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Employment of fuzzy logic in the control of the inverted pendulum

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Abstract

This paper presents the systematic design of a fuzzy logic controller for a car-pendulum mechanical system, well known in the literature as the inverted pendulum problem. The system is non-linear and inherently unstable. The objective of the controller is to maintain the pendulum in its inherently unstable position (vertical and upward oriented) using only fuzzy logic techniques. The proposed method controls the pendulum on the assumption of a track of infinite length. The validity of the approach has experimentally been verified and the results show the viability of the method. This work is part of a study on non-conventional control techniques applied to dynamic systems.

1 Introduction

The car-pendulum is a fairly popular system for validation of newly developed control methods. It is non-linear, inherently unstable and shows the dynamics of coupled bodies. A number of techniques has been applied to control the system [1], [2], [3]. More recently control methods based on fuzzy logic concepts have also been proposed for the same purpose [4], [5], [6]. For example, the approach in [6] is based on a hybrid solution, i.e. it combines a fuzzy controller and a linear state-feedback controller. The fuzzy controller is used to take the pendulum from the resting position (downward and stable) to the surrounds of the desired position; while the state-feedback controller stabilizes the pendulum in the inverted position. Furthermore, the car-track used is of finite and limited length. In this paper a different solution is proposed with the following main characteristics:

- A systematic design method.
- Only fuzzy controllers are employed.
- The car-track has no restriction with respect to its length (initial assumption).

The remainder of the paper is organized as follows: Section 2 describes the problem; section 3 is devoted to the methodology of designing the controller; section 4 shows some experimental results and section 5 presents the conclusions. As usually, the paper terminates with a list of references.

2 Problem description

The system to be considered has: (1) a car that can move along a track of limited length; (2) a pendulum attached to the car, that has one degree of freedom and can swing on the plane defined by figure 1a; (3) a system that moves the car consisting of a dc motor, an amplifier and a belt-pulley transmission system. The mechanical system is also schematically shown in figure 1a. For the purpose of modeling the system it is assumed that the car-pendulum is a rigid body and that the dynamic response of the motor circuit is sufficiently fast, that is, without delay. The angular position of the pendulum and the car position on the track are measured by sensors. Figure 1b is a block diagram of the controlled system.

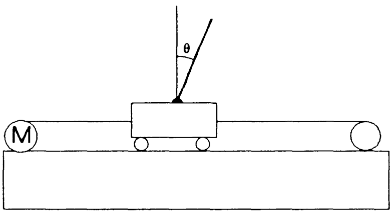

Figure 1a -The car-pendulum system

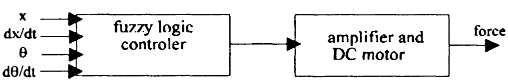

Figure 1b - Block diagram of the car-pendulum system

3 Controller design and application

The design of a fuzzy logic controller (FLC) requires the specification of a rule base and a knowledge base that express the membership functions of the fuzzy sets for the input and for the output variables. Both bases should be prepared according to the knowledge of the process to be controlled. The lack of procedures is a limiting factor in the design of fuzzy controllers in spite of the availability of knowledge of an expert.

Acquiring knowledge and its representation can be accomplished in two ways: (1) knowledge acquired from human expertise; and (2) the comprehension of underlying physical laws. The knowledge extracted should be organized in the form of rules as: "**if** conditions **then** action". The fuzzy control rules relate the state variables of the process as antecedents and the control variables as consequents. The formulation of fuzzy control rules can be obtained by means of heuristics. The most common way evokes the verbalization of the human knowledge. Another way of doing it consists of questioning the expert in such a way as to form a prototype of fuzzy control rules for a particular domain of application. The use of trial and error in these procedures is usually required.

The design of fuzzy controllers for systems of multiple inputs and one output consists in the determination of a set of fuzzy control rules, where each rule is of the form: "R_i : if x is A_i and y is B_i then z is C_i" , where x, y and z are linguistic variables representing two output variables of the process (input to the controller) and one input variable of the process (output to the controller). A_i, B_i and C_i are linguistic values of the linguistic variables x, y and z on the universe of discourse U, V and W respectively, with $i = 1, 2, ..., n$; and the connective **or** make the connection of rules inside the set of rules.

The design problem consists in the determination of the antecedents and the consequents of the control rules. For the determination of the antecedents of the rules it is required to have the information about the input to the rules and the fuzzy partition of the input space, that is, the determination of the number of fuzzy sets for each variable and the form of the membership functions. The consequent of the rule corresponds to the control input of the controlled process.

From this point on the design of the FLC will be divided into three steps, as follows:

(a) Determination of the linguistic variables;
(b) Determination of the number of sets for each variable;
(c) Determination of the membership functions for each fuzzy set;

a) Determination of linguistic variables.

The choice of the state variables of the process and the control variables of the fuzzy control rules is the essence of a FLC. Typically, the linguistic variables are the state, the state error, the derivative of the state error and the integral of the state error of the controlled process. The universe of the discourse of a variable should cover the whole interval in which the variable is observable. This assures that any measured value of the variable will be represented in a fuzzy set. In this work, the universe of discourse of the variables is continuous, however, it could as well be of discrete nature.

b) Determination of the number of fuzzy sets for each variable.

A linguistic variable in the antecedent of a fuzzy rule forms a space of fuzzy input with relation to a certain universe of discourse; while the consequent of a

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fuzzy rule forms a space of fuzzy output. For each linguistic variable a fuzzy set defined in the same universe of discourse is associated. One typical example is shown in figure 2 representing two partitions of the same universe of normalized discourse [-1, +1].

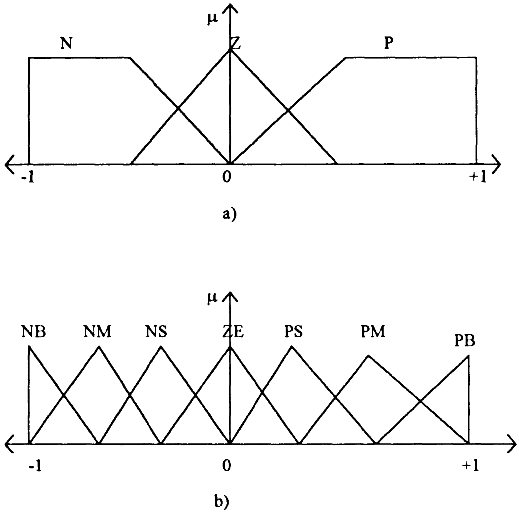

Figure 2 -Diagrammatic partitions of fuzzy sets.

c) Determination of the membership functions for each fuzzy set.

The resolution that a membership function assumes, defines the type of sensitivity that a certain fuzzy set has. Membership functions that have high resolution implies that the set of rules have large sensitivity to this fuzzy set, while the ones that have low resolution also have smaller sensitivity. The reasons why these facts happen are that when the membership function has a low resolution a large variation of the variable signal is necessary to induce small variations on the value of the membership degree. Figure 3a shows a fuzzy set A whose membership function presents high resolution and figure 3b shows the same fuzzy set A whose membership function presents low resolution.

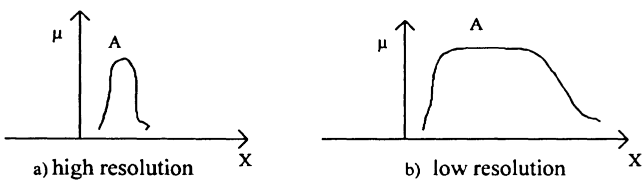

Figure 3 - membership functions with relation to resolution

Based on the previous exposure a methodology for the determination of the number of fuzzy sets and their respective membership functions will be proposed. The adopted method can be summarized as follows:

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Step 1
The minimum number of the fuzzy sets can be varied from (i = 1, 2, ..., n), where i is the number of fuzzy sets associated to each variable. Usually the number **n** never goes beyond 9 fuzzy sets, that should be distributed symmetrically around the origin. This number will depend on the application and should have a compromise between flexibility (many fuzzy sets) and simplicity (few fuzzy sets).

Step 2
Each fuzzy set should overlap the neighbors. This overlap should be between 10 and 50 per cent of the neighbor fuzzy set. Overlay between fuzzy sets plays a key role for the smooth operation of a fuzzy controller.

Step 3
The number of fuzzy sets should be greater around the desired point. For this point a recommendation could be membership functions with high resolution and of triangular shape. As the measure gets more distant membership functions with low resolution and of trapezoidal shape could be employed. Figure 4 shows one such example.

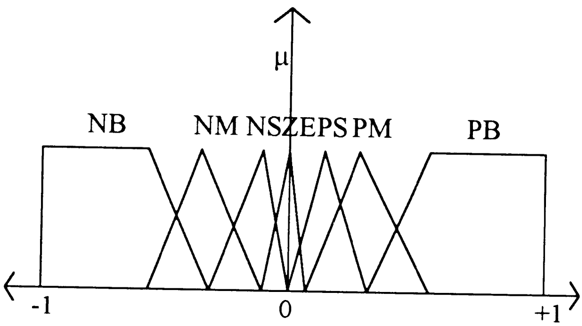

Figure 4 - Diagrammatic representation of seven fuzzy sets with their respective membership functions

Application to the case of the inverted pendulum
The rule of compositional inference Max-Min of Zadeh [9] was used for inference and the center of gravity method was employed to make the defuzzification [7].
The input variables used for the formation of the rule base have been the angular position and the angular speed. The output variable is the force (consequent of the rule) which is applied to the car. The universe of discourse of the variables has been defined as follows:

- Pendulum angle : -45° to +45°
- Angular speed: -50 to +50 °/s.
- Force: -1 to +1 (Normalized)

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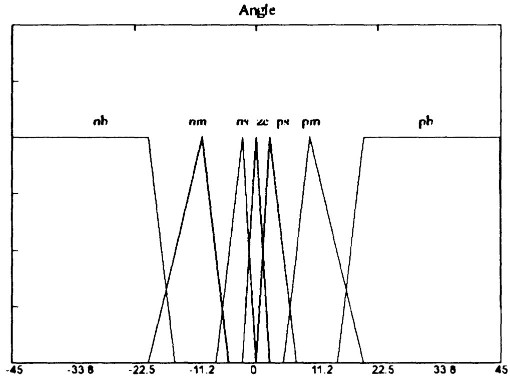
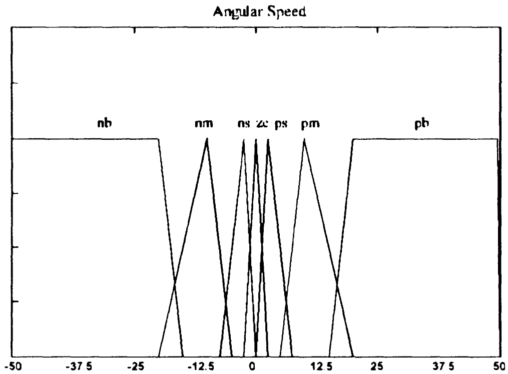
For the above variables the fuzzy sets have been defined with triangular and trapezoidal membership functions. Figure 5 shows the membership functions for this case. In table I the rule base is given for the control of the pendulum.

Antecedent Variables:





Consequent Variable:

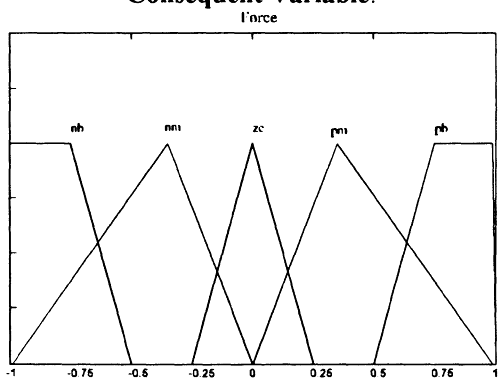


Figure 5 - Membership functions

$d\theta/dt$	θ							
	nb	**nm**	**ns**	**zn**	**zp**	**ps**	**pm**	**pb**
nb	nb	nb	nb	nb	pm	ze	pm	pb
nm	nb	nb	nb	nm	pm	pm	pm	pb
ns	nb	nb	nm	ze	ze	pm	pm	pb
ze	nb	nb	nm	ze	ze	pm	pb	pb
ps	nb	nm	nm	ze	ze	pm	pb	pb
pm	nb	nm	nm	nm	pm	pb	pb	pb
pb	nb	nm	ze	nm	pb	pb	pb	pb

table I

4 Experimental Results

The method has been implemented to control the Inverted Pendulum from Quanser Consulting Co.(Toronto - Canada) and by using the acquisition board PC Lab++ from National Instruments. The fuzzy logic algorithm has been run on a compatible IBM PC. The main system characteristics are given in table II.

parameters	values
mass of the car (M)	0.45 Kg
mass of the pendulum (m)	0.21 Kg
length of the pendulum (l)	0.61 m

Table II

The knowledge base (rules and input and output membership functions) is described in a text storage device and is read by an interpreter program developed in C [8]. The experimental results are shown in figure 6.

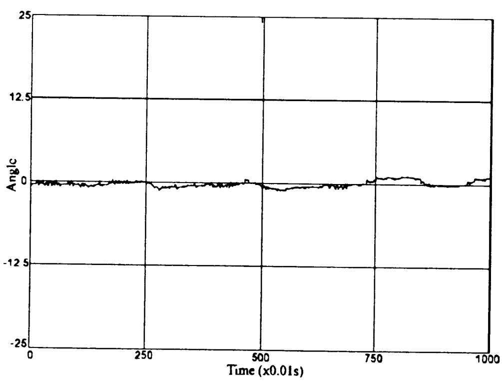


Figure 6a - Angle

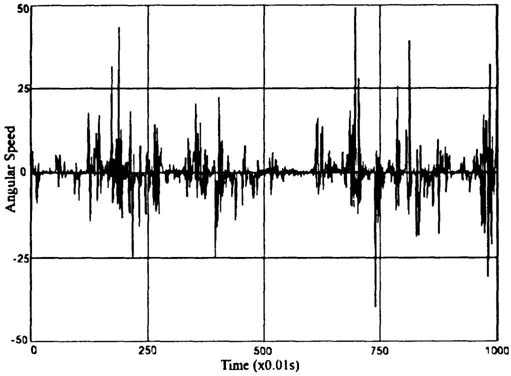


Figure 6b - Angular Speed

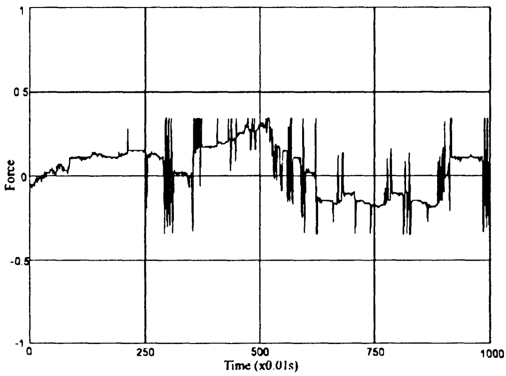


Figure 6c - Force

It is important to note that the relatively short car-track of the inverted pendulum employed for this experiment [-35 cm,+35 cm] imposes a rather severe demand on the design of the FLC.

In spite of it, the results in figure 6 demonstrate acceptable system performance provided that there are no changes in membership functions and/or parameter values. Qualitatively, these results correspond with those previously obtained by simulation [10].

5 Conclusion

Based on the initial assumption of unlimited car-track length, the control of the inverted pendulum has been studied.

Simulation results have shown that position control of the pendulum is robust and tuning is easily accomplished for a sufficiently long car-track.

When a more limited length of the car-track is employed, as in the case of the experimental set-up described in this paper, system performance could still be acceptable. However, the fuzzy controller offers no robustness and the system becomes unstable if small changes in parameter values and/or membership functions occur. The instability in turn causes tedious tuning of the controller. It can be concluded that the proposed design method is viable provided the car-track is not too short.

References

[1] Mori S., Nishihara H. & Furuta K., "Control of unstable mechanical system control of pendulum", *Int. J. Control*, vol. 23, n° 5 , pp. 673 - 692, 1976.

[2] Furuta K., Kajiwara H. & Kosuge K., "Digital control of a double inverted pendulum on a inclined rail", *Int. J. Control*, vol. 32, n° 5, pp. 907 - 924, 1980.

[3] Farwig H. M. Zu & Unbehauen H., "Discrete computer control of a triple inverted pendulum", *Optimal control applications & methods"*, vol. 11, pp. 157-171, 1990.

[4] Kosko B., *"Networks and Fuzzy Systems"*, Englewood Cliffs, N. J.,Prentice Hall, 1992.

[5] Murphy P., *"Fuzzy logic smoothes system control"*, I&CS (Control Software) pp. 45 - 49, March 1992.

[6] Lin C. E. & Sheu Y., "A hybrid-control approach for pendulum-car control", *IEEE Transactions on Ind. Electronics* vol. 39, no. 3- pp. 208 - 214, 1992.

[7] Lee C. C., "Fuzzy logic in control systems: fuzzy logic controller - part I and II" *IEEE Transactions on Syst. Man and Cyber*, pp. 404 - 435, 1991.

[8] Viot G., "Fuzzy logic in C", *Dr. Dobb's Journal*, February 1993.

[9] Zadeh L. A., "Fuzzy Sets", *Information and Control*, vol. 8, pp. 338-353, 1965.

[10] Krohling, R. A., *"Non Conventional Control Algorithms: Study of a Case"*, Master Thesis (in Portuguese), UFES, Vitoria, Brazil, 1994.