxdydz = \bar{\mathbf{v}}\bar{T}$$

where $3\theta/2$ is the turbulent kinetic energy in a small volume dV_s, δ_{ik} is Kronecker symbol, $\delta_{ik} = 0$, when $i \neq k$, $\delta_{ik} = 1$, when $i = k$. The model of the turbulent flow with random parameters can be written as follows:

$$\frac{\partial \bar{w}}{\partial \eta} = \eta \frac{\partial \bar{\Phi}}{\partial \eta} \tag{8}$$

$$h\frac{\partial \bar{u}}{\partial t} + W\frac{\partial \bar{u}}{\partial \eta} - \frac{\eta h_x}{\rho}\frac{\partial \bar{p}}{\partial \eta} + \frac{h}{\rho}\frac{\partial p}{\partial x} = \frac{\mu}{\rho h}\frac{\partial}{\partial \eta}(1+n^2\eta^2)\frac{\partial \bar{u}}{\partial \eta}$$
$$-\frac{\mu\eta}{\rho h}\left(h_x\frac{\partial \bar{\Phi}}{\partial \eta} + n^2\frac{\partial \bar{u}}{\partial \eta}\right) + hK_x$$

$$h\frac{\partial \bar{v}}{\partial t} + W\frac{\partial \bar{v}}{\partial \eta} - \frac{\eta h_y}{\rho}\frac{\partial \bar{p}}{\partial \eta} + \frac{h}{\rho}\frac{\partial p}{\partial y} = \frac{\mu}{\rho h}\frac{\partial}{\partial \eta}(1+n^2\eta^2)\frac{\partial \bar{v}}{\partial \eta}$$
$$-\frac{\mu\eta}{\rho h}\left(h_y\frac{\partial \bar{\Phi}}{\partial \eta} + n^2\frac{\partial \bar{v}}{\partial \eta}\right) + hK_y$$

$$h\frac{\partial \bar{w}}{\partial t} + W\frac{\partial \bar{w}}{\partial \eta} + \frac{1}{\rho}\frac{\partial \bar{p}}{\partial \eta} = \frac{\mu}{\rho h}\frac{\partial}{\partial \eta}(1+n^2\eta^2)\frac{\partial \bar{w}}{\partial \eta} + \frac{\mu}{\rho h}\left(\frac{\partial \bar{\Phi}}{\partial \eta} - \eta n^2\frac{\partial \bar{w}}{\partial \eta}\right) + hB_z + hK_z$$

$$h\frac{\partial \bar{T}}{\partial t} + W\frac{\partial \bar{T}}{\partial \eta} = \frac{\nu}{Prh}\frac{\partial}{\partial \eta}(1+n^2\eta^2)\frac{\partial \bar{T}}{\partial \eta} - \frac{\nu\eta n^2}{Prh}\frac{\partial \bar{T}}{\partial \eta}$$

Where $W = \bar{w} - \eta\bar{\Phi}$, $\bar{\Phi} = h_t + h_x\bar{u} + h_y\bar{v}$; $\partial p/\partial x = p_{ox}$, $\partial p/\partial y = p_{oy}$, p_{oi} is the horizontal pressure gradient which is considered to be a constant in the entire flow volume; $n = \sqrt{h_x^2 + h_y^2}$; $\mathbf{K} = -2[\mathbf{\Omega}.\mathbf{v}]$ are the Coriolis forces; $\mathbf{B} = \frac{\mathbf{g}e}{\rho}(\rho - \rho_e)$ are the buoyancy forces.

Note that the model of the turbulent boundary layer over the rough surface (8) does not contain indefinite values like Reynolds stress owing to hypothesis (7), and that in this model the Reynolds stress is given by

$$\tau_{ik}(z) = \int \rho(\bar{u}_i(\eta)\bar{u}_k(\eta) + \theta\delta_{ik})f_s dr dh dh_t dh_x dh_y - \rho U_i(z)U_k(z)$$

If we put $\frac{\partial \bar{u}}{\partial t} = \frac{\partial \bar{v}}{\partial t} = \frac{\partial \bar{w}}{\partial t} = \frac{\partial \bar{T}}{\partial t} = 0$ in the system (8), we shall receive the system of equations describing the turbulent steady flow:

$$\frac{\partial \bar{w}}{\partial \eta} = \eta \frac{\partial \bar{\Phi}}{\partial \eta} \tag{9}$$

$$\frac{\partial \bar{p}}{\partial \eta} = \frac{2\nu\rho}{h}\frac{\partial \bar{\Phi}}{\partial \eta} + \frac{h(h_x p_{0x} + h_y p_{0y})\eta}{1+n^2\eta^2} + \frac{\rho h(B_z + K_z)}{1+n^2\eta^2} - \frac{\rho h \eta(h_x K_x + h_y K_y)}{1+n^2\eta^2}$$

$$W\frac{\partial \bar{u}}{\partial \eta} - \frac{\eta h_x}{\rho}\frac{\partial \bar{p}}{\partial \eta} + \frac{h}{\rho}\frac{\partial p}{\partial x} = \frac{\nu}{h}\frac{\partial}{\partial \eta}(1+n^2\eta^2)\frac{\partial \bar{u}}{\partial \eta} - \frac{\nu\eta}{h}\left(h_x\frac{\partial \bar{\Phi}}{\partial \eta} + n^2\frac{\partial \bar{u}}{\partial \eta}\right) + hK_x$$

$$W\frac{\partial \bar{v}}{\partial \eta} - \frac{\eta h_y}{\rho}\frac{\partial \bar{p}}{\partial \eta} + \frac{h}{\rho}\frac{\partial p}{\partial y} = \frac{\nu}{h}\frac{\partial}{\partial \eta}(1+n^2\eta^2)\frac{\partial \bar{v}}{\partial \eta} - \frac{\nu\eta}{h}\left(h_y\frac{\partial \bar{\Phi}}{\partial \eta} + n^2\frac{\partial \bar{v}}{\partial \eta}\right) + hK_y$$

$$W\frac{\partial \bar{T}}{\partial \eta} = \frac{\nu}{Prh}\frac{\partial}{\partial \eta}(1+n^2\eta^2)\frac{\partial \bar{T}}{\partial \eta} - \frac{\nu\eta n^2}{Prh}\frac{\partial \bar{T}}{\partial \eta}$$

Where $B_z = g_e\beta_e(\bar{T} - T_e)$, $K_x = 2\Omega(\bar{v}\sin\varphi_e - \bar{w}\cos\varphi_e)$, $K_y = -2\Omega\bar{u}\sin\varphi_e$, $K_z = 2\Omega\bar{u}\cos\varphi_e$. The second equation (9) is the pressure integral obtained owing to the symmetry of the system (8). Let us determine dimensionless values: $\lambda^+ = hu_*/n\nu$, $\delta = h_t/nu_*$, $\sin\beta = h_y/n$, $\cos\beta = h_x/n$,

$$\phi = u^+\cos\beta + v^+\sin\beta + \delta, \quad \psi = u^+\sin\beta - v^+\cos\beta, \quad \chi = \xi\phi - w^+$$

where $\xi = z/\lambda$, $\lambda = h/n$, $u^+ = \bar{u}/u_*$, $v^+ = \bar{v}/u_*$, $w^+ = \bar{w}/u_*$, u_* is the dynamical velocity computed from the value of friction stress on a wall, $u_* = \sqrt{\tau_w/\rho}$. The following equations may be derived from (9):

$$\frac{d\chi}{d\xi} = \phi \tag{10}$$

$$(1+\xi^2)\frac{d^2\phi}{d\xi^2} + (\lambda^+\chi + 2\xi)\frac{d\phi}{d\xi} = \lambda^+\pi_1 - \lambda^+k_1 - \xi\pi_\xi$$

$$(1+\xi^2)\frac{d^2\psi}{d\xi^2} + (\lambda^+\chi + \xi)\frac{d\psi}{d\xi} = \lambda^+\pi_2 - \lambda^+k_2$$

$$(1+\xi^2)\frac{d^2T^+}{d\xi^2} + (Pr\lambda^+\chi + \xi)\frac{dT^+}{d\xi} = 0$$

$$\pi_\xi = \frac{\xi\pi_1 - \xi k_1 + \lambda^+\omega_0 k_z + \lambda^+ B_0(T^+ - T_e^+)}{(1+\xi^2)}$$

where $\pi_1 = \lambda^+(\pi_x \cos\beta + \pi_y \sin\beta)$, $k_1 = \lambda^+\omega_0(k_x \cos\beta + k_y \sin\beta)$

$$\pi_2 = \lambda^+(\pi_x \sin\beta - \pi_y \cos\beta), \quad k_2 = \lambda^+\omega_0(k_x \sin\beta - k_y \cos\beta)$$

$$k_x = v^+ \sin\varphi_e - w^+ \cos\varphi_e, \quad k_y = -u^+ \sin\varphi_e, \quad k_z = u^+ \cos\varphi_e$$

$\pi_x = p_{0x}\nu/\rho u_*^3$, $\pi_y = p_{0y}\nu/\rho u_*^3$, $\omega_0 = 2\Omega\nu/u_*^2$, $B_0 = g_e\beta_e T_*\nu/u_*^3$, $T^+ = \bar{T}/T_*$, $T_e^+ = T_e/T_*, T_* = q_g/\rho_{eg} c_p u_*$, c_p is the specific heat at constant pressure of the gas, ρ_{eg} is an average gas density over the ground, q_g is the heat flux from the ground to the air.

5 General results

The model (10) depend only on combinations of the random parameters $\lambda = h/n$, $\beta = \arctan(h_x/h_z)$, $\delta = h_t/nU_0$, the latter are not evidently dependent on the choice of the scale h . That is why this model is true for the turbulent flow over the surface which main roughness scale is comparable with a tree, a building, or a hill. Therefore it can be applied to solve the problem of the atmosphere turbulent flow over a natural large scale roughness .
The system of equations (10) was studied by usual methods and as a result it was established, that the limited solutions exist everywhere at $\chi < 0$, the solution diverges at $\chi = 0$ as a logarithmic function at $\xi \to \infty$. Thus, it is possible to put $\chi = 0$ in the equations (10) for the coordination of the solutions with known results of the turbulent flow theory. In this case the system of equations (10) transforms into a linear system. The general solution of the homogeneous differential equation system (10) (i.e. at $k_i = 0$, $\pi_i = 0$, $\chi = 0$) is given by:

$$\phi = \alpha_1 \arctan(\xi) + c_1 \quad , \psi = \alpha_2 \ln(\xi + \sqrt{1+\xi^2}) + c_2 \tag{11}$$

$$T^+ = \alpha_3 \ln(\xi + \sqrt{1+\xi^2}) + c_3$$

where α_i, c_i are some arbitrary constants. It is possible to construct the general solution of the linear equation system which follows from (10) at $\chi = 0$ on the basis of the solutions (11) .

5.1 Logarithmic profile over smooth surface

The predicted (theoretical) mean velocity and temperature profiles over a smooth surface are given by

$$U^+ = \frac{1}{\kappa}\ln z^+ + (\lambda^+ - \frac{1}{\kappa})\arctan\frac{z^+}{\lambda^+} + \frac{1}{\kappa}\ln(1 + \sqrt{1 + (\lambda^+/z^+)^2}) \tag{12}$$

$$t^+ = \frac{1}{\kappa_h}\ln z^+ + \frac{1}{\kappa_h}\ln(\kappa_h Pr) + \frac{1}{\kappa_h}\ln(1 + \sqrt{1 + (\kappa_h Pr z^+)^{-2}})$$

where $U^+ =< u^+ >; z^+ = zu_*/\nu$, $t^+ = (T_g - < T >)/T_*$, κ, κ_h are arbitrary constants. The standard expressions of the mean velocity and temperature logarithmic profiles over a smooth surface are given by (see Cebeci & Bradshaw [6])

$$U^+ = \frac{1}{\kappa}\ln z^+ + c_0 \ , \quad t^+ = \frac{1}{\kappa_h}\ln z^+ + c_h(Pr) \tag{13}$$

where $\kappa, \kappa_h, c_0, c_h(Pr)$ are the empirical values. The best correlation expressions (12) with the experimental correlated curves (13) were under conditions: $\lambda^+ = 7.71$ that corresponds to $\kappa = .41$, $c_0 = 5.$ or $\lambda^+ = 8.26$ that corresponds to $\kappa = .4$, $c_0 = 5.5$.

5.2 Mean velocity profile in pressure gradient

It results from (10) that

$$< u^+ >= \frac{1}{\kappa}\,\mathrm{Arsh}\,\frac{z^+}{\lambda^+} + (\lambda^+ - \frac{1}{\kappa})\arctan\frac{z^+}{\lambda^+}$$

$$+\frac{\lambda^+\pi_x}{2\kappa}\,\mathrm{Arsh}^2\,\frac{z^+}{\lambda^+} + (\lambda^+ - \frac{1}{\kappa})\frac{\lambda^+\pi_x}{2}\arctan^2\frac{z^+}{\lambda^+} \tag{14}$$

where $\mathrm{Arsh}(x) = \ln(x) + \ln(1 + \sqrt{1 + x^{-2}})$. The expressions (14) may be agreed with the velocity defect law in a pressure gradient [6].

5.3 Logarithmic profile over rough surface

The predicted (theoretical) mean velocity and temperature profiles over a rough surface are given by

$$< U^+ >_r =< U^+(z^+) - U^+(r^+) >_r \ , \quad < t^+ >_r =< t^+(z^+) - t^+(r^+) >_r \tag{15}$$

where $r^+ = ru_*/\nu$, and U^+, t^+ are given by the equation (12), the index r shows that a value is averaged over the random parameter r. Therefore, when $z^+ \gg \lambda^+$ we have

$$< U^+ >_r = \frac{1}{\kappa}\ln z^+ + c_0 - \Delta U^+ \ , < t^+ >_r = \frac{1}{\kappa_h}\ln z^+ - \frac{1}{\kappa_h}\ln d_r^+ + C_h$$

where $d_r^+ = d_r u_*/\nu$, d_r is the characteristic scale of roughness projections,

$$\Delta U^+ = c_0 + \frac{1}{\kappa}\ln d_r^+ + \frac{1}{\kappa} < \ln\frac{r}{d_r} >_r + (\lambda^+ - \frac{1}{\kappa})(< \arctan\frac{r^+}{\lambda^+} >_r - \frac{\pi}{2})$$

$$+\frac{1}{\kappa} < \ln(1 + \sqrt{1 + (\lambda^+/r^+)^2}) >_r - \ln 2$$

$$C_h = -\frac{1}{\kappa_h} < \ln\frac{r}{d_r} >_r - \frac{1}{\kappa_h} < \ln(1 + \sqrt{1 + (\kappa_h r^+ Pr)^{-2}}) >_r \tag{16}$$

The additive model for dynamical roughness (6) together the strong inequality $r \gg \tilde{h}$ is true almost everywhere for the natural large scale roughness. Thus, in this case $r^+/\lambda^+ \approx n$ and the mean flow velocity depends on characteristic value of an inclination of ledges, which well be estimated as $n \sim d_r/L_b = 1/\lambda_{rB}$ (L_b is spacing between roughness elements , λ_{rB} is the pitch to height ratio or Bettermann's roughness density parameter [7]), and on value $< \ln(r) > \approx \ln d_r - \Gamma_1 \ln \lambda_{rD} + \Gamma_2$ where $\lambda_{rD} = L_b/L_r$ is the pitch to width ratio or Dvorak's roughness density parameter [8] , L_r is the length of roughness element; $\Gamma_{1,2}$ are some constants (theoretical result is that $\Gamma_1 = 1$ for two-dimensional roughness and $\Gamma_1 = 2$ for three-dimensional roughness). By the appropriate choice of the parameters, the expressions (15) – (16) may be agreed with the experimental correlations of roughness density effect about which were reported by Dvorak [8] and Sigal & Danberg [9].

5.4 Turbulent natural convection over rough surface

The main equation of the turbulent natural convection may be derived from the system (10):

$$(1+\xi^2)\frac{d^2w^+}{d\xi^2} + (\xi - \frac{1}{\xi})\frac{dw^+}{d\xi} + \frac{\lambda^{+2}\xi^2 B_0(\Delta T^+ - t^+)}{(1+\xi^2)} = 0 \qquad (17)$$

where $\Delta T^+ = (T_g - T_e)/T_*$, $T_* = q_g/\rho_{eg}c_p w_*$, $w^+ = \bar{w}/w_*$, w_* is the convection velocity scale. t^+ value is determined from the equations :

$$t^+ = t_s^+(\xi) - t_s^+(\xi_r) \qquad (18)$$

$$t_s^+(\xi) = \frac{1}{\kappa_h}\ln\xi + \frac{1}{\kappa_h}\ln(\lambda^+\kappa_h Pr) + \frac{1}{\kappa_h}\ln(1+\sqrt{1+(\kappa_h Prz^+)^{-2}})$$

where $\lambda^+ = \lambda w_*/\nu$, $\xi_r = r/\lambda$. Hence $t_s^+(\xi_r) = 0$ for a smooth surface. Let $T_e = T_{e0}$ and $\xi^2 \gg 1$ in the equation (17) , then the general solution of the turbulent natural convection over a smooth surface is given by

$$w_s^+ = w_0^+ + \frac{1}{\kappa_c}\ln\xi - \frac{\lambda^{+2}B_0\Delta T_s^+}{2}\ln^2\xi + \frac{\lambda^{+2}B_0}{6\kappa_h}\ln^3\xi \qquad (19)$$

where w_0^+, κ_c are arbitrary constants; $\Delta T_s^+ = \Delta T^+ - \frac{1}{\kappa_h}\ln(\lambda^+\kappa_h Pr)$. Consequently, the general solution for the turbulent convection over a rough surface at $\xi^2 \gg 1$ is given by

$$w^+ = w_r^+(\xi) - w_r^+(\xi_r) \qquad (20)$$

$$w_r^+ = \frac{1}{\kappa_c}\ln\xi - \frac{\lambda^{+2}B_0\Delta T_r^+}{2}\ln^2\xi + \frac{\lambda^{+2}B_0}{6\kappa_h}\ln^3\xi$$

where $\Delta T_r^+ = \Delta T^+ - \frac{1}{\kappa_h}\ln(1+\sqrt{1+(\kappa_h r^+ Pr)^{-2}})$.

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The random amplitudes of the horizontal velocity are given by $u^+ = \phi\cos\beta, v^+ = \phi\sin\beta$ where ϕ one find from the equation: $\xi d\phi/d\xi = dw^+/d\xi$. Therefore $u^+ \approx u_0^+ + w^+\cos\beta/\xi$, $v^+ \approx v_0^+ + w^+\sin\beta/\xi$ where u_0^+, v_0^+ are velocity components values in a free atmosphere. Note, the expressions (19-20) are similar to the mean velocity profile in the turbulent natural convection along a vertical flat plate [6].

5.5 Ekman layer over large scale roughness

The main equation of the turbulent rotating flow over a large scale roughness can be written as follows:

$$\left((1+\xi^2)\frac{d^2}{d\xi^2} + 2\xi\frac{d}{d\xi}\right)\left((1+\xi^2)\frac{d^2}{d\xi^2} + \xi\frac{d}{d\xi}\right)\tilde{\psi} + \frac{f^2\tilde{\psi}}{1+\xi^2} = 0 \qquad (21)$$

where $\tilde{\psi} = \psi - \psi_H$, $\psi_H = U_0/u_* \sin(\beta - \gamma_e)$; $f = \lambda^{+2}\omega_0 \sin\varphi_e$. The classical result is followed from (21) for a smooth surface, when $\xi \ll 1$:

$$\frac{d^4\tilde{\psi}}{d\xi^4} + f^2\tilde{\psi} = 0$$

It's well known , that the classical solution for Ekman layer is $\tilde{\psi} = \tilde{\psi}_0 \exp(-\sqrt{f/2}(1+i)\xi)$. But $\xi \geq 1$ for a rough surface and the classical result is not valid. Let $\tilde{\psi} = \tilde{\psi}(s)$ in (21), where $s = \ln(\xi/f)$ and if $\xi^2 \gg 1$ the asymptotic equation for Ekman layer over a large scale roughness is given by:

$$\frac{d^4\tilde{\psi}}{ds^4} + \frac{d^3\tilde{\psi}}{ds^3} + e^{-2s}\tilde{\psi} = 0 \qquad (22)$$

When $s \gg 1$ one can find the velocity defect law for Ekman layer over a large scale roughness:

$$\tilde{\psi} = \tilde{\psi}_f(\xi(z)) - \tilde{\psi}_f(\xi(H_e))$$

where

$$\tilde{\psi}_f = \psi_1\frac{f}{\xi} + \psi_2\ln\frac{\xi}{f} + \psi_3\ln^2\frac{\xi}{f} + \psi_4$$

Here ψ_i are some constants that should be determined from the inner layer solution.

6 Conclusions

The similarity theory and the model of a turbulent flow over a natural large scale roughness in pressure and temperature gradients are developed. The buoyancy forces and the planetary rotation are taken into account in the model.

The general results such as the logarithmic mean velocity and temperature profiles are obtained for a turbulent flow over a rough surface. The temperature and velocity profiles for a turbulent natural convection over a large scale roughness are obtained. It was established that the classical solution for Ekman layer over a large scale roughness is not valid. The velocity defect law for Ekman layer over a rough surface has been obtained.

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